

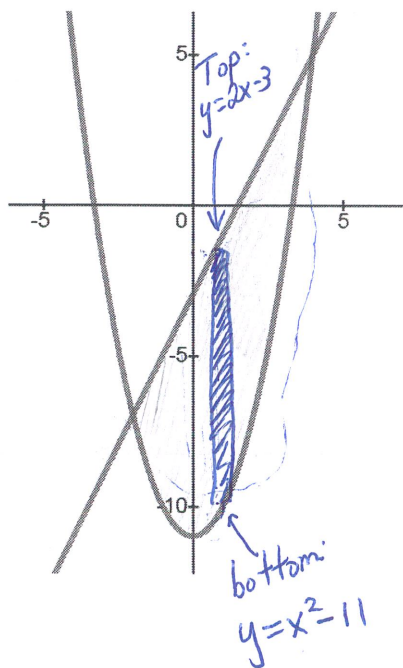
Ch. 7.1 Area between Curves  
Notes/Homework

$$\text{Area} = \int_{x_1}^{x_2} (\text{Top graph} - \text{Bottom graph}) dx$$

Key

- Steps: 1) Find intersection (bounds) by setting equations equal and solving for x.  
2) Identify Top graph and bottom graph: Set up to find Area using Integral Notation  
3) Evaluate bounds to find the Area of enclosed region between the 2 graphs or  
4) Enter into calculator to find Volume. (TI-84: Math 9 → FnInt or TI-36X Pro: 2<sup>nd</sup> → e)

Example 1: Find area between the 2 graphs:  $y = x^2 - 11$  and  $y = 2x - 3$



\* find bounds:  $x^2 - 11 = 2x - 3$

set equal to 0  
(factor and solve)

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

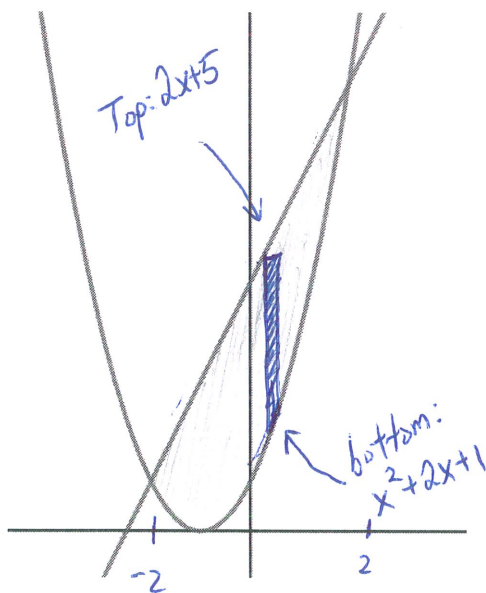
$x = 4, x = -2$  ← bounds for Integral

$$\text{Area} = \int_{-2}^4 \overbrace{(2x - 3)}^{\text{Top}} - \overbrace{(x^2 - 11)}^{\text{Bottom}} dx$$

Math → 9 → FnInt

Area = 36 units<sup>2</sup>

Example 2: Find the area between the 2 graphs:  $y = x^2 + 2x + 1$  and  $y = 2x + 5$



\* find bounds:  $x^2 + 2x + 1 = 2x + 5$

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0$$

$x = 2, -2$  ← bounds for Integral

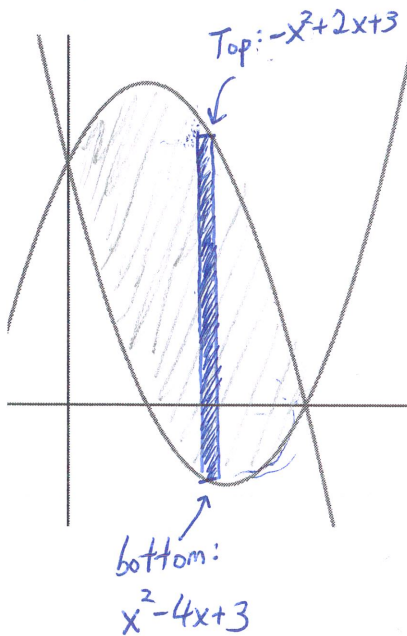
$$\text{Area} = \int_{-2}^2 \overbrace{2x + 5}^{\text{Top}} - \overbrace{(x^2 + 2x + 1)}^{\text{Bottom}} dx$$

Area =  $\frac{32}{3}$  units<sup>2</sup>

$$\text{Area} = \int_{x_1}^{x_2} (\text{Top graph} - \text{Bottom graph}) dx$$

- Steps:** 1) Find intersection (bounds) by setting equations equal and solving for  $x$ .  
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**Example 3:** Find the area between the 2 graphs:  $y = x^2 - 4x + 3$  and  $y = -x^2 + 2x + 3$



\*find bounds:  $x^2 - 4x + 3 = -x^2 + 2x + 3$

$$2x^2 - 6x = 0$$

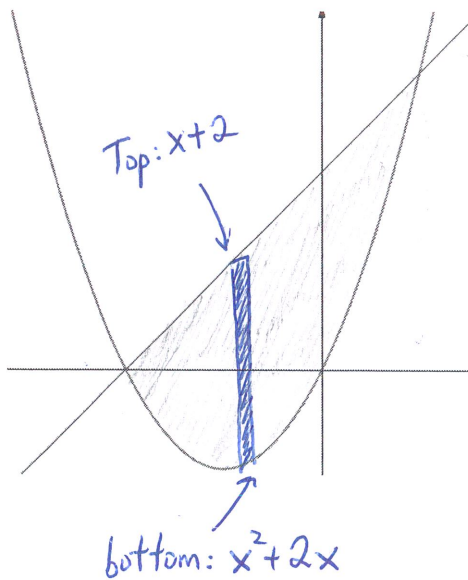
$$2x(x-3) = 0$$

$$x=0, x=3$$

$$\text{Area} = \int_0^3 \overbrace{-x^2 + 2x + 3}^{\text{Top}} - \overbrace{(x^2 - 4x + 3)}^{\text{Bottom}} dx$$

$$\boxed{\text{Area} = 9} \text{ units}^2$$

**Example 4:** Find the area between the 2 graphs:  $y = x^2 + 2x$  and  $y = x + 2$



\*find bounds:  $x^2 + 2x = x + 2$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, x = 1$$

$$\text{Area} = \int_{-2}^1 \overbrace{x+2}^{\text{Top}} - \overbrace{(x^2 + 2x)}^{\text{Bottom}} dx$$

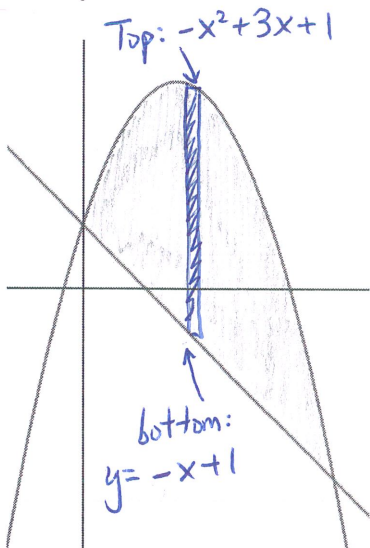
$$\boxed{\text{Area} = 4.5} \text{ units}^2$$

or  $9/2$

$$\text{Area} = \int_{x_1}^{x_2} (\text{Top graph} - \text{Bottom graph}) dx$$

- Steps:** 1) Find intersection (bounds) by setting equations equal and solving for  $x$ .  
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**Example 5:** Find the area between the 2 graphs:  $y = -x^2 + 3x + 1$  and  $y = -x + 1$



\*find bounds:  $-x + 1 = -x^2 + 3x + 1$

$$x^2 - 4x = 0$$

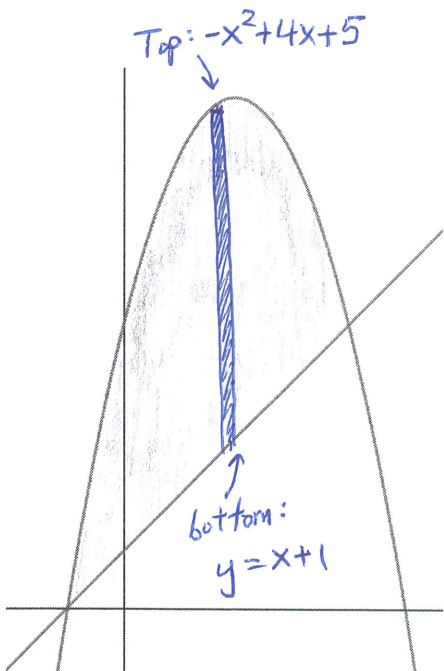
$$x(x - 4) = 0$$

$$\underline{x = 0, x = 4}$$

$$\text{Area} = \int_0^4 \overbrace{-x^2 + 3x + 1}^{\text{Top}} - \overbrace{(-x + 1)}^{\text{Bottom}} dx$$

$$\boxed{\text{Area} = \frac{32}{3} \text{ units}^2}$$

**Example 6:** Find the area between the 2 graphs:  $y = -x^2 + 4x + 5$  and  $y = x + 1$



\*find bounds:  $x + 1 = -x^2 + 4x + 5$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\underline{x = -1, 4}$$

$$\text{Area} = \int_{-1}^4 \overbrace{-x^2 + 4x + 5}^{\text{Top}} - \overbrace{(x + 1)}^{\text{Bottom}} dx$$

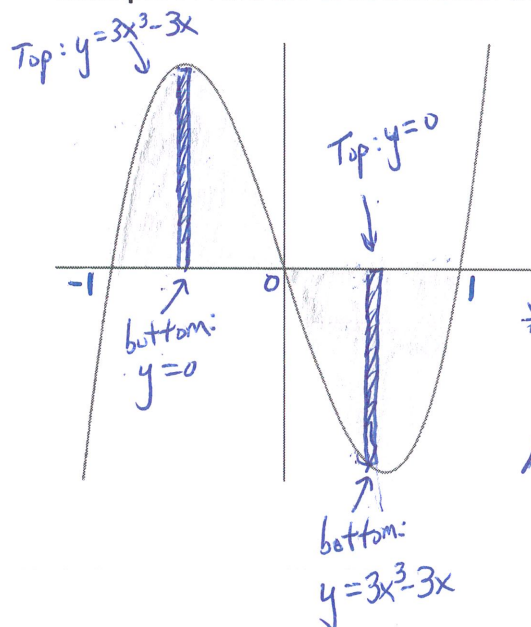
$$\boxed{\text{Area} = \frac{125}{6} \text{ units}^2}$$



$$\text{Area} = \int_{x_1}^{x_2} (\text{Top graph} - \text{Bottom graph}) dx$$

- Steps:** 1) Find intersection (bounds) by setting equations equal and solving for  $x$ .  
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**Example 7:** Find the area between the 2 graphs:  $y = 3x^3 - 3x$  and  $y = 0$



\* find bounds:  $3x^3 - 3x = 0$

$$3x(x^2 - 1) = 0$$

$$3x(x+1)(x-1) = 0$$

$$\underline{x = 0, -1, 1}$$

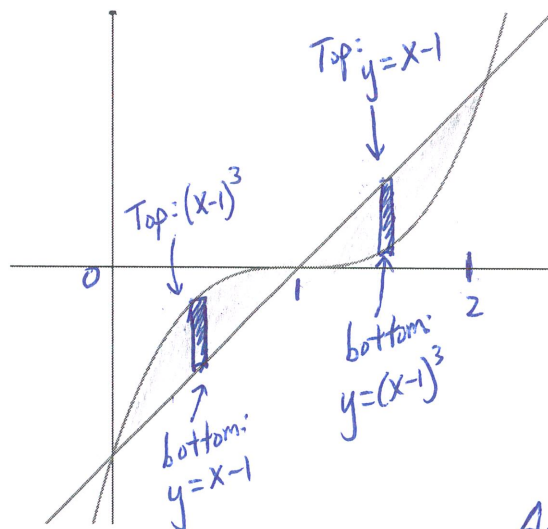
\* Create 2 separate integrals since top and bottom graphs alternated in the 2 regions.

$$\text{Area} = \int_{-1}^0 \overbrace{3x^3 - 3x}^{\text{Top}} - \underbrace{(0)}_{\text{Bottom}} dx + \int_0^1 \overbrace{0}^{\text{Top}} - \underbrace{(3x^3 - 3x)}_{\text{Bottom}} dx$$

$$0.75 + 0.75$$

$$\boxed{\text{Area} = 1.5 \text{ units}^2}$$

**Example 8:** Find the area between the 2 graphs:  $y = (x-1)^3$  and  $y = (x-1)$



\* find bounds:  $\underbrace{(x-1)^3}_{(x-1)} = \underbrace{(x-1)}_{(x-1)}$

$$(x-1)^2 = 1$$

$$x^2 - 2x + 1 = 1$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$\underline{x = 0, x = 2, x = 1}$$

\* 2 separate definite integrals needed:

$$\text{Area} = \int_0^1 (x-1)^3 - (x-1) dx + \int_1^2 (x-1) - (x-1)^3 dx$$

$$\text{Area} = 0.25 + 0.25$$

$$\boxed{\text{Area} = 0.5 \text{ units}^2}$$



## Instructions to use Online Calculator for Definite Integrals

- 1) Google "online definite integral calculator" to take you to website <https://www.integral-calculator.com/>
- 2) Use the Calculator Interface to enter in the Integrand Bar
- 3) Enter in the Bounds of Integration
- 4) and Click on "Go!"
- 5) Scroll to the bottom of page see Value of Definite Integral

The screenshot shows the Integral Calculator website interface. At the top, there is a logo with the text "Help slow down the virus!" and the title "Integral Calculator" with the tagline "Calculate integrals online - with steps and graphing!". There are links for "Also check the Derivative Calculator!" and language options: "Calculadora de Integrales en español", "Integralrechner auf Deutsch", and "Калькулятор Интегралов на Русском".

The main input area contains a text box with the expression  $(2x^2 - 4x) - (x + 1)$  and a "Go!" button. Below this, it says "This will be calculated:" and shows the definite integral  $\int_1^3 (2x^2 - 4x - (x + 1)) dx$ . A red arrow points to the input box with the handwritten note "Enter Expression here". Another red arrow points to the bounds of the integral with the handwritten note "Bounds of Integration".

On the right side, there are tabs for "About", "Help", "Examples", and "Options". Under "Options", there are several checkboxes: "Integrate numerically only?", "Simplify expressions?", "Simplify all roots?", "Use complex domain (C)?", and "Keep decimals?".

At the bottom of the page, the result is displayed as  $\frac{14}{3}$  and "Approximation: -4.666666666666667". A red arrow points to this result with the handwritten note "After clicking 'Go' Scroll to bottom of page to find value". A "Simplify" button is located below the approximation.