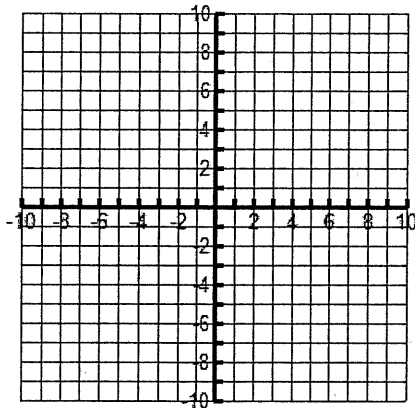


CCGPS Analytic Geometry – Parabola Notes Day 3

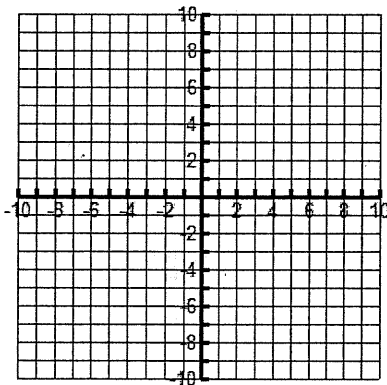
Equations for Parabolas: up/down parabolas: $(x - h)^2 = 4p(y - k)$ right/left parabolas: $(y - k)^2 = 4p(x - h)$

Example 1: Find the equation of a parabola given the vertex at (3, 3) and a focus at (3, 0). Graph the information given:



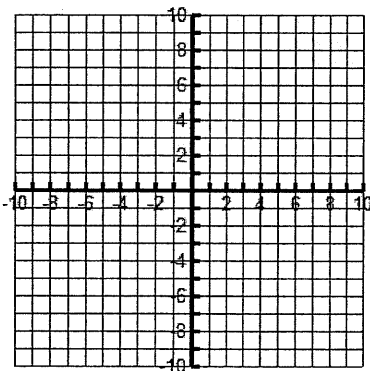
Since the parabola opens _____, the equation we need to use in standard form is:
 Since the vertex is given we know h and k and can substitute h and k into the equation:
 We can also find p .
 The equation for the parabola is:
 Graph the parabola and put in the directrix.

Example 2: Find the equation for a parabola given a focus at (3, 5) and a directrix at $x = -1$. Graph the parabola.



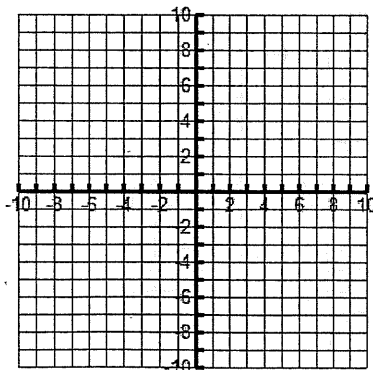
Since the parabola opens _____, the equation we need to use in standard form is:
 We can use the information given to find h and k which is the _____ of the parabola.
 Substitute h and k into the equation:
 We can also find p . The equation for the parabola is:
 Graph the parabola and put in the directrix.

Example 3: Find the equation and graph the parabola that has a vertex at (-4, -2) and the directrix $x = -2$.



Example 4: Find the equation of a parabola that opens left or right, has the vertex (4, 1) and contains the point (5, -1).

- 1) Write the standard form of a left/right opening parabola.
- 2) Substitute in the vertex (h, k).
- 3) Substitute in the given point for x and y . Now solve for p .
- 4) Go back to step #2 and write the standard form including the p value.



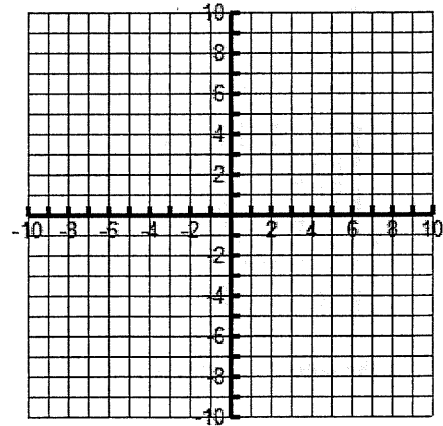
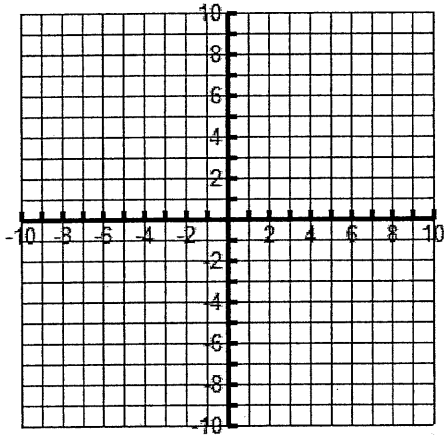
Equations for Parabolas: up/down parabolas: $(x - h)^2 = 4p(y - k)$

right/left parabolas: $(y - k)^2 = 4p(x - h)$

Practice: Find the equation of the parabola with the given characteristics:

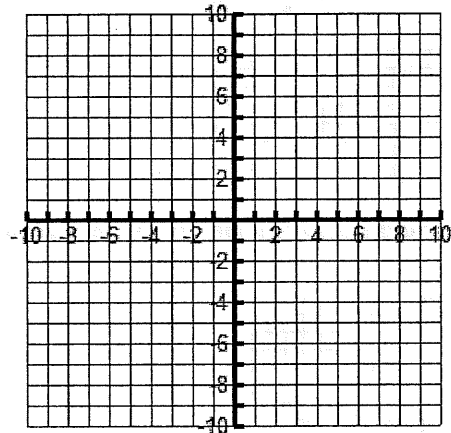
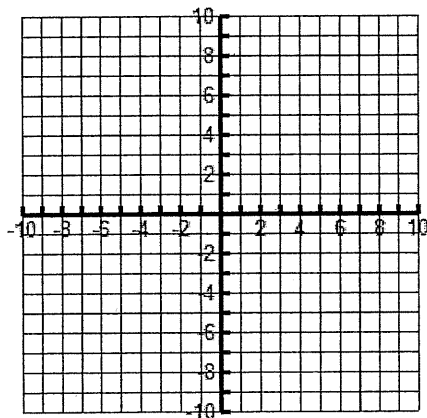
1. Vertex: (0, 6)
Focus: (-3, 6)

2. Directrix: $y = -4$
Focus: (0, 4)



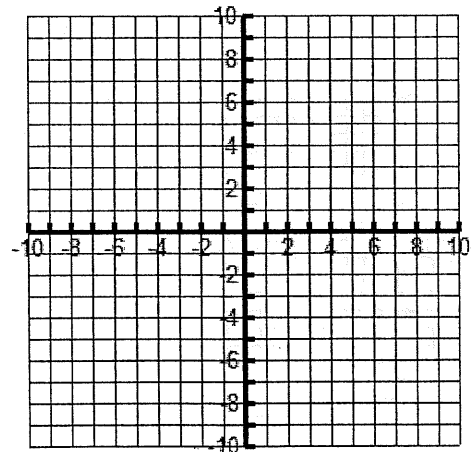
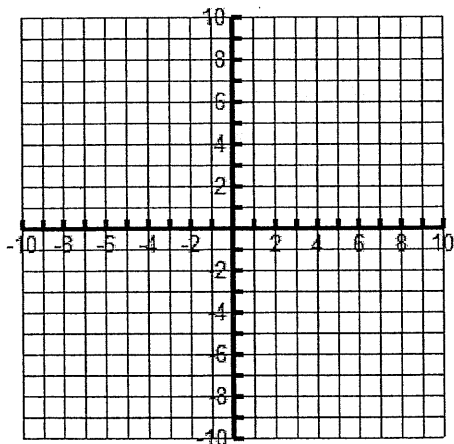
3. Opens left or right
Vertex: (7, 6)
Passes through: (-11, 9)

4. Vertex (8, -1)
y-intercept: -17



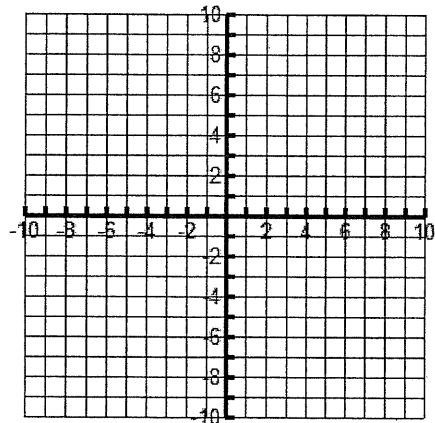
5. Focus: (3, 0)
Directrix: $x = -3$

6. Vertex: (5, -2)
Directrix: $y = -5$



Write the standard form of the equation of each parabola. Sketch the graph and show the items in the list.

1. The vertex is at $(-5, 1)$, and the focus is at $(-1, 1)$.



Equation: _____

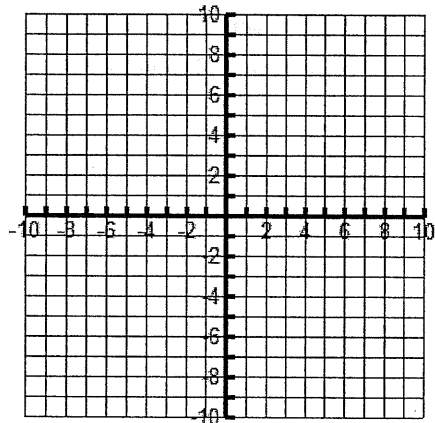
Axis of Symmetry: _____

Directrix: _____

Vertex: _____ Focus: _____

Focal width: _____ $p =$ _____

2. The equation of the axis of symmetry is $y = 2$, the focus is at $(0, 2)$, and $p = -3$.



Equation: _____

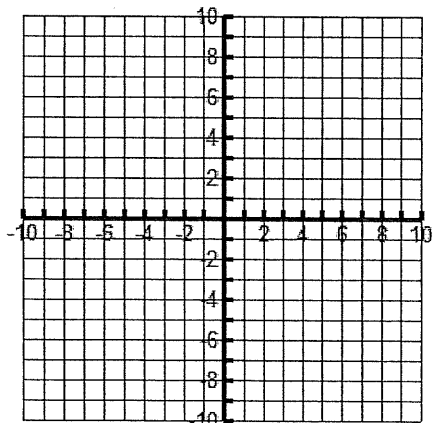
Axis of Symmetry: _____

Directrix: _____

Vertex: _____ Focus: _____

Focal width: _____ $p =$ _____

3. The parabola passes through the point at $(-3, 1)$, has its vertex at $(-2, -3)$, and opens to the left.



Equation: _____

Axis of Symmetry: _____

Directrix: _____

Vertex: _____ Focus: _____

Focal width: _____ $p =$ _____

4. The focus is at $(3, 2)$, the distance from the focus to the vertex is 2 units, and the function has a minimum.

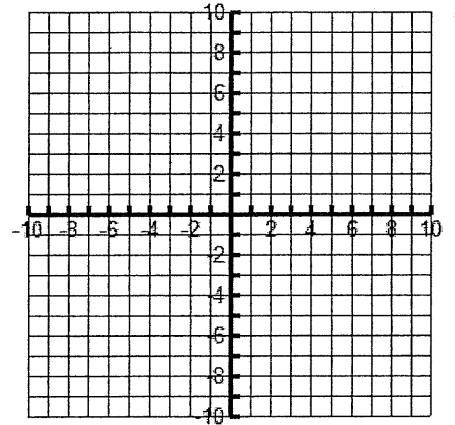
Equation: _____

Axis of Symmetry: _____

Directrix: _____

Vertex: _____ Focus: _____

Focal width: _____ $p =$ _____



5. The arch in Freedom Park has a parabolic shape. Its height is 25 feet and its base is 30 feet wide. Find an equation, which models this shape, using the x -axis to represent the ground. Hint: Use 1 square = 2 feet and put origin on lower left corner of graph.

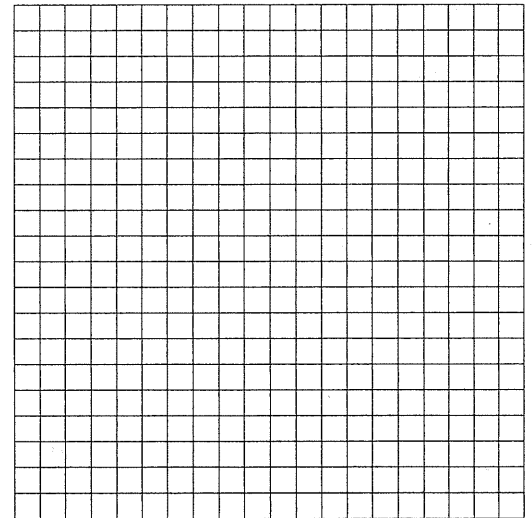
Equation: _____

Axis of Symmetry: _____

Directrix: _____

Vertex: _____ Focus: _____

Focal width: _____ $p =$ _____



Key

Example 1: Find the equation of a parabola given the vertex at (3, 3) and a focus at (3, 0). Graph the information given:

Since the parabola opens down, the equation we need to use in standard form is:

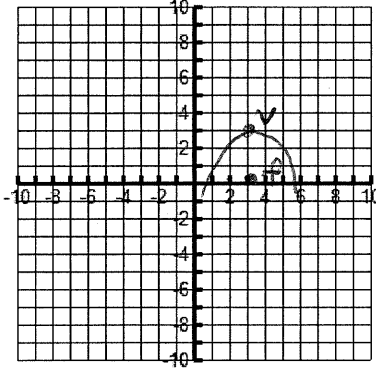
$$y = -x^2$$

Since the vertex is given we know h and k and can substitute h and k into the equation:

We can also find p . $p = -3$ $(x-h)^2 = 4p(y-k)$

The equation for the parabola is: $(x-h)^2 = -12(y-k)$

Graph the parabola and put in the directrix.



Example 2: Find the equation for a parabola given a focus at (3, 5) and a directrix at $x = -1$. Graph the parabola.

Since the parabola opens _____, the equation we need to use in standard form is:

$$p = 2 \quad (y-k)^2 = 4p(x-h)$$

We can use the information given to find h and k which is the (1, 5) of the parabola.

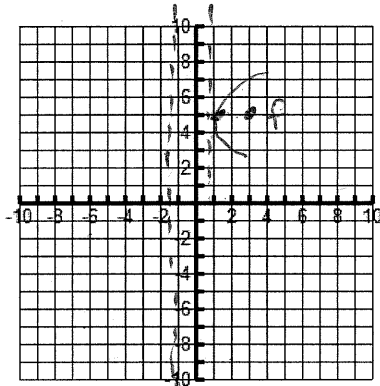
$$(y-5)^2 = 8(x-1)$$

Substitute h and k into the equation:

We can also find p .

The equation for the parabola is:

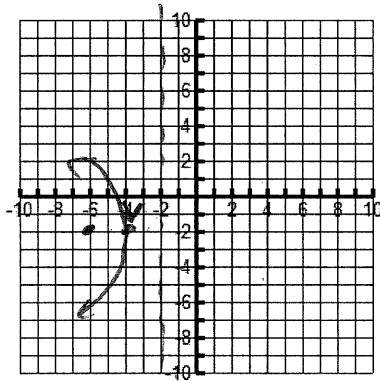
Graph the parabola and put in the directrix.



Example 3: Find the equation and graph the parabola that has a vertex at (-4, -2) and the directrix $x = -2$.

$$p = -2 \quad (y-k)^2 = 4p(x-h)$$

$$(y+2)^2 = -8(x+4)$$



Example 4: Find the equation of a parabola that opens left or right, has the vertex (4, 1) and contains the point (5, -1).

1) Write the standard form of a left/right opening parabola.

$$(y-k)^2 = 4p(x-h) \quad (-1-1)^2 = 4p(5-4)$$

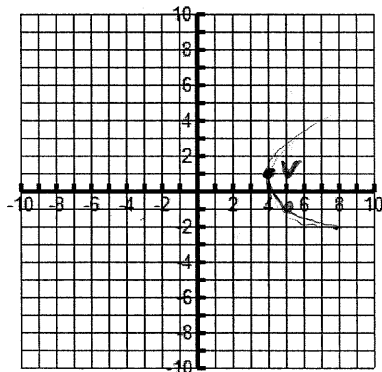
2) Substitute in the vertex (h, k).

$$4 = 4p \quad p = 1$$

3) Substitute in the given point for x and y . Now solve for p .

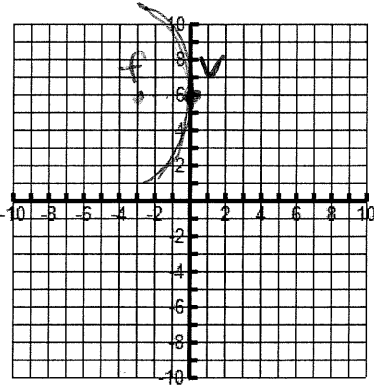
$$(y-1)^2 = 4(x-4)$$

4) Go back to step #2 and write the standard form including the p value.



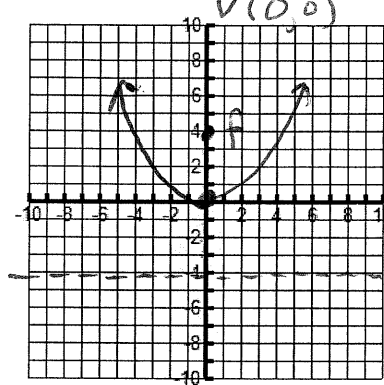
Practice: Find the equation of the parabola with the given characteristics:

1. Vertex: (0, 6)
Focus: (-3, 6)



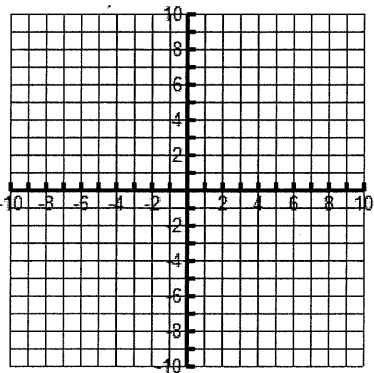
$p = -3$
 $(y-k)^2 = 4p(x-h)$
 $(y-6)^2 = -12(x-0)$

2. Directrix: $y = -4$
Focus: (0, 4)



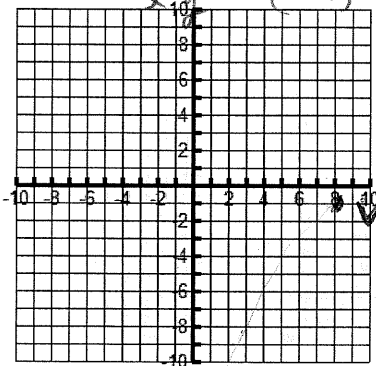
$(x-h)^2 = 4p(y-k)$
 $p = 4$
 $(x-0)^2 = 16(y-0)$

3. Opens left or right
Vertex: (7, 6)
Passes through: (-11, 9)



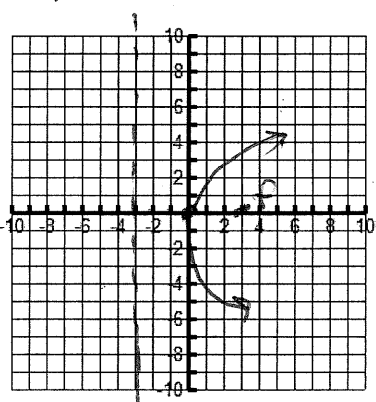
$(y-k)^2 = 4p(x-h)$

4. Vertex (8, -1)
y-intercept: -17



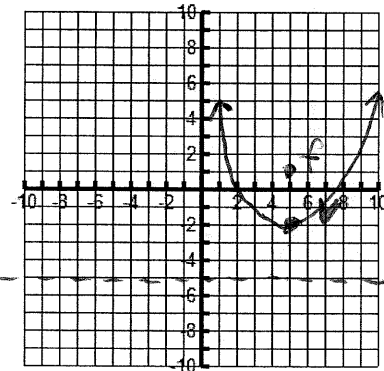
opens down
 $(x-h)^2 = 4p(y-k)$
 $(0-8)^2 = 4p(-17+1)$
 $64 = 4p(-16)$
 $-1 = p$

5. Focus: (3, 0)
Directrix: $x = -3$



vertex (0, 0)
 $(y-k)^2 = 4p(x-h)$
 $(y-0)^2 = 12(x-0)$
 $p = 3$

6. Vertex: (5, -2)
Directrix: $y = -5$



$(x-h)^2 = 4p(y-k)$
 $(x-5)^2 = 12(y+2)$
 $p = 3$

Write the standard form of the equation of each parabola. Sketch the graph and show the items in the list.

1. The vertex is at $(-5, 1)$, and the focus is at $(-1, 1)$. $p = 4$

Equation: $(y-1)^2 = 16(x+5)$

Axis of Symmetry: $y = 1$

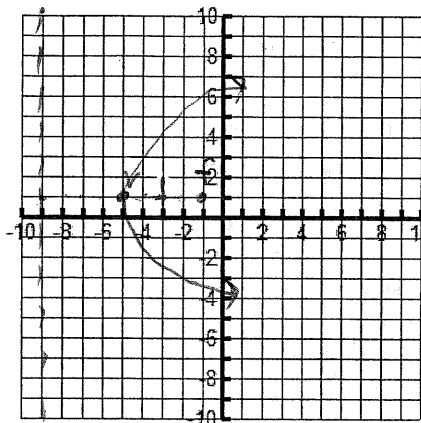
Directrix: $x = -9$

Vertex: $(-5, 1)$

Focus: $(-1, 1)$

Focal width: 16

$p = 4$



2. The equation of the axis of symmetry is $y = 2$, the focus is at $(0, 2)$, and $p = -3$.

Equation: $(y-2)^2 = 12(x-3)$

Axis of Symmetry: $y = 2$

Directrix: $x = 6$

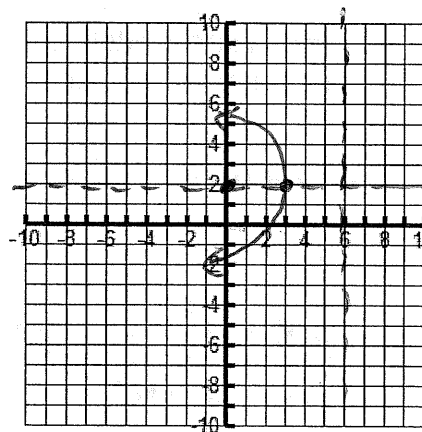
Vertex: $(3, 2)$

Focus: $(0, 2)$

Focal width: 12

$p = -3$

$4p = 12$



3. The parabola passes through the point at $(-3, 1)$, has its vertex at $(-2, -3)$, and opens to the left.

$$\begin{aligned} (y-k)^2 &= 4p(x-h) & 4^2 &= 4p(-1) \\ (1+3)^2 &= 4p(-3+2) & 16 &= -4p & p &= -4 \end{aligned}$$

Equation: $(y+3)^2 = -16(x+2)$

Axis of Symmetry: $y = -3$

Directrix: $x = 2$

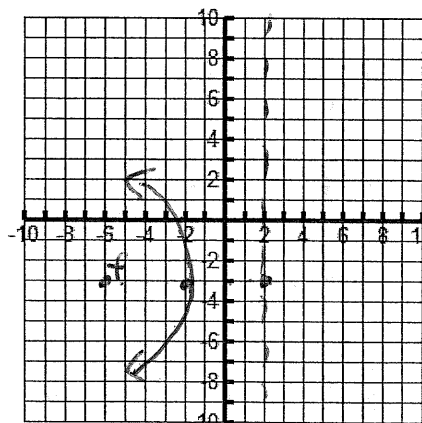
Vertex: $(-2, -3)$

Focus: $(-6, -3)$

Focal width: 16

$p = -4$

$4p$



4. The focus is at (3, 2), the distance from the focus to the vertex is 2 units, and the function has a minimum. *opens up*



Equation: $(x-3)^2 = 8(y-0)$

Axis of Symmetry: $x=3$

Directrix: $y=-2$

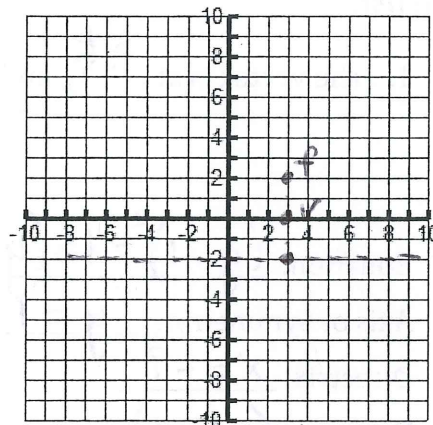
Vertex: $(3, 0)$

Focus: $(3, 2)$

Focal width: 8

$p = 2$

$4p = 8$



5. The arch in Freedom Park has a parabolic shape. Its height is 25 feet and its base is 30 feet wide. Find an equation, which models this shape, using the x-axis to represent the ground. Hint: Use 1 square = 2 feet and put origin on lower left corner of graph.

Equation: $(x-15)^2 = -\frac{15}{2}(y-30)$

Axis of Symmetry: $x=+15$

Directrix: $y = 25 + \frac{15}{8}$

Vertex: $(15, 30)$

Focus: $(15, 25 - \frac{15}{8})$

Focal width: _____

$p = -\frac{15}{8}$

point (x, y)

$(x-h)^2 = 4p(y-k)$

$(0-15)^2 = 4p(0-30)$

$-\frac{15}{2} = 4p$

$-\frac{15}{8} = p$

