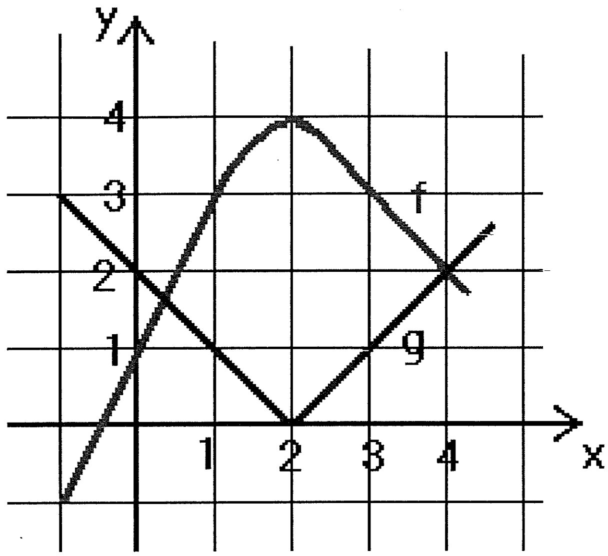
AB Calculus Ch. 2 Derivatives Morning Test Review

1. Let h(t) = f [g(t)]. Find h'(1)

2. Let z(t) = f(t)g(t). Find z'(3)



3.

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

4. Find $\frac{d^2y}{dx^2}$ for $f(x) = \frac{x-1}{x+2}$

5. The position of the particle traveling along a straight line is x(t) = $t^3 - 9t^2 + 15t + 3$ for $t \geq 0$

a) Find when the particle is moving to the left.	b) Is speed increasing or decreasing at t = 2?
c) Find avg. acceleration in interval [0, 2]	d) Where is the particle located when the velocity is zero?
e) Find particle's displacement from t = 0 to t = 3	f) Find particle's distance from t = 0 to t = 3
g) Is the velocity increasing or decreasing at t=2?	h) Find the velocity and position when acceleration is zero.

6. Find $\frac{dy}{dx}$ for $f(x) = \sqrt{x}(x^3-7)^4$.

7. Find $\frac{dy}{dx}$ for $f(x) = \frac{\sqrt{2-x^2}}{4+3x}$.

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Key

*chain Rule $f'[g(t)]\cdot g'(t)$

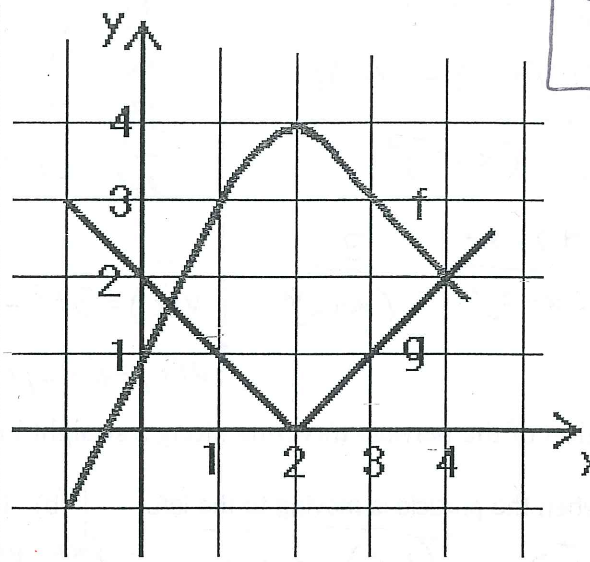
1. Let h(t) = f [g(t)]. Find h'(1)

$h'(t) = f'[g(t)]\cdot g'(t)$

$h'(1) = f'[g(1)]\cdot g'(1)$

$= f'[1]\cdot g'(1)$

$= (1)(-1) = \boxed{-1}$



2. Let z(t) = f(t)g(t). Find z'(3)

$z'(t) = f'(t)g(t) + f(t)g'(t)$

$z'(3) = f'(3)g(3) + f(3)g'(3)$

$= (-1)(1) + (3)(1)$

$= -1 + 3$

$= \boxed{2}$

3.

*implicit, product Rule

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

$6y^2\frac{dy}{dx} + 12xy + 6x^2\left(\frac{dy}{dx}\right) - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\frac{dy}{dx} = \frac{24x - 12xy}{6y^2 + 6x^2 + 6} = \boxed{\frac{4x - 2xy}{x^2 + y^2 + 1}}$

4. Find $\frac{d^2y}{dx^2}$ for $f(x) = \frac{x-1}{x+2}$

$$\frac{dy}{dx} = \frac{(1)(x+2)-(x-1)(1)}{(x+2)^2}$$

$$= \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

$$\frac{dy}{dx} = 3(x+2)^{-2}$$

$$\frac{d^2y}{dx^2} = 3(-2)(x+2)^{-3}(1) = \boxed{\frac{-6}{(x+2)^3}}$$

$v(t) = 3t^2 - 18t + 15 \rightarrow 3(t^2-6t+5) \rightarrow 3(t-5)(t-1)$

$a(t) = 6t - 18$

5. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$ for $t \geq 0$

a) Find when the particle is moving to the left.

$+ \quad - \quad +$

$0 \quad 1 \quad 5$

$\boxed{(1,5) \text{ since } v(t)<0}$

b) Is speed increasing or decreasing at t = 2?

Since $v(2)<0$ and $a(2)<0$ have same signs, speed increasing at $t=2$.

c) Find avg. acceleration in in interval [0, 2]

$$= \frac{v(2)-v(0)}{2-0} = \frac{-9-15}{2-0} = \frac{-24}{2}$$

$v(2) = -9$

$v(0) = 15$

$= \boxed{-12 \text{ units/s}^2}$

d) Where is the particle located when the velocity is zero?

$v(t)=0$ at $t=1,5$

$\boxed{s(1) = 10 \quad s(5) = -22}$

e) Find particle's displacement from t = 0 to t = 3

$s(3) = -6$

$s(0) = 3$

$s(3) - s(0)$

$-6 - 3 = \boxed{-9}$

f) Find particle's distance from t = 0 to t = 3

$s(0) = 3$

> 7

$s(1) = 10$

> 16

$s(3) = -6$

$7 + 16 = \boxed{23}$

g) Is the velocity increasing or decreasing at t=2?

*Is $a(t) > 0$ or $a(t) < 0$

$a(2) = -6$, so since $a(2) < 0$, velocity decreasing at $t=2$.

h) Find the velocity and position when acceleration is zero.

$a(t) = 6t - 18$

$0 = 6(t-3)$

$t = 3$

$\boxed{s(3) = -6 \quad v(3) = -12}$

6. Find $\frac{dy}{dx}$ for $f(x) = \sqrt{x}(x^3-7)^4$

product, chain (first)

$f'g + fg'$

$$(x)^{1/2}(x^3-7)^4$$

$$\tfrac{1}{2}(x)^{-1/2}(x^3-7)^4 + x^{1/2} \cdot 4(x^3-7)^3(3x^2)$$

$$= \boxed{\frac{(x^3-7)^4}{2\sqrt{x}} + 12x^{5/2}(x^3-7)^3}$$

7. Find $\frac{dy}{dx}$ for $f(x) = \frac{\sqrt{2-x^2}}{4+3x}$ $= \frac{(2-x^2)^{1/2}}{4+3x}$

* quotient (first)

* chain

$$f'(x) = \frac{\tfrac{1}{2}(2-x^2)^{-1/2}(-2x)(4+3x) - (2-x^2)^{1/2}(3)}{(4+3x)^2}$$

$$f'(x) = \frac{\frac{-x(4+3x)}{\sqrt{2-x^2}} - 3\sqrt{2-x^2}}{(4+3x)^2} \cdot \frac{(2-x^2)^{1/2}}{(2-x^2)^{1/2}}$$

$$= \frac{-4x - 3x^2 - 3(2-x^2)}{(4+3x)^2(2-x^2)^{1/2}} = \frac{-4x-3x^2-6+3x^2}{(4+3x)^2(2-x^2)^{1/2}}$$

$$= \boxed{\frac{-4x-6}{(4+3x)^2(2-x^2)^{1/2}}}$$