

CCGPS Analytic Geometry Systems Day 2: Solving Quadratic-Linear Systems of Equations

Steps:

1. First, solve for one of the variables in the linear equation.
2. Next, we substitute for that variable in the quadratic equation, and solve the resulting equation.
3. There are now have two values for x , but we still need to find the matching values for y .
4. Find the y -values by substituting each value of x into the linear equation.
5. Now we have 2 possible solutions for the system: We need to check each solution in each equation.
6. We finally have our solution set for this linear quadratic system.

Solve the systems of equations.

Example 1: $x^2 + y^2 = 26$
 $y = x + 4$

1. $x^2 + y^2 = 25$
 $x + y = 5$

2. $x^2 + y^2 = 16$
 $x + y + 4 = 0$

3. $x^2 + y^2 = 29$
 $x = y - 3$

4. $x^2 + y^2 = 52$
 $y - x = 2$

5. $x^2 + y^2 = 45$
 $y = 2x$

6. $x^2 - 2x + y^2 - 2y = 6$
 $x + y = 2$

7. $x^2 + y^2 = 4$
 $y = x + 2$

8. $x^2 - 2x + y^2 - 2y = 6$
 $y + x = 2$

CCGPS Analytic Geometry Systems Day 2: Solving Quadratic-Linear Systems of Equations

Steps:

1. First, solve for one of the variables in the linear equation.
2. Next, we substitute for that variable in the quadratic equation, and solve the resulting equation.
3. There are now have two values for x, but we still need to find the matching values for y.
4. Find the y-values by substituting each value of x into the linear equation.
5. Now we have 2 possible solutions for the system: We need to check each solution in each equation.
6. We finally have our solution set for this linear quadratic system.

Solve the systems of equations.

Example 1: $x^2 + y^2 = 26$
 $y = x + 4$

$x^2 + (x+4)^2 = 26$

$x^2 + (x+4)(x+4) = 26$

$x^2 + x^2 + 8x + 16 = 26$

$2x^2 + 8x - 10 = 0$

1. $x^2 + y^2 = 25$
 $x + y = 5$

$x = 5 - y$

$(5-y)^2 + y^2 = 25$

$25 - 10y + y^2 + y^2 = 25$

$2y^2 - 10y = 0$

$2y(y-5) = 0$

$y = 0, 5$

$2(x^2 + 4x - 5) = 0$

$\begin{array}{r} 5 \quad -5 \\ \times \quad -1 \\ \hline 1 \quad 4 \end{array}$

$2(x+5)(x-1) = 0$

$x = -5, x = 1$

$y = x + 4$
plug $x = -5$

$y = -5 + 4 = -1$

$(-5, -1)$

$y = x + 4$
plug $x = 1$

$y = 1 + 4 = 5$

$(1, 5)$

2. $x^2 + y^2 = 16$
 $x + y + 4 = 0$

$y = 4 - x$

$(x^2) + (4-x)^2 = 16$

$x^2 + 16 + 8x + x^2 = 16$

$2x^2 + 8x = 0$

$2x(x+4) = 0$

$x = 0, -4$

$x + y + 4 = 0$
plug $x = 0$

$0 + y + 4 = 0$

$y = -4$

$(0, -4)$

$x + y + 4 = 0$
plug $x = -4$

$-4 + y + 4 = 0$

$y = 0$

$(-4, 0)$

3. $x^2 + y^2 = 29$
 $x = y - 3$

$(y-3)^2 + y^2 = 29$

$y^2 - 6y + 9 + y^2 = 29$

$2y^2 - 6y - 20 = 0$

$2(y^2 - 3y - 10) = 0$

$\begin{array}{r} -5 \quad -10 \\ \times \quad 2 \\ \hline 1 \quad -3 \end{array}$

$2(y-5)(y+2) = 0$
 $y = 5, -2$

$x = y - 3$
plug $y = 5$
 $x = 5 - 3 = 2$

$(2, 5)$

$x = y - 3$
plug $y = -2$
 $x = -2 - 3 = -5$

$(-5, -2)$

4. $x^2 + y^2 = 52$
 $y - x = 2$

$y = x + 2$

$x^2 + (x+2)^2 = 52$

$x^2 + x^2 + 4x + 4 - 52 = 0$

$2x^2 + 4x - 48 = 0$

$2(x^2 + 2x - 24) = 0$

$\begin{array}{r} +6 \quad -24 \\ \times \quad -4 \\ \hline 1 \quad 2 \end{array}$

$2(x+6)(x-4) = 0$
 $x = -6, 4$

$y - x = 2$
plug $x = 4$

$y - 4 = 2$

$y = 6$

$(4, 6)$

$y - x = 2$
plug $x = -6$

$y + 6 = 2$

$y = -4$

$(-6, -4)$

$$5. \quad x^2 + y^2 = 45$$

$$y = 2x$$

$$x^2 + (2x)^2 = 45$$

$$x^2 + 4x^2 - 45 = 0$$

$$5x^2 - 45 = 0$$

$$5(x^2 - 9) = 0$$

$$x = 3, -3$$

$y = 2x$ plug $x = 3$ $y = 6$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(3, 6)</div>	$y = 2x$ plug $x = -3$ $y = -6$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(-3, -6)</div>
--	--

$$6. \quad x^2 - 2x + y^2 - 2y = 6$$

$$x + y = 2 \quad y = 2 - x$$

$$x^2 - 2x + (2-x)^2 - 2(2-x) = 6$$

$$x^2 - 2x + 4 - 4x + x^2 - 4 + 2x - 6 = 0$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x-3)(x+1) = 0$$

$$\begin{array}{r} -3 \quad 1 \\ \times \\ 1 \quad -2 \quad 1 \end{array}$$

$$x = 3, -1$$

$x + y = 2$ plug $x = 3$ $3 + y = 2$ $y = -1$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(3, -1)</div>	$x + y = 2$ plug $x = -1$ $-1 + y = 2$ $y = 3$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(-1, 3)</div>
--	---

$$7. \quad x^2 + y^2 = 4$$

$$y = x + 2$$

$$x^2 + (x+2)^2 = 4$$

$$x^2 + (x^2 + 4x + 4) = 4$$

$$2x^2 + 4x + 4 - 4 = 0$$

$$2x^2 + 4x = 0$$

$$2x(x+2) = 0$$

$$x = 0, x = -2$$

$y = x + 2$ plug $x = 0$ $y = 0 + 2 = 2$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(0, 2)</div>	$y = x + 2$ plug $x = -2$ $y = -2 + 2 = 0$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(-2, 0)</div>
---	--

$$8. \quad x^2 - 2x + y^2 - 2y = 6$$

$$y + x = 2$$

$$y = 2 - x$$

$$x^2 - 2x + (2-x)^2 - 2(2-x) = 6$$

$$x^2 - 2x + 4 - 4x + x^2 - 4 + 2x - 6 = 0$$

$$2x^2 - 4x - 6 = 0 \quad 2(x^2 - 2x - 3) = 0$$

$$\begin{array}{r} -3 \quad 1 \\ \times \\ 1 \quad -2 \quad 1 \end{array}$$

$$2(x-3)(x+1) = 0$$

$$x = 3, x = -1$$

$y + x = 2$ plug $x = 3$ $y + 3 = 2$ $y = -1$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(3, -1)</div>	$y + x = 2$ plug $x = -1$ $y - 1 = 2$ $y = 3$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">(-1, 3)</div>
--	--