Contest # 6 — *Answers & Solutions* — 3/17/20

Problem 6-1

Since 99999/11 leaves a remainder of 9, the largest 5-digit integer divisible by 11 is $99999 - 9 = \boxed{99990}$.

Problem 6-2

What is the largest perfect square less than 2020? Since $\sqrt{2020} \approx 44.9$, the largest perfect square less than 2020 is $44^2 = 1936$. Therefore, $2020 - n = 1936$, and $n = \boxed{84}$.

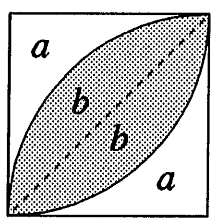
Problem 6-3

Of the first 8 tosses, exactly 1 will be "heads"; and the ninth toss must also be "heads." Only 8 permutations of the first 8 tosses contain 1 "heads" (7 "tails" and 1 "heads" can differ only in which toss is "heads"). The probability is $8 \times (\frac{1}{2})^1(\frac{1}{2})^7(\frac{1}{2}) = \frac{1}{2^6} = \boxed{\frac{1}{64}}$.

Problem 6-4

Method I:

From the area of 2 quarter-circles (each with radius 2 and area $a+2b$), subtract the area of the square, $2a+2b$. The result is $2b$, the shaded region's area. Its area is $\pi+\pi-4 = \boxed{2\pi-4}$.



Method II:

A quarter-circle $(a+2b)$ minus a right triangle $(a+b)$ is $b = \pi-2$. Double that to get $2b = 2\pi-4$, the area of the shaded region.

Problem 6-5

If $Y = 200y$ & $X = 200x$, then $Y = 200y = 200x^2 = 200\left(\frac{X}{200}\right)^2 = \frac{1}{200}X^2$, so $a = \boxed{\frac{1}{200}}$.

Problem 6-6

If $x = \log_4 a = \log_{10} b = \log_{25}(a+b)$, we can conclude that $4^x = a$, $10^x = b$ and $25^x = a+b$. It follows that $4^x+10^x = 25^x = a+b$, and $\frac{a}{b} = \frac{4^x}{10^x} = \left(\frac{2}{5}\right)^x$. Also, $\frac{b}{a+b} = \frac{10^x}{25^x} = \left(\frac{2}{5}\right)^x$. Therefore, $\frac{a}{b} = \frac{b}{a+b}$, or $\frac{b}{a} = \frac{a}{b}+1$. Now, if we let $c = \frac{a}{b}$, then we get that $\frac{1}{c} = c+1$. The positive value of $c = \frac{a}{b}$ is the positive solution of $c^2+c-1 = 0$. That value of $c = \frac{a}{b}$ is $\boxed{\frac{-1+\sqrt{5}}{2}}$.

[**NOTE:** The value of x is $\log_{2/5}(c)$, which, to seven decimal places, is 0.5251737 . . . .]

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