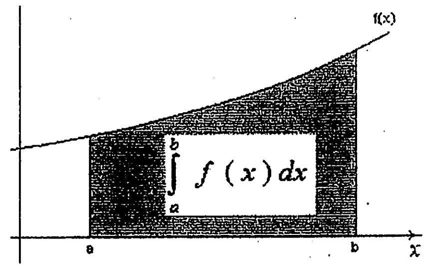
1

**Calculus** **Chapters 4.3/4.4a** **1st Fundamental Theorem of Calculus**

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is the antiderivative of f.




Recall:

*The general derivative is a **slope-finding function** or formula : (ex. f '(x) = 2x + 1)

*The specific derivative is the **actual slope** at a point (ex: f '(3) = 7)

Likewise...

The indefinite integral is an **Area-Finding Function** or formula (Ex: $\int 2xdx = x^2 + C$ )

The definite integral is the **Actual Area** of the region for an interval (Ex: $\int_1^3 2xdx = 8$ )

*If a function is **continuous** on a closed interval, then the function is able to be integrated on that interval

**Class Examples:**

1. Evaluate $\int_1^4 (3x^2 + 4x - 1)dx$

**NOTE: For definite integrals, we don't need to worry about the constant of integration " +C ". It will always wash out.

2. Evaluate $\int_{-2}^1 2xdx$

2

**Integral Properties:**

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (given that c is between a and b)

**Example 3:** If $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, find the below:

a) $\int_0^6 f(x)dx$

b) $\int_6^3 f(x)dx$

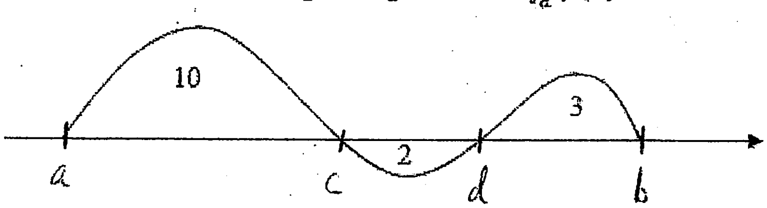
c) $\int_3^3 f(x)dx$

d) $\int_3^6 (-5f(x)+3)dx$

**Ex. 4:** If $\int_3^8 f'(x)dx = 10$ and $f(8) = 6$, find $f(3)$.

*Reminder that the FFTC can be used as an equation solving tool to find the value of an antiderivative at a specific point. Recall that $\int_a^b f'(x)dx = f(b) - f(a)$

**Ex. 5:** The area for each region is given. Find $\int_a^b f(x)dx$



# Non-AP Calculus Ch. 4.3-4.4 Definite Integrals Classwork Problems

**41. Using Properties of Definite Integrals** Given

$$\int_0^5 f(x)\,dx = 10 \quad \text{and} \quad \int_5^7 f(x)\,dx = 3$$

evaluate

(a) $\int_0^7 f(x)\,dx.$

(b) $\int_5^0 f(x)\,dx.$

(c) $\int_5^5 f(x)\,dx.$

(d) $\int_0^5 3f(x)\,dx.$

**42. Using Properties of Definite Integrals** Given

$$\int_0^3 f(x)\,dx = 4 \quad \text{and} \quad \int_3^6 f(x)\,dx = -1$$

evaluate

(a) $\int_0^6 f(x)\,dx.$

(b) $\int_6^3 f(x)\,dx.$

(c) $\int_3^3 f(x)\,dx.$

(d) $\int_3^6 -5f(x)\,dx.$

**43. Using Properties of Definite Integrals** Given

$$\int_2^6 f(x)\,dx = 10 \quad \text{and} \quad \int_2^6 g(x)\,dx = -2$$

evaluate

(a) $\int_2^6 [f(x) + g(x)]\,dx.$

(b) $\int_2^6 [g(x) - f(x)]\,dx.$

(c) $\int_2^6 2g(x)\,dx.$

(d) $\int_2^6 3f(x)\,dx.$

**9. Think About It** Consider the function $f$ that is continuous on the interval $[-5, 5]$ and for which

$$\int_0^5 f(x)\,dx = 4.$$

Evaluate each integral.

(a) $\int_0^5 [f(x) + 2]\,dx$

④

**Evaluating a Definite Integral** **In Exercises 5–34, evaluate the definite integral. Use a graphing utility to verify your result.**

**8.** $\int_{-1}^{2} (7 - 3t)\, dt$

**10.** $\int_{1}^{2} (6x^2 - 3x)\, dx$

**13.** $\int_{1}^{2} \left(\frac{3}{x^2} - 1\right) dx$

**16.** $\int_{-8}^{8} x^{1/3}\, dx$

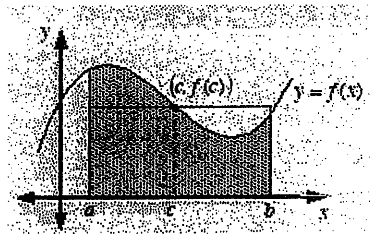
**17.** $\int_{-1}^{1} (\sqrt[3]{t} - 2)\, dt$

**18.** $\int_{1}^{8} \sqrt{\frac{2}{x}}\, dx$

**Calculus** **Chapter 4.4b** **Average Value Theorem (MVT for Integrals)**

…unction $f$ is integrable on the closed interval [a, b], then the average value of $f$ on the interval is

$$f(c)=\frac{1}{b-a}\int_a^b f(x)dx$$



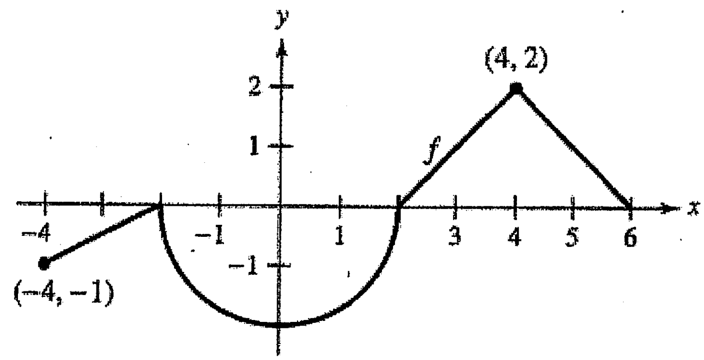

*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

**Example 1:** a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

6

## 4.3-4.4a Definite Integrals Using Graph Classwork Problems

**47. Think About It** The graph of $f$ consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.




(a) $\int_0^2 f(x)\,dx$

(b) $\int_2^6 f(x)\,dx$

(c) $\int_{-4}^2 f(x)\,dx$

(d) $\int_{-4}^6 f(x)\,dx$

(e) $\int_{-4}^6 |f(x)|\,dx$

(f) $\int_{-4}^6 [f(x) + 2]\,dx$

## Chapter 4.4b Average Value Theorem Classwork Problems

**Finding the Average Value of a Function** In Exercises 51–56, find the average value of the function over the given interval and all values of $x$ in the interval for which the function equals its average value.

**51.** $f(x) = 9 - x^2, \quad [-3, 3]$

**52.** $f(x) = \dfrac{4(x^2 + 1)}{x^2}, \quad [1, 3]$

**53.** $f(x) = x^3, \quad [0, 1]$

**54.** $f(x) = 4x^3 - 3x^2, \quad [0, 1]$

**55.** $f(x) = \sin x, \quad [0, \pi]$

**56.** $f(x) = \cos x, \quad \left[0, \frac{\pi}{2}\right]$

8

## 4.5a U-Substitution Classwork Problems

**Finding an Indefinite Integral** **In Exercises 5–26, find the indefinite integral and check the result by differentiation.**

**6.** $\int (x^2 - 9)^3(2x)\, dx$

**7.** $\int \sqrt{25 - x^2}\,(-2x)\, dx$

**8.** $\int \sqrt[3]{3 - 4x^2}(-8x)\, dx$

**15.** $\int 5x \sqrt[3]{1 - x^2}\, dx$

**20.** $\int \frac{6x^2}{(4x^3 - 9)^3}\, dx$

**22.** $\int \frac{x^3}{\sqrt{1 + x^4}}\, dx$

23. $\int \left(1 + \frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$

24. $\int \left[x^2 + \frac{1}{(3x)^2}\right] dx$

33. $\int \pi \sin \pi x \, dx$

36. $\int \csc^2\left(\frac{x}{2}\right) dx$

37. $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} \, d\theta$

40. $\int \sqrt{\tan x} \sec^2 x \, dx$

41. $\int \frac{\csc^2 x}{\cot^3 x} \, dx$

42. $\int \frac{\sin x}{\cos^3 x} \, dx$

10

## 4.5b U-Substitution Change of Variable

**Change of Variables** **In Exercises 47–54, find the indefinite integral by the method shown in Example 5.**

**47.** $\int x\sqrt{x+6}\,dx$

**48.** $\int x\sqrt{3x-4}\,dx$

**49.** $\int x^2\sqrt{1-x}\,dx$

**50.** $\int (x+1)\sqrt{2-x}\,dx$

**Calculus** **Chapter 4.5a** **U-Substitution Method for Indefinite Integrals**

U-Substitution is the reverse of the derivative chain rule. We want to look for a function and its derivative to be in the integral.

Example 1:

*Suppose* $f(x) = \sin(3x)$

| | |
|---|---|
| $f'(x) = \cos(3x) \cdot 3$<br>$f'(x) = 3\cos(3x)$ | This means that:<br>$\int 3\cos(3x)dx = \sin(3x) + C$ |

**U-Substitution Steps:**

1. Assign the 'u' value to the expression inside the parentheses.
2. Find the derivative of u: $\frac{du}{dx}$
3. Solve for dx.
4. Rewrite the integral in terms of u and du. (Check to make sure no 'x' or 'dx' remains)
5. Evaluate the Integral
6. Write the answer in terms of x.
7. **Not all Integral problems require U-Substitution. Check first to see if expansion/rewriting problem will allow problem to only need the Power Rule**

**Ex. 2:** $\int x(x^2+1)^{15}dx$

**Ex. 3:** $\int x^2 \sec^2(2x^3)dx$

**Ex. 4:** $\int x^3\sqrt{5-x^4}\,dx$

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**Ex. 5:** $\int \tan^5 x \sec^2 x \, dx$

**Ex. 6:** $\int (3-y)\left(\frac{1}{\sqrt{y}}\right)dy$

**Change of Variable U-Substitution Method:**

**Ex. 7:** $\int x\sqrt{x+3}\,dx$

**Ex. 8:** $\int x^2\sqrt{2-x}\,dx$

**Calculus** **Chapter 4.5b** **U-Substitution Method for Definite Integrals**

U-Substitution with definite integrals: Be sure the bounds matches the variable of the problem

Ex. 1: $\int_{1}^{2} 2x(x^2 - 2)^3\, dx$

Ex. 2: $\int_{1}^{5} \frac{x}{\sqrt{2x-1}} dx$

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## Integrals of Odd and Even Functions

Review: Suppose $\int_{10}^{3} f(x)dx = 9$ and $\int_{-1}^{3} f(x)dx = 5$, find $\int_{-1}^{10} f(x)dx$

Even/Odd Rules:

**Even:** $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

**Odd:** $\int_{-a}^{a} f(x)dx = 0$

**Ex. 3:** Suppose g(x) is an even function where $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

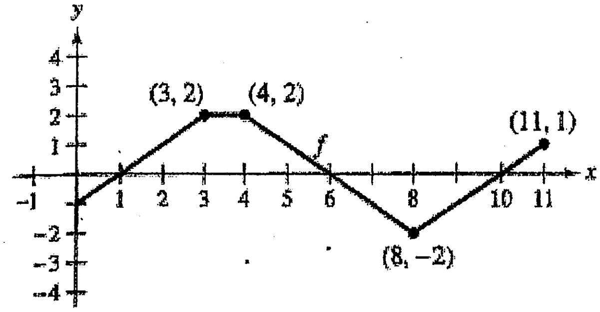
(Sketch a possible graph using the above given information)

**Ex. 4:** Same as Example 3, but g(x) is an odd function: $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

**Ex. 5:** If f(x) is even and $\int_{3}^{6} f(x)dx = 7$ and $\int_{-6}^{3} f(x)dx = 12$, find $\int_{0}^{6} f(x)dx$

# Ch. 4.3-4.4 Definite Integrals Selected Homework

**Think About It** The graph of $f$ consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.




(a) $\int_0^1 -f(x)\,dx$

(b) $\int_3^4 3f(x)\,dx$

(c) $\int_0^7 f(x)\,dx$

(d) $\int_5^{11} f(x)\,dx$

(e) $\int_0^{11} f(x)\,dx$

(f) $\int_4^{10} f(x)\,dx$

**Think About It** Consider the function $f$ that is continuous on the interval $[-5, 5]$ and for which

$$\int_0^5 f(x)\,dx = 4.$$

Evaluate each integral.

(a) $\int_0^5 [f(x) + 2]\,dx$

(b) $\int_{-2}^3 f(x + 2)\,dx$

(c) $\int_{-5}^5 f(x)\,dx$ ($f$ is even.)

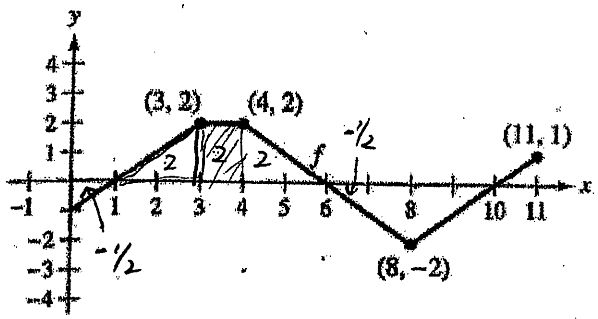
(d) $\int_{-5}^5 f(x)\,dx$ ($f$ is odd.)

16 -16

# Ch. 4.3-4.4 Definite Integrals Selected Homework

4.3

48) **Think About It** The graph of $f$ consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.




(a) $\int_0^1 -f(x)\,dx$ $= -(-\tfrac{1}{2}) = \boxed{\tfrac{1}{2}}$

(b) $\int_3^4 3f(x)\,dx$ $3\int_3^4 f(x)\,dx = 3(2) = \boxed{6}$

(c) $\int_0^7 f(x)\,dx$ $6 - 1 = \boxed{5}$

(d) $\int_5^{11} f(x)\,dx$ $-4 + 1 = \boxed{-3}$

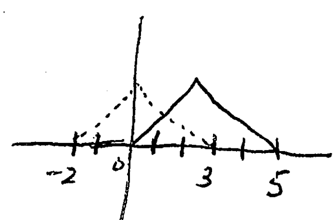
(e) $\int_0^{11} f(x)\,dx$ $-\tfrac{1}{2} + 6 - 4 + \tfrac{1}{2} = \boxed{2}$

(f) $\int_4^{10} f(x)\,dx$ $2 - 4 = \boxed{-2}$

49. **Think About It** Consider the function $f$ that is continuous on the interval $[-5, 5]$ and for which

$$\int_0^5 f(x)\,dx = 4.$$

$2x\Big]_0^5 = 2(5) - 2(0) = 10$




Evaluate each integral.

(a) $\int_0^5 [f(x) + 2]\,dx = \int_0^5 f(x)\,dx + \int_0^5 2\,dx$
$= 4 + 10 = \boxed{14}$

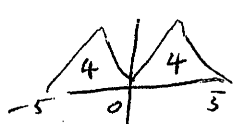
(b) $\int_{-2}^3 f(x+2)\,dx =$

$u = x+2$ $\frac{du}{dx} = 1$

convert bounds:
$x = -2,\ u = x+2 = -2+2 = 0$
$x = 3,\ u = x+2 = 3+2 = 5$

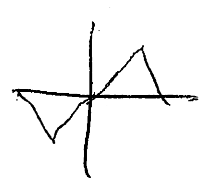
$\int_0^5 f(u)\,du = \boxed{4}$

(c) $\int_{-5}^5 f(x)\,dx$ ($f$ is even.)




$= \boxed{8}$

(d) $\int_{-5}^5 f(x)\,dx$ ($f$ is odd.)



$= \boxed{0}$