Mini-Mathletes 2017: Multiple Choice Solutions

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Problem 1: If Nicole throws the ball up three times farther than her friend, that means the friend throws it up 1/3 as far as Nicole. So, 96/3 = **32 (D)**.

Problem 2: 2 and 5 are both prime numbers that have a product of 10. If a number is multiplied by another number with a units digit of zero, the product will end in zero. Therefore, the product of all prime numbers will end in **0 (A)**.

Problem 3: Let the number of shoes Maddie's first and second brothers have be F and S, respectively. Since Maddie has 9 pairs of shoes, $F = 9 - 3 = 6$. Then, $S = 2(6) = 12$. So the total number of shoes Maddie and her brothers have altogether is $9 + 6 + 12 =$ **27 (B)**.

Problem 4: Harry has 4 sisters (including Harriet) and 5 brothers. Harriet has only 3 sisters (S) because she herself is a girl. Because Harry is a boy, Harriet has 6 brothers (B). The product of these numbers is $3(6) =$ **18 (D)**.

Problem 5: The formula for finding the area of a circle is πr^2, where r is the radius. If the area of the circle must be less than 50π square units, this can be represented with the inequality $\pi r^2 \leq 50\pi$. Dividing both sides by π leaves $r^2 \leq 50$. Since r must be a positive integer, the sum of the possible values of r is the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 =$ **28 (C)**.

Problem 6: We need to find the least common multiple (LCM) of the first four composite integers. We test for composite numbers: 1, 2, and 3 are not composite, but 4 is. 5 and 7 are prime, but 6 and 8 are composite. 9 is our fourth composite number. Finally, we find the LCM of 4, 6, 8, and 9 which leads us to an answer of **72 (C)**.

Problem 7: We are given the lengths of the hypotenuse and one leg for both triangles. To find the areas of the triangles, we must find the other leg length using the Pythagorean Theorem.

Triangle 1: $13^2 - 5^2 = b^2$

$$b^2 = 144 \rightarrow b = 12 \text{ (ignore negative solution)}$$

$$\text{Area} = \frac{12(5)}{2} = 30$$

Triangle 2: $10^2 - 8^2 = b^2$

$$b^2 = 36 \rightarrow b = 6 \text{ (ignore negative solution)}$$

$$\text{Area} = \frac{6(8)}{2} = 24$$

Thus, the ratio of the area of the smaller triangle to the area of the larger triangle is 24/30 = **4/5 (C)**.

<u>Problem 8:</u> Because these particular numbers are small, reasonable, and we are given that they are positive integers, the best methods to use are guess-and-check and logic. We know that we can't have any positive integer greater than 8 because then the sum of the squares would exceed 80. Our first combination of number to try that has a sum of 12 is 4 and 8. We find that the sum of their squares, $16 + 64$ does indeed add to 80. Thus the difference between these numbers is $8 - 4 =$ **4 (A)**.

<u>Problem 9:</u> Denote Caleb's age as C, his sister's age as S, and his brother's age as B. $C + S + B = 32$. It is given that Caleb is six years older than his sister, so $S = C - 6$. Similarly, $B = C - 4$. Substituting these new values in for the original S and B, we have $C + C - 6 + C - 4 = 32$. Solving for C gives us 14. But we are not finished! The problem asks for Caleb's age five years from now. So, $C + 5$ gives us a final answer of **19 (E)**.

<u>Problem 10:</u> The fence of a 60-foot by 20-foot garden has perimeter $2(60) + 2(20) = 160$. Using the exact same length of fencing, we must create a square. $160/4 = 40$ gives us the side length of this square. We then subtract the area of the original garden (1200) from the area of the new garden (1600), which leads to **400 (C)**.

<u>Problem 11:</u> We know our smallest possible product will be negative, because we have a negative number in the set. In order to obtain a negative product, we must multiply either two positives and one negative together, or three negatives together. (-8)(-6)(-4) is smaller than any combination of two positives and one negative, so our answer is **-192 (C)**.

<u>Problem 12:</u> Let Hanna's allowance be H, Lucy's allowance be L, and Aria's allowance be A. H/L = 4/7 and L/A = 10/11. Lucy's allowance is the same in both cases. We must find some way to rewrite both fractions so the denominator of the first fraction is equal to the numerator of the second fraction. The LCM of 7 and 10 is 70. If we multiply the first fraction's numerator and denominator each by ten, we get 40/70, and still maintain the same value of the fraction. Similarly, we can multiply the second fraction by 7 and obtain 70/77, while keeping the original value of the fraction. If H/L = 40/70 and L/A = 70/77, then H/A = **40/77 (B)**.

<u>Problem 13:</u> We test each letter by vertically cutting it in half and seeing if the two parts are mirror images of each other. The first **8 (D)** letters of MATHMATTERS have a vertical line of symmetry.

<u>Problem 14:</u> Let the width be w. We are given that the length is twice the width, so the length is $2w$. The perimeter of the rectangle is $2(w) + 2(2w) = 90 \rightarrow w = 15$. To find the area of the garden, we compute $15(30) =$ **450 (B)**.

<u>Problem 15:</u> There are 11 total integers from 40 to 50 inclusive. There are only three primes: 41, 43, and 47. Therefore, there are 8 numbers which are not prime in our set. The probability of *not* selecting a prime number from 40 to 50 inclusive is **8/11 (D)**.

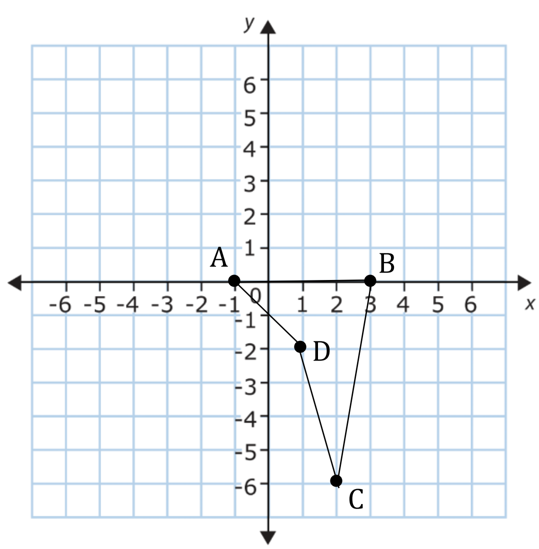
<u>Problem 16:</u> Since each block has six faces, this would give us a total of $6(7) = 42$ faces if the blocks were not connected. In order to account for the sides that are glued together we need to count how many sides are hidden by the glue connections (keep in mind that at each glue connection, two sides are hidden). There are 4 glue connections on the bottom row, 1 glue connection on the top row and 2 glue connections between the top and bottom rows. That is a total of $4 + 1 + 2 = 7$ glue connections, or $(7)(2) = 14$ faces that have been hidden. That leaves us with a total of $42 - 14 = 28$ exposed faces. Each face has area 2, so the total surface area is $28(2) =$ **56 (A)**.

Problem 17: We see the pattern: every integer in a position one more than a multiple of three is 3, every integer in a position two more than a multiple of three is 6, and every integer in a position of a multiple of three is 9. 131 and 113 are both two more than a multiple of three, so both terms are 6. $6(6) =$ **36 (C)**.

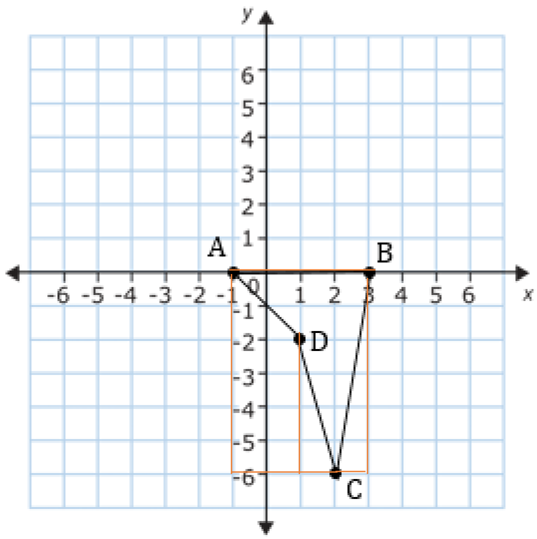
Problem 18: There are ${}_5C_2$ (read 5 choose 2) ways to select a group of two people from five people. In other words, there are 5 ways to pick the first person in the group, and 4 ways to pick the second person. But group *AB*, for example, is the same as group *BA*, so we see that we have counted each group twice. Thus, there are 10 ways to for a group of two people for pictures. We see there are ${}^5C_3$ ways to pick a group of 3 people from five people. Like before, there are 5 choices for the first person, 4 choices for the second person, and 3 choices for the third person. However, this time we have overcounted by a factor of 6 because there are six ways to arrange group *ABC*. So, the number of ways to group three people together is 10. Lastly, there is only 1 way to pick a group of five people; that is to select every person. Therefore, $10 + 10 + 1$ yields **21 (C)** possibilities for taking pictures.

Problem 19: Including Shay and Ashley, there are 72 total people we need to account for. It is given that each person will consume an average of three cookies. This means there needs to be $3(72) = 216$ cookies made. 216 cookies is 18 dozen cookies. We also know that each batch of a dozen cookies requires 7/6 cups of flour. $(7/6)(18) =$ **21 (E)** total cups of flour.

Problem 20: First we draw the quadrilateral on a coordinate grid.



There is no formula for finding the area of this kind of figure, so we have to try something else. If we enclose this figure inside of a rectangle, we can subtract out the areas of the unwanted parts (areas that we know how to find) from the area of the big rectangle. Inside this rectangle, we have figure ABCD, two triangles, and a trapezoid.



The area of the large rectangle is $6(4) = 24$ square units. Next, to find the area of the trapezoid, we compute $\frac{(4+6)(2)}{2} = 10$. The areas of the 1 by 4 and 1 by 6 triangles are 2 and 3, respectively. Thus, the total area of the unwanted regions is $10 + 2 + 3 = 15$. Subtracting this from 24 yields **9 (A)**.

Problem 21: To compute volume, we must compute 'length × width × height'. The volume of the prism (with integer side lengths) is 23 cubic units, so therefore 23 must be the product of three integers. Because 23 is a prime number, the only combination of numbers that have a product of 23 are 23, 1, and 1. Now, we must find the surface area of each face of our 23 by 1 by 1 rectangular prism. The sum of the areas of the six faces is $23 + 23 + 23 + 23 + 1 + 1 = 4(23) + 2(1) =$ **94 (B)**.

Problem 22: The six free throws that Max missed are 30% of the total number of free throws that he took. Our next step is to set up an equation: $6 = .3x \rightarrow x = 20$. Since x is the total number of free throws that Max took, he made $.7(20) =$ **14 (B)**.

Problem 23: The ratio of cold drinks to hot drinks sold at The Brew is 7:8 (part-to-part ratio). So the ratio of the number of cold drinks sold to the total number of drinks purchased is 7:15 (part-to-whole ratio) $\rightarrow \left(\frac{7}{15}\right)(480) =$ **224 (D)**.

Problem 24: The only right triangle with integer side lengths and area 6 has side lengths of 3, 4, and 5. This triangle has perimeter 12. The area of the second right triangle is 25 times the first. If two triangles are similar, then the ratio of their areas is the square of the scale factor. Because the square root of 25 is an integer, we know that the triangles are similar; the side lengths of the larger triangle are $\sqrt{25} = 5$ times the side lengths of the smaller triangle. Therefore, we find that the side lengths of the larger triangle are 15, 20, and 25. The difference in perimeter of the two triangles is $(15 + 20 + 25) - (3 + 4 + 5) =$ **48 (E)**.

Problem 25: It takes me 15 minutes to swim a mile, 4 minutes to bike a mile, and 7.5 minutes to run a mile. In my triathlon, I need to complete 1 mile of swimming, 25 miles of biking, and 6 miles running. So, it would take me $1(15) + 25(4) + 6(7.5) =$ **160 (E)**.

Problem 26: There are 12 choices for the first scoop, and 11 choices for the second scoop (because it can't be the same flavor as the first). We don't divide by 2: since order of the scoops matters, vanilla and chocolate, for example, is different from chocolate and vanilla. Then there are 4 choices for the topping. Computing $12 \times 11 \times 4$ gives us a total of **528 (A)** different cones.

Problem 27: $\angle DBC = \angle BDC = 70^\circ$ because they are the base angles of isosceles triangle BDC. $\angle BDC$ and $\angle ADC$ form a linear pair. So, $\angle ADC = 110^\circ$. Triangle ADC is also an isosceles triangle because sides AD and CD are congruent. So, the measure of $\angle BAC = \frac{180-110}{2} =$ **35° (E)**.

Problem 28: Each pair of numbers next to each other in the expression has a sum of -1. For instance, 1 – 2 = -1, 3 – 4 = -1, 5 – 6 = -1, etc. There are $2016/2 = 1008$ such pairs. This means that the sum of the first 2016 numbers is -1008. But we still need to add 2017, so our final answer is $-1008 + 2017 =$ **1009 (D)**.

Problem 29: There are 6 choices for the first letter, 5 choices for the second letter, and so on until there is only one choice remaining for the last letter. So, it may seem like there are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways to arrange the letters in PRETTY. But the two T's in this word are the same. We counted every possible order of letters, but also counted again when we switched the places of the two T's (meaning we have counted each order twice). Therefore, we have overcounted by a factor of 2: $720/2 =$ **360 (A)**.

Problem 30: When all the points 1 inch away from the vertices are plotted and connected, we form four quarter-circles all with radius 1 inch. The shaded border also consists of four 3 inch $\times$ 1 inch rectangles. The sum of the areas of the rectangles are $4(3) = 12$. Four quarter-circles form one circle with radius 1 inch, so the area of the quarter-circle regions is π. To find the total area of the border, we add the area of the circle and the areas of the four rectangles together, yielding a final answer of **$\pi + 12$ (B)**.