Key

AP MC Review WS #2 : Chapter 6 Differential Equations Practice Problems

Verifying Solutions for Differential Equations:

*Recall: $\frac{d}{dx}e^{u} = e^{u} \cdot u'$

0. The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

$\boxed{\text{(A)}}$ $y'' - 3y' - 15 = 0$

(B) $y'' - 3y' + 15 = 0$

(C) $y'' - y' - 5 = 0$

(D) $y'' - y' + 5 = 0$

*since all answer choices involve y' and y'', let's find the 2 derivative equations:

$y' = e^{3x} \cdot 3 - 5 \rightarrow y' = 3e^{3x} - 5$

$y'' = 3e^{3x} \cdot 3 \rightarrow y'' = 9e^{3x}$

*Test answer choices through substitution to find match for original equation:

$y'' - 3y' - 15 = 0$

$9e^{3x} - 3(3e^{3x} - 5) - 15 = 0$

$9e^{3x} - 9e^{3x} + 15 - 15 = 0$ ✓ $\boxed{A}$

1. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

$\boxed{\text{(C)}}$ $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

*cross multiply, separate variables:

$\frac{dy}{dx} = \frac{y \sec^2 x}{1}$

$dy = y \sec^2 x \, dx$

$\frac{dy}{y} = \sec^2 x \, dx$

*$\int \sec^2 u \, du = \tan u + C$

$\int \frac{1}{y} dy = \int \sec^2 x \, dx$

$e^{\ln|y|} = e^{\tan x + C}$

$|y| = e^{\tan x} \cdot e^{C}$

$y = Ce^{\tan x}$

use (0,5) to find C

$5 = Ce^{\tan 0} \rightarrow 5 = Ce^{0}$

$\underline{\underline{C = 5}}$

$\boxed{y = 5e^{\tan x}}$

2. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

$\boxed{\text{(C)}}$ $y = -\ln(-e^x + 5)$

(D) $y = -\ln(e^x + 3)$

(E) $y = \ln(e^x + 3)$

* Cross multiply, separate variables:

$\frac{dy}{dx} = \frac{e^{y+x}}{1}$

$dy = e^{y+x}dx$

$dy = e^y \cdot e^x \cdot dx$

$\frac{dy}{e^y} = e^x dx$

$\int e^{-y} dy = \int e^x dx$

$u = -y$ | $dy = -du$

$\frac{du}{dy} = -1$

$\int e^u(-du) = \int e^x dx$

$-\int e^u du$

$-e^u$

$-1(-e^{-y} = e^x + C)$

$e^{-y} = -e^x + C$

$\ln e^{-y} = \ln(-e^x + C)$

$-y = \ln(-e^x + C)$

$y = -\ln(-e^x + C)$

use $(0, -\ln 4)$ to find C

$-\ln 4 = -\ln(-e^0 + C)$

$-\ln 4 = -\ln(-1 + C)$

$4 = -1 + C$

$5 = C$

$\boxed{y = -\ln(-e^x + 5)}$

3. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

$\boxed{\text{(D)}}$ $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

* curve with slope $2x+3$ means $\frac{dy}{dx} = 2x+3$

solve differential equation:

This is not a "find equation of tangent line" problem.

$\frac{dy}{dx} = \frac{2x+3}{1}$

$dy = (2x+3)dx$

$\int 1\, dy = \int 2x+3\, dx$

$y = \frac{2x^2}{2} + 3x + C$

$y = x^2 + 3x + C$

use $(1, 2)$ to find C

$2 = (1)^2 + 3(1) + C$

$2 = 4 + C$

$-2 = C$

$\boxed{y = x^2 + 3x - 2}$

4. For what value of k, if any, will $y = k\sin(5x) + 2\cos(4x)$ be a solution to the differential equation $y'' + 16y = -27\sin(5x)$?

(A) -27

(B) $-\frac{9}{5}$

$\boxed{\text{(C)}}$ 3

(D) There is no such value of k.

* we need to first find y''

$$y = k\sin(5x) + 2\cos(4x)$$

$$y' = k\cdot\cos(5x)\cdot 5 + 2\cdot -\sin(4x)\cdot 4 \rightarrow y' = 5k\cos(5x) - 8\sin(4x)$$

$$y'' = 5k\cdot -\sin(5x)\cdot 5 - 8\cos(4x)\cdot 4 \rightarrow y'' = -25k\sin(5x) - 32\cos(4x)$$

* make substitution into differential equation:

$$y'' + 16y = -27\sin(5x)$$

$$-25k\sin(5x) - 32\cos(4x) + 16(k\sin 5x + 2\cos 4x) = -27\sin(5x)$$

* combine like terms:

$$-25k\sin(5x) - \cancel{32\cos 4x} + 16k\sin(5x) + \cancel{32\cos 4x} = -27\sin(5x)$$

$$-9k\sin(5x) = -27\sin(5x)$$

$$k = \frac{-27\cancel{\sin(5x)}}{-9\cancel{\sin(5x)}} \rightarrow \boxed{k=3}$$

5. Of the following, which are solutions to the differential equation $y'' - 10y' + 9y = 0$?

I. $y = 2\sin(3x)$
II. $y = 5e^x$
III. $y = Ce^{9x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

$\boxed{\text{(D)}}$ II and III only

~~I~~. $y = 2\sin(3x)$

$$y' = 2\cos(3x)\cdot 3 \rightarrow 6\cos(3x)$$

$$y'' = -6\sin(3x)\cdot 3 \rightarrow -18\sin(3x)$$

$$y'' - 10y' + 9y = 0$$

$$-18\sin(3x) - 10(6\cos 3x) + 9(2\sin(3x)) \neq 0$$

✓ II. $y = 5e^x$

$$y' = 5e^x\cdot 1 \rightarrow 5e^x$$

$$y'' = 5e^x\cdot 1 = 5e^x$$

$$y'' - 10y' + 9y = 0$$

$$5e^x - 10(5e^x) + 9(5e^x) = 0$$

$$50e^x - 50e^x = 0 \checkmark$$

✓ III. $y = Ce^{9x}$

$$y' = Ce^{9x}\cdot 9 \rightarrow 9Ce^{9x}$$

$$y'' = 9Ce^{9x}\cdot 9 \rightarrow 81Ce^{9x}$$

$$y'' - 10y' + 9y = 0$$

$$81Ce^{9x} - 10(9Ce^{9x}) + 9(Ce^{9x}) = 0$$

$$-9Ce^{9x} + 9Ce^{9x} = 0 \checkmark$$

6. For what value of k, if any, is $y = e^{2x} + ke^{-3x}$ a solution to the differential equation $4y - y'' = 10e^{-3x}$?

$\boxed{\text{(A)}}$ -2

(B) $\frac{10}{3}$

(C) 10

(D) There is no such value of k.

*First, find equation for y''

$y = e^{2x} + ke^{-3x}$

$y' = e^{2x} \cdot 2 + ke^{-3x}(-3) \rightarrow 2e^{2x} - 3ke^{-3x}$

$y'' = 2e^{2x} \cdot 2 - 3ke^{-3x} \cdot -3 \rightarrow 4e^{2x} + 9ke^{-3x}$

*Next, make substitutions into differential equation.

$4y - y'' = 10e^{-3x}$

$4(e^{2x} + ke^{-3x}) - (4e^{2x} + 9ke^{-3x}) = 10e^{-3x}$

$\cancel{4e^{2x}} + 4ke^{-3x} - \cancel{4e^{2x}} - 9ke^{-3x} = 10e^{-3x}$

$-5ke^{-3x} = 10e^{-3x}$

$k = \frac{10\cancel{e^{-3x}}}{-5\cancel{e^{-3x}}}$

$\boxed{k = -2}$

7. For what value of k, if any, will $y = ke^{-2x} + 4\cos(3x)$ be a solution to the differential equation $y'' + 9y = 26e^{-2x}$?

$\boxed{\text{(A)}}$ 2

(B) $\frac{13}{5}$

(C) 26

(D) There is no such value of k.

*First, find equation for y''

$y = ke^{-2x} + 4\cos(3x)$

$y' = ke^{-2x}(-2) - 4\sin(3x) \cdot 3 \rightarrow -2ke^{-2x} - 12\sin(3x)$

$y'' = -2ke^{-2x}(-2) - 12\cos(3x) \cdot 3 \rightarrow 4ke^{-2x} - 36\cos(3x)$

*Next, make substitutions into differential equation:

$y'' + 9y = 26e^{-2x}$

$4ke^{-2x} - 36\cos(3x) + 9(ke^{-2x} + 4\cos(3x)) = 26e^{-2x}$

$4ke^{-2x} - \cancel{36\cos 3x} + 9ke^{-2x} + \cancel{36\cos 3x} = 26e^{-2x}$

$13ke^{-2x} = 26e^{-2x}$

$k = \frac{26\cancel{e^{-2x}}}{13\cancel{e^{-2x}}}$

$\boxed{k = 2}$

8. Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

I. $y = 2\sin(4x)$
II. $y = 3e^{2x}$
III. $y = Ce^{4x}$, where C is a constant.

(A) I only

(B) II only

(C) III only

$\boxed{\text{(D)}}$ II and III only

II. $y = 3e^{2x}$
$y' = 3e^{2x} \cdot 2 = 6e^{2x}$
$y'' = 6e^{2x} \cdot 2 = 12e^{2x}$
$y'' - 6y' + 8y = 0$
$12e^{2x} - 6(6e^{2x}) + 8(3e^{2x}) = 12e^{2x} - 36e^{2x} + 24e^{2x} = 0$ ✓

III. $y = Ce^{4x}$
$y' = 4Ce^{4x}$
$y'' = 16Ce^{4x}$
$y'' - 6y' + 8y = 0$
$16Ce^{4x} - 6(4Ce^{4x}) + 8Ce^{4x} = 0$ ✓

* First find equation for y''

✗ I. $y = 2\sin 4x$
$y' = 8\cos 4x$
$y'' = -32\sin 4x$
$y'' - 6y' + 8y = 0$

$-32\sin 4x - 6(8\cos 4x) + 8(2\sin 4x) \neq 0$
* Not a solution ✗

9. For what value of k, if any, is $y = e^{-2x} + ke^{4x}$ a solution to the differential equation $y - \frac{y''}{4} = 5e^{4x}$?

$\boxed{\text{(A)}}$ $-\frac{5}{3}$

(B) $\frac{20}{3}$

(C) 5

(D) There is no such value of k.

* First find y'' equation
$y = e^{-2x} + ke^{4x}$
$y' = e^{-2x} \cdot -2 + ke^{4x} \cdot 4 \rightarrow -2e^{-2x} + 4ke^{4x}$
$y'' = -2e^{-2x}(-2) + 4ke^{4x}(4) \rightarrow 4e^{-2x} + 16ke^{4x}$

* Next, make substitutions into differential equation.

$y - \frac{y''}{4} = 5e^{4x}$
$y - \frac{1}{4}(y'') = 5e^{4x}$
$e^{-2x} + ke^{4x} - \frac{1}{4}(4e^{-2x} + 16ke^{4x}) = 5e^{4x}$
$\cancel{e^{-2x}} + 1ke^{4x} - \cancel{e^{-2x}} - 4ke^{4x} = 5e^{4x}$
$-3ke^{4x} = 5e^{4x}$
$k = \frac{5\cancel{e^{4x}}}{-3\cancel{e^{4x}}}$
$\boxed{k = -\frac{5}{3}}$