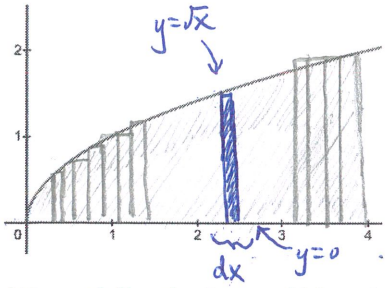
Key

Calculus Ch. 7.2a: Volume by Disc Method

Recall finding area under the curve $y = \sqrt{x}$ between [0, 4]. $Area = \int_a^b (Top\ graph - bottom\ graph)dx$



$$Area = \int_0^4 (\sqrt{x} - 0)\,dx \rightarrow \boxed{Area = 5.333\ units^2}$$

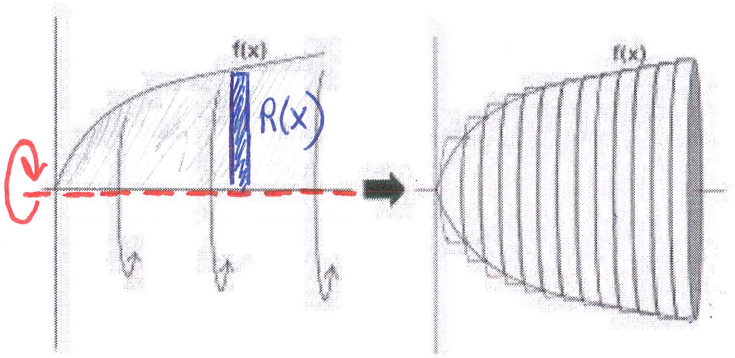
height, width

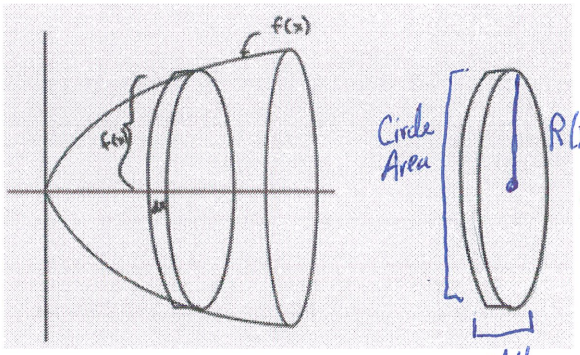
Rectangle Area = width x height

*Essentially, the Integral Notation allows us to add infinite numbers of differently sized rectangles to form area calculation.

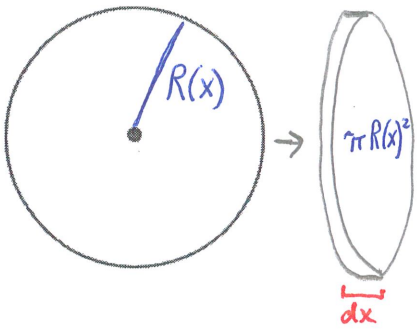
*Integral Calculus allows for accumulation of rectangles to form full Area of specified region

With **Disc Method**, we are going to take this region created by f(x) and the x-axis and rotate this function 360° around the x-axis. What shapes do you see if we were to separate the resulting object into thin slices? circular discs





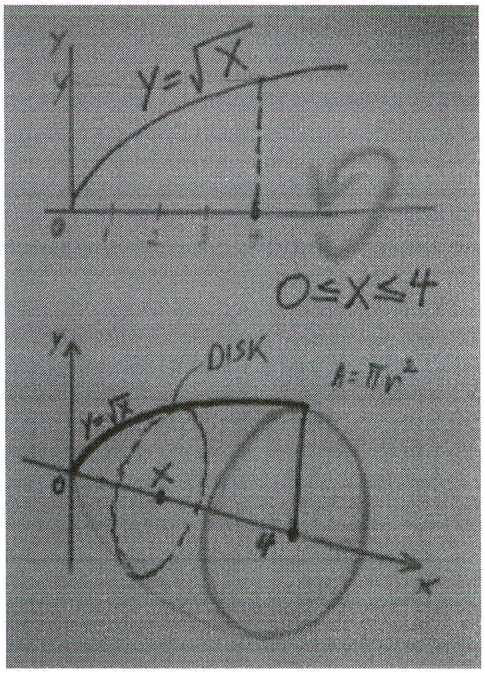
Area of Circle is πr^2



$\pi R(x)^2$

$\pi R(x)^2\,dx$

Area, width



$$\textbf{Volume (Disc Method)}: V = \int_a^b \pi R(x)^2\,dx$$

Integral Calculus: Accumulating and stacking circular discs to form the full Volume of curved object

Volume (Disc Method): $V = \pi \int_a^b [R(x)^2]dx$

Disc Method Steps:

a) Draw dotted line across the x-axis to indicate location of Axis of Revolution **(AOR)**

b) Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region [R(x) = Top graph − bottom]

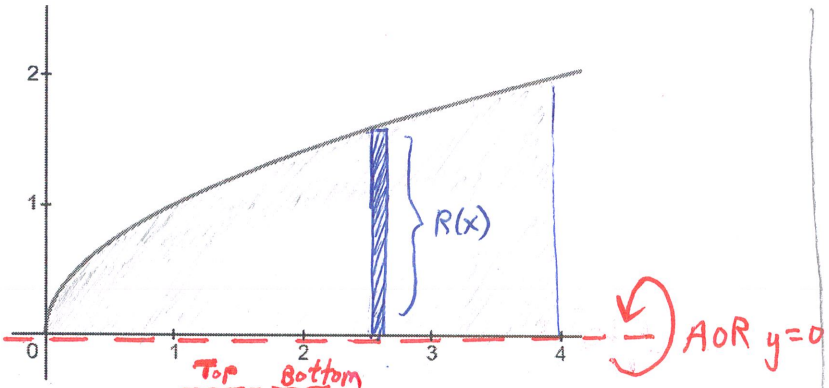
c) Identify the left and right bounds (a and b). If needed, set the equations equal to find bounds.

d) Enter expressions for R(x) and bounds into Disc Method Integral Notation.

e) Enter into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2nd → e)

Example 1: Find the volume of the solid formed by rotating the curve $y = \sqrt{x}$ around the x-axis between [0, 4]

AOR: y=0



AOR y=0

Top − Bottom

$R(x) = \sqrt{x} - 0 = \sqrt{x}$

$V = \pi \int_a^b [R(x)]^2 dx$

$V = \pi \int_0^4 [\sqrt{x}]^2 dx$

* or plug in calculator

$V = \pi \int_0^4 x\,dx \rightarrow \pi \left[\frac{x^2}{2}\right]_0^4$

$V = \pi\left(\frac{16}{2} - 0\right) = \boxed{8\pi \text{ units}^3}$

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2) Find the volume of the solid bounded by the x-axis, y-axis, and the curve $y = 4 - x^2$ rotated about the x-axis

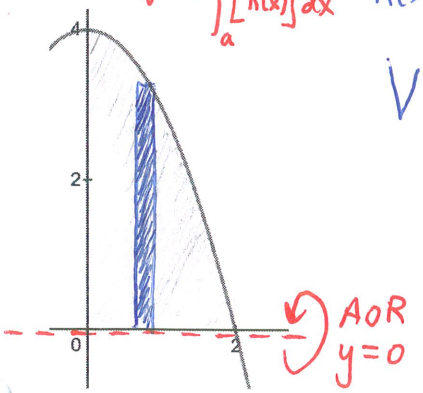
AOR y=0

$V = \pi \int_a^b [R(x)]^2 dx$

Top − Bottom

$R(x) = 4 - x^2 - (0) = 4 - x^2$

$V = \pi \int_0^2 [4 - x^2]^2 dx$



AOR y=0

$\boxed{V = \frac{256}{15}\pi \text{ units}^3}$

Volume (Disc Method): $V = \pi \int_a^b [R(x)^2]dx$

Disc Method Steps:

a) Draw dotted line across the ↑ (line of rotation) to indicate location of Axis of Revolution **(AOR)**

b) Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region $[R(x) = Top - bottom]$

c) Identify the left and right bounds (a and b). If needed, set the equations equal to find bounds.

d) Enter expressions for R(x) and bounds into Disc Method Integral Notation.

e) Enter into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2^{nd} → e)

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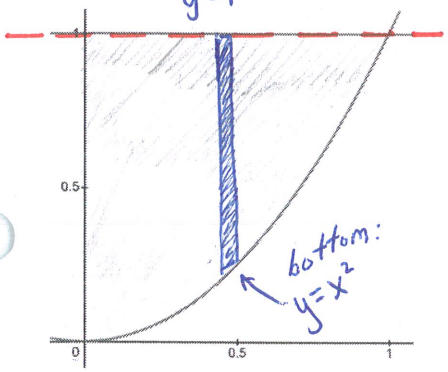
3) **Find the volume of the solid bounded by the y = 1, y-axis, and the graph $y = x^2$ rotated about the line y = 1**

AOR ↓



AOR $y = 1$

$R(x) = 1 - (x^2)$ (Top − Bottom)

$R(x) = 1 - x^2$

$V = \pi \int_0^1 [1 - x^2]^2 dx$

$V = \frac{8}{15}\pi$ units^3

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4) **Find the volume of the solid bounded by the y = 0, x = 1, and the graph $y = x^2$ rotated about the x-axis**

AOR $y = 0$ ↓

$R(x) = x^2 - 0$ (Top − bottom)

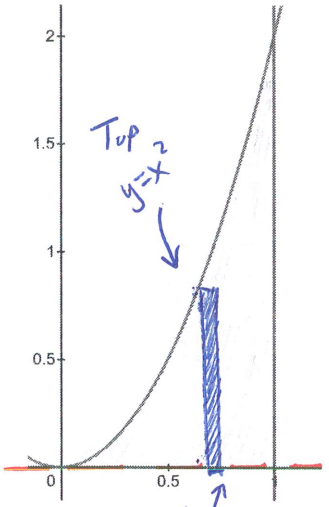
$R(x) = x^2$

↓

$V = \pi \int_a^b [R(x)]^2 dx$

$V = \pi \int_0^1 [x^2]^2 dx$

$V = 0.2\pi$ or $\frac{1}{5}\pi$ units^3



AOR: $y = 0$

Disc Method Steps: ***Volume (Disc Method)***: $V = \pi \int_a^b [R(x)^2]dx$

a) Draw dotted line across the line of rotation to indicate location of Axis of Revolution **(AOR)**

b) Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region

c) Identify the left and right bounds (a and b). If needed, set the equations equal to find bounds.

d) Enter expressions for R(x) and bounds into Disc Method Integral Notation.

e) Enter into calculator to find Volume. (TI-84: Math 9→ FnInt or TI-36X Pro: 2^{nd} → e)

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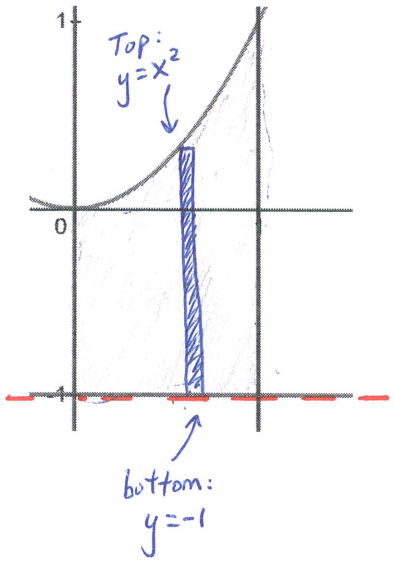
5) **Find the volume of the solid bounded by x = 1, y =- 1, y-axis, and the graph $y = x^2$ rotated about the line y = -1**

AOR: $y = -1$



Top − Bottom

$R(x) = x^2 - (-1)$

$R(x) = x^2 + 1$

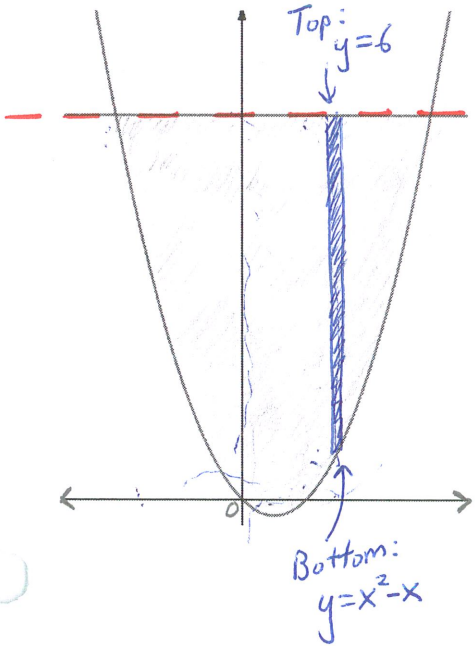
$V = \pi \int_a^b [R(x)]^2 dx$

$V = \pi \int_0^1 [x^2+1]^2 dx$

$V = \frac{28}{15}\pi$ units^3

AOR: $y = 6$

6) **Find the volume of the solid bounded by equations $y = x^2 - x$ and y = 6 rotated about the line y = 6**



Top − Bottom

$R(x) = 6 - (x^2 - x)$

$R(x) = 6 - x^2 + x$

$V = \pi \int_a^b [R(x)]^2 dx$

* set equations equal to find bounds:

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

Integral bounds: $x = 3, x = -2$

$V = \pi \int_{-2}^{3} [6 - x^2 + x]^2 dx$

$V = \frac{625}{6}\pi$ units^3

Instructions to use Online Calculator for Definite Integrals

1) **Google "online definite integral calculator" to take you to website** https://www.integral-calculator.com/
2) Use the Calculator Interface to enter in the Integrand Bar
3) Enter in the Bounds of Integration
4) and Click on "Go!"
5) Scroll to the bottom of page see Value of Definite Integral