Non-AP Calculus 3.1-3.4 Quiz Review WS #1

1)

Find the absolute maximum and absolute minimum of the function f on the given interval:

$$f(x) = \frac{x^2-4}{x^2+4} \text{ on } [-4,4]$$

2)

If $f(x) = \frac{x^2-6x}{x+2}$, does Rolle's theorem apply to the function on the interval [0,6]. If yes, find the value of c defined by Rolle's theorem.

3)

Determine whether the Mean Value Theorem can be applied to $f(x) = \frac{x+1}{x}$ on $\left[\frac{1}{2}, 2\right]$.

If so, then find all values of c on (a, b) defined by MVT.

4) If $f(x) = \frac{1}{4}x^4 - 2x^3 + 6$ find the following (where appropriate):

a) Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s) with because statements.

b) Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers with because statements)

5)

Sketch a labeled graph with the following characteristics:

a) $f(-1) = 4$ and $f(2) = -1$
b) $f'(x) > 0$ if $x < -1$ and if $x > 2$
c) $f'(x) < 0$ if $-1 < x < 2$
d) POI at point (1, 2)
e) $f''(x) < 0$ if $x < 1$
f) $f''(x) > 0$ if $x > 1$

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Key

1)

Find the absolute maximum and absolute minimum (value) of the function f on the given interval:

$f(x) = \frac{x^2-4}{x^2+4}$ on [-4,4]

f(x) continuous on $[-4,4]$

Steps:
1) confirm continuity on $[a,b]$
2) find critical points from $f'(x)$
3) Test critical points, endpoints

$$f'(x) = \frac{(2x)(x^2+4)-(x^2-4)(2x)}{(x^2+4)^2} = \frac{\cancel{2x^3}+8x-\cancel{2x^3}+8x}{(x^2+4)^2}$$

$$\boxed{f'(x) = \frac{16x}{(x^2+4)^2}}$$

$16x=0$

$\boxed{x=0}$

$(x^2+4)^2 \neq 0$

$f(-4) = \frac{(-4)^2-4}{4^2+4} = \frac{3}{5}$

$f(4) = \frac{(4)^2-4}{4^2+4} = \frac{3}{5}$

$f(0) = -1$

Abs. min is -1 at $x=0$

Abs. max is $\frac{3}{5}$ at $x=4, x=-4$

2)

If $f(x) = \frac{x^2-6x}{x+2}$, does Rolle's theorem apply to the function on the interval [0,6]. If yes, find the value of c defined by Rolle's theorem. VA: $x=-2$

f(x) continuous on $[0,6]$

f(x) differentiable on $(0,6)$

$f(0)=0$

$f(6)=0$ ✓

* set $f'(x)=0$

$$f'(x) = \frac{(2x-6)(x+2)-(x^2-6x)(1)}{(x+2)^2} = \frac{2x^2-2x-12-x^2+6x}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-12}{(x+2)^2}$$

* set $x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6, x=2$

$\boxed{c=2}$

3)

Determine whether the Mean Value Theorem can be applied to $f(x) = \frac{x+1}{x}$ on $\left[\frac{1}{2}, 2\right]$.

If so, then find all values of c on (a, b) defined by MVT. VA: $x=0$

f(x) continuous $[1/2, 2]$

f(x) differentiable $(1/2, 2)$

* set $f'(c) = \frac{f(b)-f(a)}{b-a}$

$f(\frac{1}{2}) = \frac{1/2+1}{1/2} = 3$

$f(2) = \frac{2+1}{2} = \frac{3}{2}$

$\text{slope} = \frac{3-3/2}{1/2-2} = \frac{3/2}{-3/2} = \boxed{-1}$ ← target slope

$$f'(x) = \frac{(1)(x)-(x+1)(1)}{x^2} = \frac{x-x-1}{x^2} = \frac{-1}{x^2}$$

$$\boxed{f'(x) = \frac{-1}{x^2}}$$

$-1 = \frac{-1}{x^2}$

$-x^2=-1$

$x^2=1$

$x=\pm 1$

$c=1, \cancel{c=-1}$

$\boxed{c=1}$

4) If $f(x) = \frac{1}{4}x^4 - 2x^3 + 6$ find the following (where appropriate):

a) Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s) with because statements.

$f'(x) = \frac{1}{4} \cdot 4x^3 - 6x^2$

$f'(x) = x^3 - 6x^2$

$f'(x) = x^2(x-6)$

$0 = x^2(x-6)$

$x^2 = 0$ | $x - 6 = 0$

$\boxed{x=0}$ | $\boxed{x=6}$

$f'(x)$: $-$ (at -1) | 0 | $-$ (at 1) | 6 | $+$ (at 7)

Relative minimum at $(6, f(6))$ b/c $f'(x)$ changes from $-$ to $+$

$f(x)$ decreasing $(-\infty, 0), (0, 6)$ b/c $f'(x) < 0$

$f(x)$ increasing $(6, \infty)$ b/c $f'(x) > 0$

b) Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers with because statements)

$f''(x) = 3x^2 - 12x$

$f''(x) = 3x(x-4)$

$0 = 3x(x-4)$

$3x = 0$ | $x - 4 = 0$

$\boxed{x=0}$ | $\boxed{x=4}$

$f''(x)$: $+$ (at -1) $\cup$ | 0 | $-$ (at 1) $\cap$ | 4 | $+$ (at 5) $\cup$

$f(x)$ concave up $(-\infty, 0), (4, \infty)$ b/c $f''(x) > 0$

$f(x)$ concave down $(0, 4)$ b/c $f''(x) < 0$

POI at $(0, f(0))$ and $(4, f(4))$ b/c $f''(x)$ change signs

5)

Sketch a labeled graph with the following characteristics:

a) f(-1) = 4 and f(2) = -1

b) f'(x) > 0 if x < -1 and if x > 2 (slope is positive left of 1 and right of $x=2$)

c) f'(x) < 0 if -1 < x < 2 negative slope between $-1 < x < 2$

d) POI at point (1, 2)

e) f''(x) < 0 if x < 1 concave down left of $x=1$

f) f''(x) > 0 if x > 1 concave up right of $x=1$

$f'(x)$: $+$ | -1 | $-$ | 2 | $+$

$f''(x)$: $-$ | 1 | $+$