Problem 3-1

No Pythagorean triple has hypotenuse 18 or 19. The 12-16-20 (or 16-12-20) triple has the property we seek. That day can be written as $\boxed{\text{Dec. 16, 2020}}$ or as either 12-16-20 or 16-12-20.

Problem 3-2

A fraction like $a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d}}}$ is called a *continued fraction*. Use a "split and flip" approach. Split off the integer part to get $46/35 = 1 + 11/35$. Now flip the fraction to get $46/35 = 1 + 1/(35/11)$. Repeat the procedure with $35/11 = 3 + 2/11 = 3 + 1/(11/2)$. Finally, since $11/2 = 5 + 1/2$, the complete expansion is $46/35 = 1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{2}}}$, from which $d = \boxed{2}$.

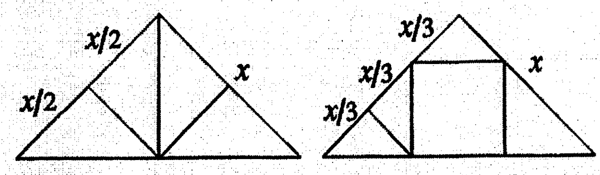
Problem 3-3

The first 5 palindromes greater than 9 999 are 10 001, 10 101, 10 201, 10 301, and $\boxed{10\,401}$.

Problem 3-4

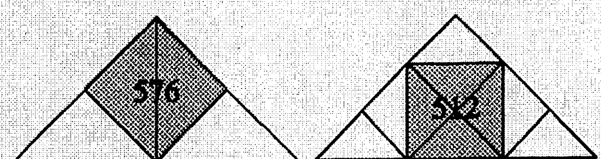
Let $y = \sqrt[3]{x}$, from which $x = y^3$. Substituting into the given equation, $(y^3)^y = (y^3)(y)$, or $y^{3y} = y^4$. Either both bases are equal to 1 or the exponents are equal. If $y = 1$, then $x = 1$. If not, then $3y = 4$, and $y = 4/3$, from which $x = 64/27$. Both check, so the two possible values of x are $\boxed{\frac{64}{27}, 1}$.

Problem 3-5



Call the original isosceles right triangle T. In the diagrams, let the length of each leg of T be x. In the left hand diagram, the vertical diagonal of the square partitions T into 4 congruent isosceles right triangles. Since each leg of T is x, the area of the square is $x^2/4$. In the other diagram, draw the segment shown to create 3 congruent isosceles right triangles, each with hypotenuse-length $x\sqrt{2}/3$. That's the length of a side of the square with area $2x^2/9$. Finally, since $x^2/4 = 576$, $x = 48$ and $2x^2/9 = \boxed{512}$ is the area of the smaller square. (The ratio of the squares' areas is 9 to 8.)

A solution without words is shown below.



$576 = (1/2)\triangle$ $\qquad$ $512 = (4/9)\triangle$

Problem 3-6

The sum of the polynomial's coefficients is $P(1) = 8$, so 8 is an upper bound for the value of any coefficient. In particular, the coefficients are single-digit integers. When written in base 10, $P(10)$ is the sequence of the polynomial's coefficients. For example, If $P(x) = 3x^2 + 4x + 2$, then $P(1) = 9$ and $P(10) = 342$. Here, $P(10) = 2312$, so $P(x) = 2x^3+3x^2+x+2$. Hence, $P(2) = 2\times2^3+3\times2^2+2+2 = 16+12+2+2 = \boxed{32}$.