Contest # 6 *Answers & Solutions* 3/16/21

Problem 6-1

The fraction will achieve its greatest value when its denominator is minimized. The denominator is minimized when $x = 0$, so the greatest possible value of the fraction is $\frac{4042}{2} = \boxed{2021}$.

Problem 6-2

Extending the common side of the 15-gons creates 2 congruent exterior angles. Since the degree-measure of each exterior angle of a regular 15-gon is $\frac{360}{15} = 24$, the angle x has degree-measure $\boxed{48, \text{ or } 48°}$.

Problem 6-3

Since we're told that $\log_{10}(2000!) = 5735.52\ldots$, it follows that $10^{5735} < 2000! < 10^{5736}$. Integers between 10^n and 10^{n+1} have $n+1$ digits, so the number of digits in the expansion of 2000! is $\boxed{5736}$.

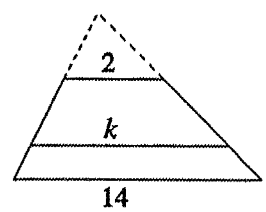
Problem 6-4

If we add 2 to any value of x that satisfies the first equation, we'll have a value of x that satisfies the second equation. Since the solutions of the first equation are 1, 2, 3, the solutions of the second equation are $1+2$, $2+2$, $3+2$, or more simply $\boxed{3, 4, 5}$.

Problem 6-5

Let x represent the number kilometers per liter that the first car gets, and let y represent the capacity of the first car's gas tank, in liters. We are asked to determine the value of xy, in kilometers. We are told that $(x+6)(y-3) = xy$, so $x = 2y-6$. We are also told that $(x-6)(y+6) = xy$, so $x = y+6$. Solving, $y = 12$, $x = 18$, and $xy = \boxed{216}$.

Problem 6-6



Extend the legs of the trapezoid until they meet as shown in the diagram. The 3 triangles in the diagram are all similar, so the ratio of their areas = the square of the ratio of corresponding bases. Therefore, $(2/14)^2 = (1/7)^2 = 1/49 =$ (area of smallest $\triangle$)/(area of biggest $\triangle$). If x is the area of the smallest $\triangle$, then the biggest triangle's area is $49x$. As a consequence, the area of trapezoid T is $49x-x = 48x$ and each of the two smaller trapezoids has area $48x/2 = 24x$. Since the square of the ratio of corresponding side-lengths of similar triangles equals the ratio of the areas of the triangles, it follows that $(2/k)^2 = (x)/(24x+x) = 1/25$, so $k = \boxed{10}$.