AP Calculus AB **Quiz Review 3.1-3.4 Worksheet #3**

1) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 2x^2$ on the interval [-1, 3] . State theorem and conditions

2) If $f(x) = x^2 - 4x$ on [-1,5], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

3) If $g(x) = x^3 + x$ on [-2, 1], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

4) If $f(x) = \frac{4}{3}x^3 + \frac{7}{2}x^2 - 2x + 1$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

5) If $f(x) = x^4 - 4x^3 + 1$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

6) Sketch a labeled graph of a function, f, with the following characteristics:
$f(0) = 4$, $f(6) = 0$, $f(2) = 0$, $f(4) = 2$
$f'(x) < 0$ for $x < 2$ or $x > 4$
$f'(2)$ does not exist
$f'(4) = 0$
$f'(x) > 0$ for $2 < x < 4$
$f''(x) < 0$ for $x \neq 2$

Key

AP Calculus AB **Quiz Review 3.1-3.4 Worksheet #3**

1) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 2x^2$ on the interval [-1, 3] . State theorem and conditions

$f(x)$ continuous $[-1, 3]$

$f'(x) = 3x^2 + 4x$

$0 = x(3x+4)$

$x = 0$ | $3x+4=0$

$x=0$, $\cancel{x=-4/3}$

outside the interval

$f(-1) = 1$

$f(0) = 0$

$f(3) = 45$

Abs max value is 45 at $x=3$

Abs min value is 0 at $x=0$

2) If $f(x) = x^2 - 4x$ on [-1,5], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

$f(x)$ continuous $[-1,5]$ $f(x)$ differentiable on $(-1,5)$

$f(-1) = 5$

$f(5) = 5$ ✓

set $f'(x) = 0$

$f'(x) = 2x - 4$

$2x - 4 = 0$

$2x = 4$

$x = 2$

$\boxed{c = 2}$

3) If $g(x) = x^3 + x$ on [-2, 1], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$ continuous $[-2,1]$ $g(x)$ differentiable $(-2,1)$

set $g'(x) = \frac{g(b)-g(a)}{b-a}$

$g'(x) = \frac{g(1)-g(-2)}{1--2}$

$g(1) = 2$

$g(-2) = -10$

$\frac{g(1)-g(-2)}{1--2} = \frac{2--10}{1--2} = \frac{12}{3} = 4$ target slope

$g'(x) = 3x^2 + 1$

$3x^2 + 1 = 4$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\cancel{c=1}$, $\boxed{c = -1}$

4) If $f(x) = \frac{4}{3}x^3 + \frac{7}{2}x^2 - 2x + 1$ find the following (where appropriate):
Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$f'(x) = \frac{4}{3} \cdot 3x^2 + \frac{7}{2} \cdot 2x - 2$

$f'(x) = 4x^2 + 7x - 2$

$f'(x) = 4x^2 + 7x - 2$

$8 \quad -8 \quad -1 \quad 4 \quad 7 \quad 4$ $f'(x) = (x+2)(x - 1/4)$

$f'(x) = (x+2)(4x-1)$

$x = -2, \; x = 1/4$

$f'(x)$: $+$ | $-$ | $+$ at -2, $1/4$; test values -3, 0, 1

$f(x)$ increasing $(-\infty, -2), (1/4, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-2, 1/4)$ b/c $f'(x) < 0$

Rel. max $(-2, f(-2))$ b/c $f'(x)$ changes from $+$ to $-$

Rel. min $(1/4, f(1/4))$ b/c $f'(x)$ changes from $-$ to $+$

6) If $f(x) = x^4 - 4x^3 + 1$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 12x(x-2)$

$0 = 12x(x-2)$

$12x = 0$ | $x - 2 = 0$

$\boxed{x = 0}$ | $\boxed{x = 2}$

$f''(x)$: $+$ | $-$ | $+$ at 0, 2; test values -1, 1, 3

$f(x)$ is concave up $(-\infty, 0), (2, \infty)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(0, 2)$ b/c $f''(x) < 0$

POI at $(0, f(0))$ and $(2, f(2))$ b/c $f''(x)$ change signs.

7) Sketch a labeled graph of a function, f, with the following characteristics:

f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2

f '(x) < 0 for x < 2 or x > 4 negative slope left of 2 and right of 4

f '(2) does not exist slope at 2 does not exist

f '(4) = 0 slope at x = 4 is zero

f '(x) > 0 for 2 < x < 4 positive slope between x = 2 and x = 4

f ''(x) < 0 for x ≠ 2 concave down everywhere except at x = 2

$f'(x)$: $-$ | $+$ | $-$ at 2, 4

$f''(x)$: $-$ | $-$ at 2