18) Evaluate $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{99}{100!}$

a) $\frac{1}{2}-\frac{1}{100!}$ b) $1-\frac{1}{100!}$ c) $1-\frac{1}{99!}$ d) $\frac{1}{2}-\frac{1}{99!}$ e) none of these

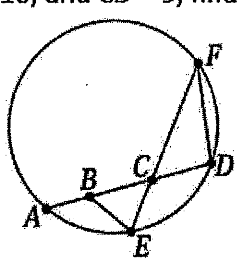
19) Suppose that 243 in base b is equal to 153 in base $b+2$, where $b>0$. What is b?

a) 5 b) 6 c) 7 d) 8 e) 9

26) Find the sum of all solutions to $|x-2016|+|x-2017|=3$

a) 3 b) 0 c) 2015 d) 3164 e) 4033

27) Circle O has three chords, AD, DF, and EF. Point E likes along the arc AD. Point C is the intersection of chords AD and EF. Point B lies on segment AC such that $EB = EC = 8$. Given $AB = 6$, $BC = 10$, and $CD = 9$, find DF.



a) $9\sqrt{3}$ b) $\frac{12+\sqrt{6}}{2}$ c) $\frac{9\sqrt{10}}{2}$ d) $8\sqrt{6}$ e) none of these

30) The area of the triangle ABC whose sides have measures a=17.6, b=11.1, c=13.1 is approximately

a) 15.88 b) 72.61 c) 81.53 d) 91.24 e) it cannot be determined

Additional Practice Problems (From Kennesaw 2012-13 part 1 test)

3. Let $a = 2^{1000}$, $b = 3^{600}$, $c = 10^{300}$. If a, b, and c are arranged from smallest to largest, then which of the following is correct?

(A) $a < b < c$ (B) $b < c < a$ (C) $c < a < b$ (D) $a < c < b$ (E) $b < a < c$

8. The ordered pair of real numbers (a, b) satisfies the following system of equations:

$$2a - b = 2$$
$$\log 2a - \log b = 2.$$

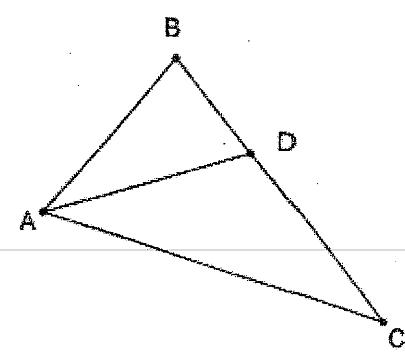
Compute $a + b$.

(A) $\frac{99}{102}$ (B) $\frac{99}{101}$ (C) $\frac{101}{99}$ (D) $\frac{102}{99}$ (E) $\frac{102}{101}$

14. Let a, b, and c be the roots of the polynomial equation $x^3 - 17x - 19 = 0$. Compute the value of $a^3 + b^3 + c^3$.

(A) 0 (B) 3 (C) 17 (D) 51 (E) 57

16. In triangle ABC, the bisector of angle A meets side $\overline{BC}$ at point D such that $\overline{AD}$ and $\overline{DC}$ are the same length. If the lengths of $\overline{AB}$ and $\overline{BC}$ are 12 and 16 inches, respectively, compute the cosine of angle ACB.



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$

18. The first three terms of a geometric sequence are the values x, y, and z, in that order. The first three terms of an arithmetic sequence are the values y, x, and z, in that order. If x, y, and z are distinct real numbers, compute the ratio of the fifth term of the geometric sequence to the fifth term of the arithmetic sequence.

(A) $\frac{1}{2}$ (B) $\frac{4}{7}$ (C) $\frac{7}{6}$ (D) $\frac{8}{5}$ (E) $\frac{19}{13}$

19. If $x > 0$ and $y > 0$ and (x, y) is a solution of the system of equations $\frac{1}{x^2}+\frac{1}{xy}=\frac{1}{9}$ and $\frac{1}{y^2}+\frac{1}{xy}=\frac{1}{16}$, compute $x+y$.

(A) $\frac{12}{5}$ (B) $\frac{16}{9}$ (C) $\frac{75}{8}$ (D) $\frac{108}{25}$ (E) $\frac{125}{12}$

20. In a right triangle, the square of the hypotenuse is four times the product of the legs. If α is the smallest angle of the triangle, compute $\tan\alpha$.

(A) $\sqrt{3}-\sqrt{2}$ (B) $\frac{\sqrt{2}}{3}$ (C) $2-\sqrt{3}$ (D) $2+\sqrt{3}$ (E) $\frac{\sqrt{3}}{2}$

21. When the number x^2 is written in base y, the value is 341. When the number y^2 is written in base x, the value is 44. Compute $x + y$.

(A) 17 (B) 19 (C) 21 (D) 23 (E) 25

23. If $|x| + x + y = 12$ and $x + |y| - y = 14$, compute the value of $\frac{x}{y}$.

(A) -2.375 (B) -1.625 (C) -1.250 (D) 1.875 (E) 2.125

25. A line drawn from the origin to the center of a circle has a slope of 2, and a tangent line to the circle drawn from the origin has a slope of 3. Compute the slope of the other tangent line drawn from the origin.

(A) $\frac{13}{9}$ (B) $\frac{11}{6}$ (C) $\frac{9}{5}$ (D) $\frac{5}{3}$ (E) $\frac{3}{2}$

Key

Luella Tournament Prep **September 7 2017** From 2016 Varsity Test

18) Evaluate $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{99}{100!}$ $+\frac{1}{100!} = 1$ so $\frac{1}{2!}+\frac{2}{3!}+\ldots+\frac{99}{100!} = \boxed{1-\frac{1}{100!}}$

a) $\frac{1}{2}-\frac{1}{100!}$ b) $1-\frac{1}{100!}$ c) $1-\frac{1}{99!}$ d) $\frac{1}{2}-\frac{1}{99!}$ e) none of these

19) Suppose that 243 in base b is equal to 153 in base $b+2$, where $b>0$. What is b?

a) 5 b) 6 c) 7 d) 8 e) 9

$243_b = 153_{(b+2)}$

$2b^2+4b+3 = 1(b+2)^2+5(b+2)+3$

$2b^2+4b = b^2+4b+4+5b+10$

$b^2-5b-14=0$

$(b-7)(b+2)=0$

$\boxed{b=7}$

26) Find the sum of all solutions to $|x-2016|+|x-2017|=3$

a) 3 b) 0 c) 2015 d) 3164 $\boxed{\text{e) } 4033}$

$x=2015$

$|2015-2016|+|2015-2017|=3$

$1 + 2 = 3$ ✓

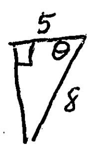
$x=2018$

$|2018-2016|+|2018-2017|=3$

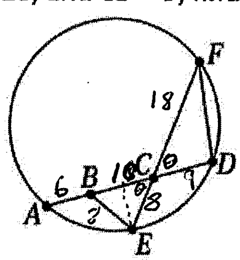
$2+1=3$ ✓

$2015+2018=\boxed{4033}$

27) Circle O has three chords, AD, DF, and EF. Point E likes along the arc AD. Point C is the intersection of chords AD and EF. Point B lies on segment AC such that $EB = EC = 8$. Given $AB = 6$, $BC = 10$, and $CD = 9$, find DF.



$\cos\theta = \frac{5}{8}$



$16 \times 9 = 8 \times CF$

$18 = CF$

$DF^2 = 9^2+18^2-2(9)(18)\cos\theta$

$DF^2 = 81+324-324\cos\theta$

$DF^2 = 405-324\cos\theta$

$DF^2 = 405-324\left(\frac{5}{8}\right)$

$DF^2 = \frac{3240-1620}{8} = \frac{405}{2}$

$DF = \sqrt{\frac{405}{2}} = \sqrt{\frac{81\cdot 5}{2}} = \frac{9\sqrt{5}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \boxed{\frac{9\sqrt{10}}{2}}$

a) $9\sqrt{3}$ b) $\frac{12+\sqrt{6}}{2}$ $\boxed{\text{c) } \frac{9\sqrt{10}}{2}}$ d) $8\sqrt{6}$ e) none of these

30) The area of the triangle ABC whose sides have measures a=17.6, b=11.1, c=13.1 is approximately

Heron's Formula

s = semi-perimeter of triangle

$A = \sqrt{s(s-a)(s-b)(s-c)}$

a) 15.88 $\boxed{\text{b) } 72.61}$ c) 81.53 d) 91.24 e) it cannot be determined

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = 17.6+11.1+13.1 = 41.8$

$= \sqrt{41.8(41.8-17.6)(41.8-11.1)(41.8-13.1)} \approx 70$

Additional Practice Problems (From Kennesaw 2012-13 part 1 test)

3. Let $a = 2^{1000/5}$, $b = 3^{600/3}$, $c = 10^{300/3}$. If a, b, and c are arranged from smallest to largest, then which of the following is correct?

(A) $a < b < c$ $\boxed{\text{(B)}}$ $b < c < a$ (C) $c < a < b$ (D) $a < c < b$ (E) $b < a < c$

$(2^{10})^{100}$ $(3^{6})^{100}$ $(10^{3})^{100}$ $b < c < a$

1024 729 1000

8. The ordered pair of real numbers (a, b) satisfies the following system of equations:

$$2a - b = 2$$
$$\log 2a - \log b = 2.$$

$\log_{10}\left(\frac{2a}{b}\right) = 2$ $10^2 = \frac{2a}{b}$ $100 = \frac{2a}{b}$

Compute $a + b$.

(A) $\frac{99}{102}$ (B) $\frac{99}{101}$ (C) $\frac{101}{99}$ $\boxed{\text{(D) } \frac{102}{99}}$ (E) $\frac{102}{101}$

$100b = 2a$

$50b = a$

$2(50b) - b = 2$

$100b - b = 2$

$99b = 2$

$b = \frac{2}{99}$

$a = \frac{100}{99}$

$\boxed{a + b = \frac{102}{99}}$

14. Let a, b, and c be the roots of the polynomial equation $x^3 - 17x - 19 = 0$. Compute the value of $a^3 + b^3 + c^3$.

$1x^3 + 0x^2 - 17x - 19 = 0$

(A) 0 (B) 3 (C) 17 (D) 51 $\boxed{\text{(E) } 57}$

$a^3 - 17a - 19 = 0$

$a^3 = 17a + 19$

$b^3 = 17b + 19$

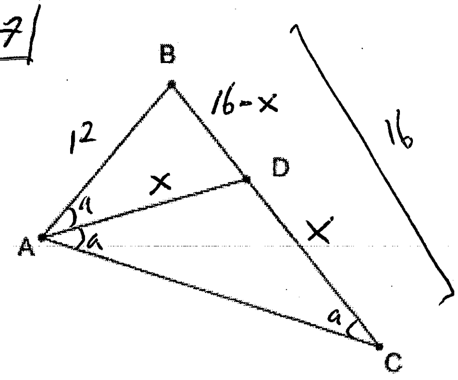
$+\ c^3 = 17c + 19$

$a + b + c = \frac{-0}{1} = 0$

$a^3 + b^3 + c^3 = 17(a + b + c) + 57$

$= 17(0) + 57 = \boxed{57}$

16. In triangle ABC, the bisector of angle A meets side $\overline{BC}$ at point D such that $\overline{AD}$ and $\overline{DC}$ are the same length. If the lengths of $\overline{AB}$ and $\overline{BC}$ are 12 and 16 inches, respectively, compute the cosine of angle ACB.



(A) $\frac{1}{2}$ $\boxed{\text{(B) } \frac{2}{3}}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$

$\frac{\sin 2a}{16} = \frac{\sin a}{12}$

$\frac{2\sin a\cos a}{16} = \frac{\sin a}{12}$

$24 \sin a \cos a = 16 \sin a$

$24 \cos a = 16$

$\cos a = \frac{16}{24} = \boxed{\frac{2}{3}}$

18. The first three terms of a geometric sequence are the values x, y, and z, in that order. The first three terms of an arithmetic sequence are the values y, x, and z, in that order. If x, y, and z are distinct real numbers, compute the ratio of the fifth term of the geometric sequence to the fifth term of the arithmetic sequence.

(A) $\frac{1}{2}$ (B) $\frac{4}{7}$ (C) $\frac{7}{6}$ $\boxed{\text{(D) } \frac{8}{5}}$ (E) $\frac{19}{13}$

Geometric sequence

$x,\ y,\ z$

$\frac{y}{x} = \frac{z}{y} \rightarrow z = \frac{y^2}{x}$

Arithmetic sequence

$\underbrace{y,\ x}_{x-y},\ \underbrace{x,\ z}_{x-y} \rightarrow z = x + (x-y)$

$z = 2x - y$

$\frac{y^2}{x} = 2x - y$

$y^2 = 2x^2 - xy$

$0 = 2x^2 - xy - y^2$

$0 = (2x+y)(x-y)$

$y = -2x,\ \cancel{y = x}$ eliminate, since x and y are distinct

Geometric sequence:

$x, y, z \rightarrow x, -2x, \frac{(-2x)^2}{x}$

$\rightarrow x, -2x, +4x, \underline{-8x}, \underline{16x}$

Arithmetic sequence

$y, x, z \rightarrow -2x, x, 4x, \underline{7x}, \underline{10x}$

Ratio $= \frac{16x}{10x} = \boxed{\frac{8}{5}}$

19. If $x > 0$ and $y > 0$ and (x, y) is a solution of the system of equations $\frac{1}{x^2} + \frac{1}{xy} = \frac{1}{9}$ and $\frac{1}{y^2} + \frac{1}{xy} = \frac{1}{16}$, compute $x + y$.

(A) $\frac{12}{5}$ (B) $\frac{16}{9}$ (C) $\frac{75}{8}$ (D) $\frac{108}{25}$ $\boxed{\text{(E) } \frac{125}{12}}$

$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = \left(\frac{1}{x} + \frac{1}{y}\right)^2 = \left(\frac{1}{9} + \frac{1}{16}\right) = \frac{16+9}{144} = \frac{25}{144} = \frac{5^2}{12^2}$

$\frac{1}{x} + \frac{1}{y} = \frac{5}{12}$

$\frac{1}{x}\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{1}{9}$

$\frac{1}{x}\left(\frac{5}{12}\right) = \frac{1}{9}$

$\frac{1}{x} = \frac{4}{15}$, $x = \frac{15}{4}$

$\frac{1}{y}\left(\frac{1}{y} + \frac{1}{x}\right) = \frac{1}{16}$

$\frac{1}{y}\left(\frac{5}{12}\right) = \frac{1}{16}$

$\frac{1}{y} = \frac{\cancel{12}^{3}}{5} \cdot \frac{1}{\cancel{16}_{4}}$

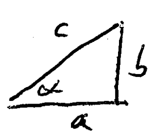
$\frac{1}{y} = \frac{3}{20}$ $y = \frac{20}{3}$

$x + y = \frac{15}{4} + \frac{20}{3} = \boxed{\frac{125}{12}}$

20. In a right triangle, the square of the hypotenuse is four times the product of the legs. If α is the smallest angle of the triangle, compute $\tan\alpha$.

(A) $\sqrt{3} - \sqrt{2}$ (B) $\frac{\sqrt{2}}{3}$ $\boxed{\text{(C) } 2 - \sqrt{3}}$ (D) $2 + \sqrt{3}$ (E) $\frac{\sqrt{3}}{2}$

$\frac{b}{a} = \underline{\qquad}$



Let $\frac{b}{a} = x$

Let $\frac{b}{a} = x$

$c^2 = 4ab$

$c^2 = a^2 + b^2$

$a^2 + b^2 = 4ab$

$a^2 - 4ab + b^2 = 0$

$\frac{a^2}{ab} - \frac{4ab}{ab} + \frac{b^2}{ab} = 0$

$\frac{a}{b} - 4 + \frac{b}{a} = 0$

$\left(\frac{1}{x} - 4 + x = 0\right)x$

$1 - 4x + x^2 = 0$

$x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$\boxed{\text{Smallest value: } 2 - \sqrt{3}}$

21. When the number x^2 is written in base y, the value is 341. When the number y^2 is written in base x, the value is 44. Compute $x+y$.

(A) 17 (B) 19 (C) 21 $\boxed{\text{(D) 23}}$ (E) 25

$3y^2+4y+1=x^2$ $\quad 44_x = 4x+4 = y^2 \rightarrow y=\sqrt{4x+4}=2\sqrt{x+1}$

$3(4x+4)+4(2\sqrt{x+1})+1=x^2$

$12x+12+8\sqrt{x+1}+1=x^2$

$12(x+1)+8\sqrt{x+1}=x^2-1$

$\frac{12(x+1)}{x+1}+\frac{8\sqrt{x+1}}{x+1}=\frac{(x+1)(x-1)}{x+1}$

$12+\frac{8}{\sqrt{x+1}}=x-1$

$\left(\frac{8}{\sqrt{x+1}}\right)^2=(x-13)^2$

$\frac{8^2}{x+1}=(x-13)^2$

$64=(x+1)(x-13)^2$

$x+1=16,\ x=15$

$y=8$

$x+y=8+15=\boxed{23}$

23. If $|x|+x+y=12$ and $x+|y|-y=14$, compute the value of $\frac{x}{y}$.
① $x>0$ ② $y<0$

$\boxed{\text{(A) }-2.375}$ (B) −1.625 (C) −1.250 (D) 1.875 (E) 2.125

$2x+y=12$ $\qquad x-2y=14$

$y=12-2x \nearrow x-2(12-2x)=14$

$x-24+4x=14$
$+24$

$5x=38$

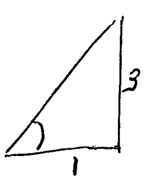
$x=\frac{38}{5}$ $\quad y=\frac{-16}{5}$

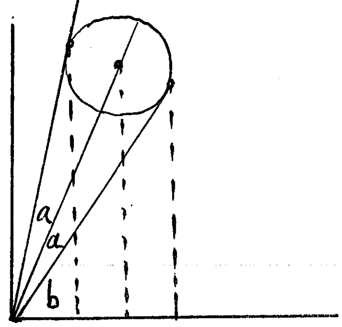
$\frac{x}{y}=\frac{38/5}{-16/5}=\frac{-38}{16}\div 2=\frac{-19}{8}\approx-2.375$

25. A line drawn from the origin to the center of a circle has a slope of 2, and a tangent line to the circle drawn from the origin has a slope of 3. Compute the slope of the other tangent line drawn from the origin.

$*\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(A) $\frac{13}{9}$ (B) $\frac{11}{6}$ (C) $\frac{9}{5}$ (D) $\frac{5}{3}$ (E) $\frac{3}{2}$





$\tan(2a+b)=\frac{3}{1}=3$

$\tan(a+b)=\frac{2}{1}=2$

$\tan(a+a+b)=\frac{\tan a+\tan(a+b)}{1-\tan a\tan(a+b)}=3$

$=\frac{\tan a+2}{1-\tan a(2)}=3$

$\tan a+2=3(1-2\tan a)$

$\tan a+2=3-6\tan a$

$7\tan a=1$

$\tan a=1/7$

$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}=2$

$\frac{\frac{1}{7}+\tan b}{1-\frac{1}{7}(\tan b)}=2$

$\frac{1}{7}+\tan b=2-\frac{2}{7}\tan b$

$\frac{9}{7}\tan b=2-\frac{1}{7}$

$\frac{9}{7}\tan b=\frac{13}{7}$

$\tan b=\frac{13}{7}\cdot\frac{7}{9}=\boxed{\frac{13}{9}}$