

**Non-AP Calculus 3.1-3.4 Quiz Review WS #1**

1)

Find the absolute maximum and absolute minimum of the function  $f$  on the given interval:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \text{ on } [-4, 4]$$

2)

If  $f(x) = \frac{x^2 - 6x}{x + 2}$ , does Rolle's theorem apply to the function on the interval  $[0, 6]$ . If yes, find the value of  $c$  defined by Rolle's theorem.

3)

Determine whether the Mean Value Theorem can be applied to  $f(x) = \frac{x+1}{x}$  on  $\left[\frac{1}{2}, 2\right]$ .

If so, then find all values of  $c$  on  $(a, b)$  defined by MVT.

4) If  $f(x) = \frac{1}{4}x^4 - 2x^3 + 6$  find the following (where appropriate):

a) Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s) with because statements.

b) Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers with because statements)

5)

Sketch a labeled graph with the following characteristics:

a)  $f(-1) = 4$  and  $f(2) = -1$

b)  $f'(x) > 0$  if  $x < -1$  and if  $x > 2$

c)  $f'(x) < 0$  if  $-1 < x < 2$

d) POI at point  $(1, 2)$

e)  $f''(x) < 0$  if  $x < 1$

f)  $f''(x) > 0$  if  $x > 1$

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Key

1)

Find the absolute maximum and absolute minimum of the function  $f$  on the given interval:

$f(x) = \frac{x^2-4}{x^2+4}$  on  $[-4,4]$

$f(x)$  continuous on  $[-4,4]$   
 $f'$  ,  $f$  ,  $-f$  ,  $f'$

Steps:

- 1) confirm continuity on  $[a,b]$
- 2) find critical points from  $f'(x)$
- 3) Test critical points, endpoints

$$f'(x) = \frac{(2x)(x^2+4) - (x^2-4)(2x)}{(x^2+4)^2} = \frac{2x^3+8x - 2x^3+8x}{(x^2+4)^2}$$

$$f'(x) = \frac{16x}{(x^2+4)^2}$$

$$16x = 0 \quad | \quad (x^2+4)^2 \neq 0$$

$$\boxed{x=0}$$

$$f(-4) = \frac{(-4)^2-4}{4^2+4} = \frac{3}{5}$$

$$f(4) = \frac{(4)^2-4}{4^2+4} = \frac{3}{5}$$

$$f(0) = -1$$

Abs. min is  $-1$  at  $x=0$   
 Abs. max is  $\frac{3}{5}$  at  $x=4, x=-4$

2)

If  $f(x) = \frac{x^2-6x}{x+2}$ , does Rolle's theorem apply to the function on the interval  $[0,6]$ . If yes, find the value of  $c$  defined by Rolle's theorem. VA:  $x=-2$

$f(x)$  continuous on  $[0,6]$   
 $f(x)$  differentiable on  $(0,6)$

$$f(0) = 0$$

$$f(6) = 0 \quad ] \checkmark$$

\* set  $f'(x) = 0$

$$f'(x) = \frac{(2x-6)(x+2) - (x^2-6x)(1)}{(x+2)^2} = \frac{2x^2-2x-12-x^2+6x}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-12}{(x+2)^2}$$

$$\boxed{c=2}$$

\* set  $x^2+4x-12=0$

$$\begin{array}{r} 6 \\ \times \\ \hline 12 \\ \times \\ \hline 12 \\ \times \\ \hline 0 \end{array} \quad (x+6)(x-2) = 0$$

$$x = -6, x = 2$$

3)

Determine whether the Mean Value Theorem can be applied to  $f(x) = \frac{x+1}{x}$  on  $[\frac{1}{2}, 2]$ .

If so, then find all values of  $c$  on  $(a, b)$  defined by MVT. VA:  $x=0$

$f(x)$  continuous  $[\frac{1}{2}, 2]$   
 $f(x)$  differentiable  $(\frac{1}{2}, 2)$

\* set  $f'(c) = \frac{f(b)-f(a)}{b-a}$

$$f(\frac{1}{2}) = \frac{\frac{1}{2}+1}{\frac{1}{2}} = 3$$

$$f(2) = \frac{2+1}{2} = \frac{3}{2}$$

$$\text{slope} = \frac{3 - \frac{3}{2}}{\frac{1}{2} - 2} = \frac{\frac{3}{2}}{-\frac{3}{2}} = \boxed{-1} \quad \leftarrow \text{target slope}$$

$$f'(x) = \frac{(1)(x) - (x+1)(1)}{x^2} = \frac{x-x-1}{x^2} = \frac{-1}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

$$-1 = \frac{-1}{x^2}$$

$$\begin{array}{l} -x^2 = -1 \\ x^2 = 1 \\ x = \pm 1 \end{array}$$

$$c = 1, c = -1$$

$$\boxed{c=1}$$

4) If  $f(x) = \frac{1}{4}x^4 - 2x^3 + 6$  find the following (where appropriate):

a) Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s) with because statements.

$$f'(x) = \frac{1}{4} \cdot 4x^3 - 6x^2$$

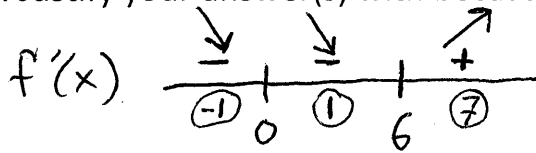
$$f'(x) = x^3 - 6x^2$$

$$f'(x) = x^2(x-6)$$

$$0 = x^2(x-6)$$

$$x^2=0 \quad | \quad x-6=0$$

$$\boxed{x=0} \quad | \quad \boxed{x=6}$$



Relative minimum at  $(6, f(6))$  b/c  $f'(x)$  changes from  $-$  to  $+$

$f(x)$  decreasing  $(-\infty, 0), (0, 6)$  b/c  $f'(x) < 0$

$f(x)$  increasing  $(6, \infty)$  b/c  $f'(x) > 0$

b) Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers with because statements)

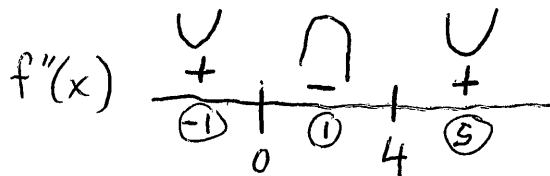
$$f''(x) = 3x^2 - 12x$$

$$f''(x) = 3x(x-4)$$

$$0 = 3x(x-4)$$

$$3x=0 \quad | \quad x-4=0$$

$$\boxed{x=0} \quad | \quad \boxed{x=4}$$



$f(x)$  concave up  $(-\infty, 0), (4, \infty)$  b/c  $f''(x) > 0$

$f(x)$  concave down  $(0, 4)$  b/c  $f''(x) < 0$

POI at  $(0, f(0))$  and  $(4, f(4))$  b/c  $f''(x)$  change signs

5)

Sketch a labeled graph with the following characteristics:

a)  $f(-1) = 4$  and  $f(2) = -1$

b)  $f'(x) > 0$  if  $x < -1$  and if  $x > 2$  (slope is positive left of  $x = -1$  and right of  $x = 2$ )

c)  $f'(x) < 0$  if  $-1 < x < 2$  negative slope between  $-1 < x < 2$

d) POI at point  $(1, 2)$

e)  $f''(x) < 0$  if  $x < 1$  concave down left of  $x = 1$

f)  $f''(x) > 0$  if  $x > 1$  concave up right of  $x = 1$

