Key

Calculus AB Course Review: Unit 5 Differential Equations MC WS #1

1. If $a(t) = 4t - 1$ and $v(1) = 3$, then $v(t)$ equals

(A) $2t^2 - t$ (B) $2t^2 - t + 1$ $\boxed{\text{(C)}}$ $2t^2 - t + 2$
(D) $2t^2 + 1$ (E) $2t^2 + 2$

$v(t) = \int a(t)\,dt$
$v(t) = \int 4t - 1\,dt$

$v(t) = \frac{4t^2}{2} - t + c$
$v(t) = 2t^2 - t + c$

$* v(1) = 3$
$3 = 2(1)^2 - (1) + c$
$3 = 1 + c$
$2 = c$

$\boxed{v(t) = 2t^2 - t + 2}$

2) If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, then

cross multiply

(A) $y^2 = 4\sqrt{x} - 7$ (B) $\ln y = 4\sqrt{x} - 8$ (C) $\ln y = \sqrt{x} - 2$
(D) $y = e^{\sqrt{x}}$ (E) $y = e^{\sqrt{x} - 2}$

$2\sqrt{x}\,dy = y\,dx$
$\frac{dy}{y} = \frac{dx}{2\sqrt{x}}$

$\int \frac{dy}{y} = \frac{1}{2}\int x^{-1/2}\,dx$
$\ln|y| = \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + c$

$e^{\ln|y|} = e^{x^{1/2} + c}$
$|y| = e^{x^{1/2}} \cdot e^{c}$
$|y| = Ce^{x^{1/2}}$

$* y(4) = 1$
$1 = Ce^{\sqrt{4}}$
$\frac{1}{e^2} = C$
$e^{-2} = C$

$y = e^{-2} e^{x^{1/2}}$
$\boxed{y = e^{\sqrt{x} - 2}}$

3) If $\frac{dy}{dx} = e^y$ and $y = 0$ when $x = 1$, then

(A) $y = \ln|x|$ (B) $y = \ln|2 - x|$ (C) $e^{-y} = 2 - x$
(D) $y = -\ln|x|$ (E) $e^{-y} = x - 2$

$\frac{dy}{dx} = \frac{e^y}{1}$
$dy = e^y\,dx$
$\frac{dy}{e^y} = dx$

$\int e^{-y}\,dy = \int 1\,dx$
$u = -y$
$\frac{du}{dy} = -1$
$du = -dy$
$dy = -du$

$\int e^u(-du) = \int 1\,dx$
$-e^{-y} = x + C$
$e^{-y} = -x + C$
$e^0 = -1 + C$
$1 = -1 + c$
$2 = C$

find C
* using (1,0)

$e^{-y} = -x + 2$
$\boxed{e^{-y} = 2 - x}$

4) If $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{9+x^2}}$ and $y = 5$ when $x = 4$, then y equals

(A) $\sqrt{9+x^2} - 5$ (B) $\boxed{\sqrt{9+x^2}}$ (C) $2\sqrt{9+x^2} - 5$

(D) $\dfrac{\sqrt{9+x^2}+5}{2}$ (E) none of these

cross multiply

$dy\sqrt{9+x^2} = x\,dx$

$\int dy = \int \dfrac{x\,dx}{\sqrt{9+x^2}}$

$y = \int \dfrac{\cancel{x}}{u^{1/2}} \cdot \dfrac{du}{2\cancel{x}}$

$u = 9+x^2$ $\quad dx = \dfrac{du}{2x}$

$\dfrac{du}{dx} = 2x$

$y = \frac{1}{2}\int u^{-1/2}\,du$

$y = \frac{1}{2}\left(\dfrac{u^{1/2}}{1/2}\right) + C$

$y = (9+x^2)^{1/2} + C$

*plug in (4,5)

$5 = \sqrt{9+4^2} + C$

$5 = \sqrt{25} + C$ $\quad \underline{0 = C}$

$y = \sqrt{9+x^2} + 0$

$\boxed{y = \sqrt{9+x^2}}$

5) Which differential equation has the slope field shown?

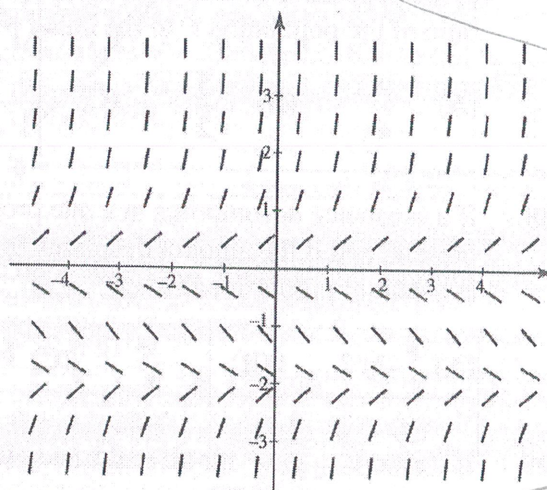
(A) $\boxed{y' = y(y+2)}$

(B) $y' = x(y+2)$

(C) $y' = xy + 2$

(D) $y' = \dfrac{x}{y+2}$

(E) $y' = \dfrac{y}{y+2}$



(slope = 0)
$y' = 0$ when $y = 0$ and when $y = -2$

6) Which function is a possible solution of the slope field shown?

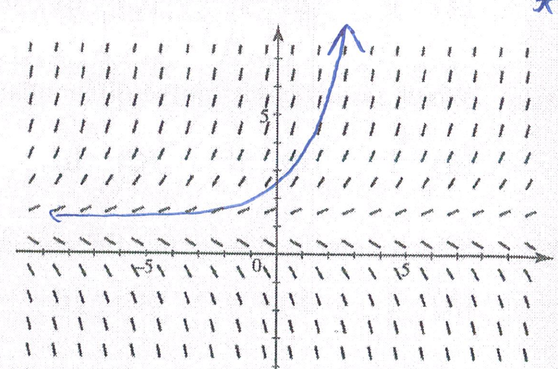
(A) $y = 1 - \dfrac{1}{x}$

(B) $y = 1 - \ln x$

(C) $y = 1 + \ln x$

(D) $\boxed{y = 1 + e^x}$

(E) $y = 1 + \tan x$



*Resembles exponential growth function
$y = e^x$

7) A cup of coffee at temperature 180°F is placed on a table in a room at 68°F. The d.e. for its temperature at time t is $\frac{dy}{dt} = -0.11(y - 68)$; $y(0) = 180$. After 10 min the temperature (in °F) of the coffee is

(A) 96 (B) 100 $\boxed{\text{(C)}}$ 105 (D) 110 (E) 115

cross multiply, *separate variables first

find C, plug in (0,180)

$dy = -0.11(y-68)\,dt$

$\int \frac{dy}{y-68} = \int -0.11\,dt$

$u = y-68$

$\frac{du}{dy} = 1 \quad dy = du$

$\int \frac{1}{u}\,du = -0.11\int 1\,dt$

$\ln|u|$

$\ln|y-68| = -0.11t + C$

$e \qquad e$

$|y-68| = e^{-0.11t} \cdot e^{c}$

$y - 68 = Ce^{-0.11t}$

$y = Ce^{-0.11t} + 68$

$180 = Ce^{0} + 68$

$112 = C$

$y = 112e^{-0.11t} + 68$

$y(10) = 112e^{-0.11(10)} + 68$

$\approx \boxed{105°F}$

8) Solutions of the differential equation $y\,dy = x\,dx$ are of the form

$\boxed{\text{(A)}}$ $x^2 - y^2 = C$ (B) $x^2 + y^2 = C$ (C) $y^2 = Cx^2$

(D) $x^2 - Cy^2 = 0$ (E) $x^2 = C - y^2$

$\int y\,dy = \int x\,dx$

$2\left(\frac{y^2}{2} = \frac{x^2}{2} + c\right)$

$y^2 = x^2 + c$

$-c = x^2 - y^2$

$\boxed{x^2 - y^2 = C}$

"C" can take on pos and negative values

9) The solution curve of $y' = y$ that passes through point (2, 3) is

(A) $y = e^x + 3$ (B) $y = \sqrt{2x+5}$ $\boxed{\text{(C)}}$ $y = 0.406e^x$

(D) $y = e^x - (e^2 + 3)$ (E) $y = e^x/(0.406)$

$\frac{dy}{dx} = y$

$dy = y\,dx$

$\int \frac{dy}{y} = \int 1\,dx$

$\ln|y| = x + c$

$e \qquad e$

$|y| = e^x \cdot e^c$

$y = Ce^x$ plug in (2,3)

$3 = Ce^2$

$\frac{3}{e^2} = C$

$C \approx 0.406$

$\boxed{y = 0.406e^x}$

10) According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32°C arrives at a mortuary where the temperature is kept at 10°C. Then the differential equation satisfied by the temperature T of the corpse t hr later is

surrounding temperature

(A) $\frac{dT}{dt} = -k(T-10)$ (B) $\frac{dT}{dt} = k(T-32)$ (C) $\frac{dT}{dt} = 32e^{-kt}$

(D) $\frac{dT}{dt} = -kT(T-10)$ (E) $\frac{dT}{dt} = kT(T-32)$

*direct proportion: $y = kx$

32°, surrounding temp

$$\frac{dT}{dt} = -k(T-10)$$

decreasing, constant of proportionality

*Intial temperature at $t=0$ is 32°C

11) If the corpse in Question 51 cools to 27°C in 1 hr, then its temperature (in °C) is given by the equation

(A) $T = 22e^{0.205t}$ (B) $T = 10e^{1.163t}$ (C) $T = 10 + 22e^{-0.258t}$

(D) $T = 32e^{-0.169t}$ (E) $T = 32 - 10e^{-0.093t}$

(time, Temperature)
(0, 32)
(1, 27)

$$\frac{dT}{dt} = -k(T-10)$$

$$dT = -k(T-10)dt$$

$$\frac{dT}{T-10} = -k\,dt$$

$$\int \frac{dT}{T-10} = -k\int 1\,dt$$

$u = T-10$

$\frac{du}{dT} = 1$ $dT = du$

$$\int \frac{du}{u} = -k\int 1\,dt$$

$$\ln|u|$$

$$\ln|T-10| = -kt + C$$

e e

$$T-10 = e^{-kt} \cdot e^{c}$$

$$T-10 = Ce^{-kt}$$

$$T = Ce^{-kt} + 10$$

$$32 = Ce^{-k(0)} + 10$$

$$22 = C$$

$$T = 22e^{-kt} + 10$$

$$27 = 22e^{-k(1)} + 10$$

$$17 = 22e^{-k}$$

$$\frac{17}{22} = e^{-k}$$

$$\ln\left(\frac{17}{22}\right) = \ln e^{-k}$$

$$-0.2578 = -k$$

$k = 0.2578$

$$\boxed{T = 22e^{-0.258t} + 10}$$