CCGPS Analytic Geometry – Dec. 5, 2014 (Fri)
Notes: Complex Numbers Day 2 – Page 11
Homework: Multiplying and Dividing Complex Numbers Worksheet

Essential Question: How do we multiply and divide complex numbers?

Consider about multiplication. What is $\left(\sqrt{-1}\right)^2$? Let's try to find a pattern to the powers of i:

R1 $i = i$
R2 $i^2 = -1$
R3 $i^3 = -i$
R4 $i^4 = 1$
$i^5 = i$
$i^6 = -1$
$i^7 = -i$
$i^8 = 1$

Notice that every 4 values, the pattern repeats. To figure out where we are in the pattern for bigger exponents, we can divide the exponent by 4 and use the remainder.

Model: For i^{22} consider $22 \div 4$. It has a remainder of 2. So, $i^{22} = i^2 = -1$

Examples:

1. $i^{12} = 1$

2. i^{26} $4\overline{)26}$ 6 R2 24 -1

3. $i^{18} = -1$ $4 \cdot 4 = 16$ R2

4. i^{11} $4\overline{)11}$ 2 R3 $\boxed{-i}$

When we multiply complex numbers, we multiply similarly to variable expressions. We can distribute and then combine like terms.

Examples:

5. $(8 + 5i)(2 - 3i)$

$16 - 24i + 10i - 15i^2$

$\boxed{31 - 14i}$

6. $(-6 + 2i)(5 - 3i)$

$-30 + 18i - 10i - 6i^2$

$-30 + 8i + 6$

$\boxed{-24 + 8i}$

7. $4(4i + 12)(3 - 2i)$

$(16i + 48)(3 - 2i)$

$48i - 32i^2 + 144 - 96i$

$\boxed{-48i + 176}$

8. $(11 - 3i)(11 + 3i)$

$121 + 33i - 33i - 9i^2$

$121 + 9 = \boxed{130}$

Notice in our last example what happened to *i*. This is because the expressions are known as <u>conjugates</u>. We will use this property to divide (or rationalize) complex numbers. REMEMBER: Whatever we multiply with the denominator, we need to also multiply with the numerator!!

Model: $\frac{3}{9-i} * \boxed{\frac{(9+i)}{(9+i)}} = \frac{27+3i}{9^2+1^2} = \frac{27}{82} + \frac{3}{82}i$

Examples: Write the following in standard form.

9. $\frac{(3-i)}{2+7i} \cdot \frac{(2-7i)}{(2-7i)}$

$\frac{6-21i-2i+14i^2}{4+49}$

$\boxed{\frac{-8-23i}{53}}$

10. $\frac{8i}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{8i+16i^2}{1+4}$

$\boxed{\frac{-16+8i}{5}}$

11. $\frac{(5+2i)}{3-i} \cdot \frac{(3+i)}{3+i} = \frac{15+5i+6i+2i^2}{9+1}$

$\frac{15+11i-2}{10}$

$\boxed{\frac{13+11i}{10}}$

12. $\frac{2-3i}{2i} \cdot \frac{i}{i} \quad \frac{2i+3}{2i^2} = \frac{2i+3}{-2} = \boxed{\frac{3+2i}{-2}}$

CCGPS Analytic Geometry

Homework: Multiplying and Dividing Complex Numbers

1. $(12 + 7i)(2 - 4i)$

$24 - 48i + 14i - 28i^2$

$24 - 34i + 28$

$\boxed{52 - 34i}$

2. $(3 + 8i)(11 - 5i)$

3. $(4i - 6i)(9i + 10)$

4. $6(1 - i)(1 + i)$

$6(1 + i^2)$

$\boxed{12}$

5. $\frac{3+2i}{1-3i} \cdot \frac{(1+3i)}{(1+3i)}$

$\frac{3 + 9i + 2i + 6i^2}{1 + 9} = \boxed{\frac{-3 + 11i}{10}}$

6. $\frac{6-4i}{4+5i} \cdot \frac{(4-5i)}{(4-5i)}$

$\frac{24 - 30i - 16i + 20i^2}{16 + 25}$

7. $\frac{2-3i}{i} \cdot \frac{i}{i}$

$\frac{2i - 3i^2}{i^2}$

$\frac{2i + 3}{-1}$

$\boxed{-(3 + 2i)}$

8. $\frac{4+5i}{2-i}$

9. $\frac{6+7i}{3+2i}$