Mini-Mathletes 2015: SOLUTIONS

Round 1: Free Response

1. Mr. Yang bakes $7 \div \frac{1}{2} = 7 \times 2/1 = 14/1$, or 14 half-dozens of cookies. Thus, the total money Mr. Yang makes is $14 \times 4.25 = 59.5 \rightarrow \boxed{\$59.50}$.

2. We conclude that the total number of people in Milton's chorus group is one less than a multiple of 5 and also one less than a multiple of 6. The lowest possible number to satisfy these two conditions is one less than the lowest common multiple of 5 and 6. Since 5 and 6 are relatively prime, there are $(5 \times 6) - 2 = \boxed{29}$ people.

3. We can write the following equations from the details given in the problem:

$$E = 3K$$
$$T = 6 + K$$
$$E = T$$

We are asked to solve for K. Substituting $3K = E = T$ in the second equation above gives $3K = 6 + K \rightarrow 2K = 6 \rightarrow K = \boxed{3}$ years old.

4. Perhaps the easiest way to solve this problem would be to set up a system of linear equations in two variables. Let's call the number of 2-point baskets Elena made x and the number of 3-point baskets y. We have:

$$2x + 3y = 29$$
$$x + y = 11$$

Multiplying the second equation by 2 and subtracting it from the first equation, we get that $y = 29 - 2(11) = \boxed{7}$.

5. We are given that the surface area of the cube is 726 cm^2, meaning that the area of each face of the cube is $726/6 = 121$ cm^2. Thus, the side length of the cube is $\sqrt{121} = 11$ cm, and the volume is $11^3 = 1331$ cm^3. The ratio of the area to volume came out to be $\frac{726}{1331} = \boxed{6/11}$.

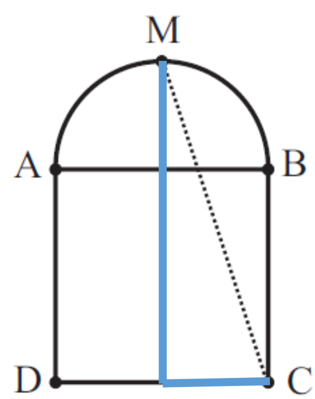
6. If the mean of seven positive integers is 16, then their sum must be $7(16) = 112$. If the sum of six of the seven integers is 108, then the removed integer has a value of $112 - 108 = \boxed{4}$.

7. The minute hand rotates 360° in 1 hour, or 60 minutes. If the minute hand has rotated 810 degrees since noon, it has completed $810°/360° = 2.25$ revolutions. Since 1 revolution is equivalent to 60 minutes, $60(2.25) = \boxed{135}$ minutes have elapsed since noon.

8. In one hour, Harrison is able to complete $1/6$ of the painting job, while Sean is able to finish $1/12$ of it. Working together, Sean and Harrison are able to accomplish $(1/6 + 1/12) = 1/4$ of the job in one hour. Thus, it will take them $\boxed{\mathbf{4}}$ hours to complete the entire painting job.

9. Note that the total area of the target is $(3^2)\pi = 9\pi$. The area of the blue region is
$$9\pi - (2^2)\pi = 5\pi.$$
Thus, the probability that the point lies within the blue region is $\frac{5\pi}{9\pi} = \boxed{\mathbf{5/9}}$.

10. Drop a perpendicular from point M to side DC. We'll call this point P. Because the radius of the semicircle is $6/2 = 3$ cm, and AD = 6 cm, MP will measure 3+6 = 9 cm. Now, we will use the Pythagorean Theorem on the right triangle shown below:



We get $(MC)^2 = (MP)^2 + (PC)^2 = 9^2 + 3^2 \rightarrow MC = \boxed{\mathbf{3\sqrt{10}}}$ centimeters.

Round 2: Multiple Choice

1. Work backwards – Amy had 7 apples before she gave Patricia 3 apples. These 7 apples were half of Amy's 14 original apples. $\boxed{E}$

2. There are a total of 10 fully shaded squares, each with an area of 2*2=4. Additionally, there are 10 triangles with area 1/2(2)(2)=2. Finally, the area of the smaller triangle plus the area of the pentagon sum to 4. Therefore, the total shaded area of the figure is 64. $\boxed{D}$

3. We reason that to have at least 2 people have birthdays in the same month, we need at least 12(2-1) + 1 = 13 people. Because if we have only 12, they can be born each in a different month. So, in general, to have at least n people have birthdays in the same month, we need at least 12(n-1) + 1 people. Because if we have only 12(n-1), they can be born in such a way that each n-1 people were born in a different month. Thus, the answer is 12*(8 -1) + 1 = 85. $\boxed{D}$

4. To make the number as small as possible, the smaller digits are placed in the higher-valued positions. To make the number even, the larger even digit 4 must be in the units digit. The smallest possible even number is 12394 and 9 is in the tens place. $\boxed{E}$

5. Be careful not to confuse numbers in problems like this: we are only concerned with the times. Starting from 10:28 AM, we add on 500 minutes, or 8 hours and 20 minutes, to get to 6:48 PM when the cookies are thawed to a reasonable temperature. Next, adding on 89 seconds, or 1 minute and 29 seconds, brings the time to 6:49:29 PM. Finally, adding 25 minutes brings the time to 7:14:29 PM. Round the time down to get 7:14 PM. $\boxed{D}$

6. It is clear that $70 = 2\times5\times7$ is the largest positive integer less than 75 that has exactly 8 distinct factors, which are: {1, 2, 5, 7, 10, 14, 35, 70}. $\boxed{A}$

7. Note that the problem say that Simeon has 1/3 of the book left to read, not that he read 1/3 of the book. Subtract 1–(1/3) and set the difference, 2/3, equal to 12/x. The equivalent fraction with 12 in the numerator is 18, representing a total of 18 days needed to read the book. Multiply 18 days by 17 pages per day to get the total of 306 pages. $\boxed{C}$

8. The mean, or average, is equal to the sum of all numbers divided by the number of numbers. If you have 7 classes, there are a total of 7 numbers. Do not be fooled by the question when it asks for the sum of 4 grades- some people tend to add an additional 4 to the denominator. For the lowest possible "successful" average, the mean must be 90%. Multiply that by 7, and subtract the sum of the three given grades, 267. The difference is the lowest possible sum of the other grades, 363. $\boxed{B}$

9. The angles will be 30, 30+x, and (30+2x) degrees. . So 30 + (30+x) + (30+2x) = 180. Solving for x gives x = 60 - 30 = 30. The largest angle will be 30 + 2*30= 90 degrees. $\boxed{A}$

10. Notice that

$$7^9 = \left(\frac{1}{343}\right)^x = \left(\frac{1}{7}\right)^{3x} = 7^{-3x}.$$

This implies that -3x = 9 and therefore x= - 9/3= -3. $\boxed{E}$

11. Let x be the number of residents. Each resident bumps his head with x-1 residents. Hence x(x-1) will be the number that each resident bumps his head with another. This will give us a double count. So we have to solve x(x-1)/2 = 45. Solving for x, we get x=10 as the positive solution. $\boxed{C}$

12. Add the two equations to get 18(x+y) = 35 + 91 = 126 = 18×7. Thus, x + y = 7. $\boxed{B}$

13. Make a table:

	1	3	5
2	$1+2=3$	$3+2=5$	$5+2=7$
4	$1+4=5$	$3+4=7$	$5+4=9$
6	$1+6=7$	$3+6=9$	$5+6=11$

The table shows that seven of the nine equally likely events have prime numbers for their outcomes. So the probability of a prime outcome is 7/9. $\boxed{D}$

14. Note that the diameter is as long as one side of the square. Therefore, the total area of the square is $16R^2$. The total unshaded area in the square is equal to the area of one circle, which is $\pi(2R)^2 = 4\pi R^2$. Subtract the area of the circle from the area of the square to get $16R^2 - 4\pi R^2$, or $4R^2(4-\pi)$. $\boxed{D}$

15. There are one odd and two even numbers showing. Because all primes other than 2 are off and the sum of an even number and an odd number is odd, the common sum must be odd. That means 2 must be opposite 59 and the common sum is 2+59 = 61. The other two hidden numbers are 61 – 44 = 17 and 61 – 38 = 23. The average of 2, 17, and 23 is (2+17+23)/3 = 42/3 = 14. $\boxed{B}$