



4) If  $f(x) = \frac{4}{3}x^3 + \frac{7}{2}x^2 - 2x + 1$  find the following (where appropriate):  
Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

5) If  $f(x) = x^4 - 4x^3 + 1$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

6) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2$$

$$f'(x) < 0 \text{ for } x < 2 \text{ or } x > 4$$

$f'(2)$  does not exist

$$f'(4) = 0$$

$$f'(x) > 0 \text{ for } 2 < x < 4$$

$$f''(x) < 0 \text{ for } x \neq 2$$

- 1) Find the value(s) of the absolute extrema of the function  $f(x) = x^3 + 2x^2$  on the interval  $[-1, 3]$ . State theorem and conditions

$f(x)$  continuous  $[-1, 3]$

$$f'(x) = 3x^2 + 4x$$

$$0 = x(3x + 4)$$

$$x = 0 \mid 3x + 4 = 0$$

$$x = 0, \quad x = -\frac{4}{3}$$

outside the interval

$$f(-1) = 1$$

$$f(0) = 0$$

$$f(3) = 45$$

Abs max value is 45  
at  $x = 3$

Abs min value is 0 at  $x = 0$

- 2) If  $f(x) = x^2 - 4x$  on  $[-1, 5]$ , determine if Rolle's Theorem can be applied. If yes, find the value(s) of  $c$  defined on Rolle's Theorem. State conditions and show steps.

$f(x)$  continuous  $[-1, 5]$   $f(x)$  differentiable on  $(-1, 5)$

$$f(-1) = 5$$

$$f(5) = 5$$

set  $f'(x) = 0$

$$f'(x) = 2x - 4$$

$$2x - 4 = 0$$

$$2x = 4$$

$$x = 2$$

$$c = 2$$

- 3) If  $g(x) = x^3 + x$  on  $[-2, 1]$ , determine if the Mean Value Theorem can be applied. If yes, find the value(s) of  $c$  defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$  continuous  $[-2, 1]$   $g(x)$  differentiable  $(-2, 1)$

$$\text{set } g'(x) = \frac{g(b) - g(a)}{b - a}$$

$$g'(x) = \frac{g(1) - g(-2)}{1 - (-2)}$$

$$g(1) = 2$$

$$g(-2) = -10$$

$$\frac{g(1) - g(-2)}{1 - (-2)} = \frac{2 - (-10)}{1 - (-2)} = \frac{12}{3} = 4$$

target slope

$$g'(x) = 3x^2 + 1$$

$$3x^2 + 1 = 4$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$$c \neq 1, \quad c = -1$$

4) If  $f(x) = \frac{4}{3}x^3 + \frac{7}{2}x^2 - 2x + 1$  find the following (where appropriate):

Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = \frac{4}{3} \cdot 3x^2 + \frac{7}{2} \cdot 2x - 2$$

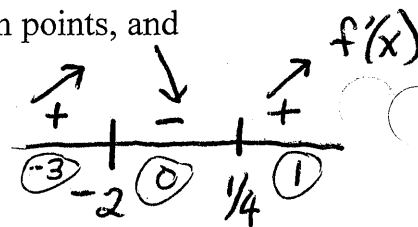
$$f'(x) = 4x^2 + 7x - 2$$

$$f''(x) = 8x + 7$$

$$f(x) = (x+2)(x-\frac{1}{4})$$

$$f'(x) = (x+2)(4x-1)$$

$$x = -2, x = \frac{1}{4}$$



$f(x)$  increasing  $(-\infty, -2), (\frac{1}{4}, \infty)$  b/c  $f'(x) > 0$

$f(x)$  decreasing  $(-2, \frac{1}{4})$  b/c  $f'(x) < 0$

Rel. max  $(-2, f(-2))$  b/c  $f'(x)$  changes from + to -

Rel. min  $(\frac{1}{4}, f(\frac{1}{4}))$  b/c  $f'(x)$  changes from - to +

6) If  $f(x) = x^4 - 4x^3 + 1$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

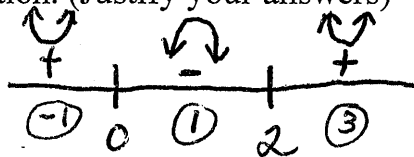
$$f''(x) = 12x(x-2)$$

$$0 = 12x(x-2)$$

$$12x = 0 \quad | \quad x - 2 = 0$$

$$x = 0 \quad | \quad x = 2$$

$$f''(x)$$



$f(x)$  is concave up  $(-\infty, 0), (2, \infty)$  b/c  $f''(x) > 0$

$f(x)$  is concave down  $(0, 2)$  b/c  $f''(x) < 0$

POI at  $(0, f(0))$  and  $(2, f(2))$  b/c  $f''(x)$  change signs.

7) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2$$

$f'(x) < 0$  for  $x < 2$  or  $x > 4$  negative slope left of 2 and right of 4

$f'(2)$  does not exist slope at 2 does not exist

$f'(4) = 0$  slope at  $x = 4$  is zero

$f'(x) > 0$  for  $2 < x < 4$  positive slope between  $x = 2$  and  $x = 4$

$f''(x) < 0$  for  $x \neq 2$  concave down everywhere except at  $x = 2$

