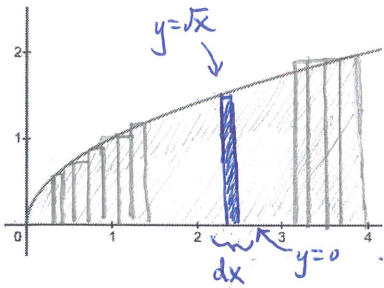


Key

Calculus Ch. 7.2a: Volume by Disc Method

Recall finding area under the curve  $y = \sqrt{x}$  between  $[0, 4]$ .  $Area = \int_a^b (Top\ graph - bottom\ graph) dx$



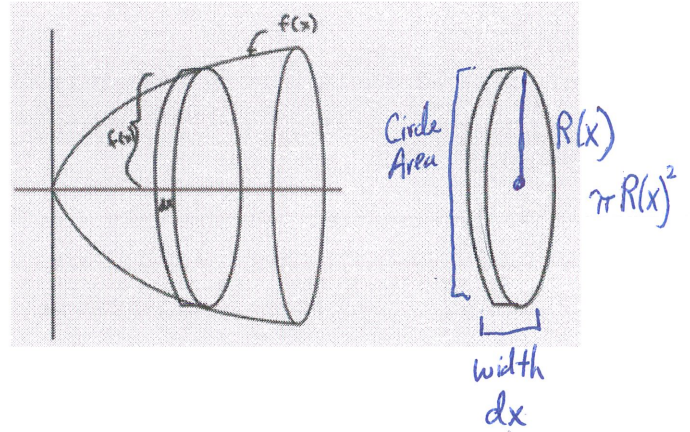
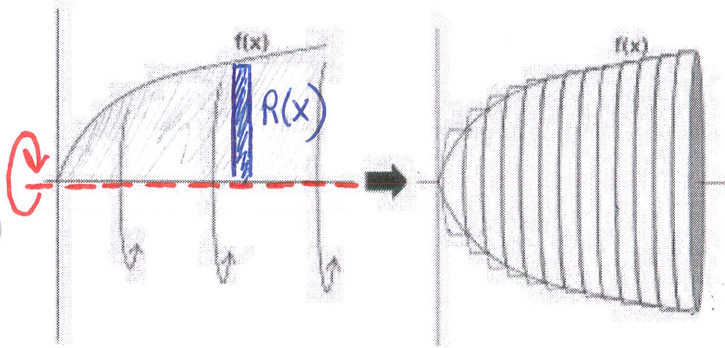
$$Area = \int_0^4 (\underbrace{\sqrt{x}}_{\text{height}} - \underbrace{0}_{\text{width}}) dx \rightarrow \boxed{Area = 5.333 \text{ units}^2}$$

Rectangle Area = width  $\times$  height

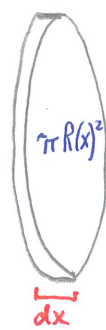
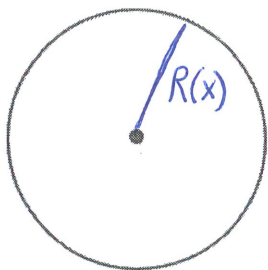
\*Essentially, the Integral Notation allows us to add infinite numbers of differently sized rectangles to form area calculation.

*\*Integral Calculus allows for accumulation of rectangles to form full Area of specified region*

With **Disc Method**, we are going to take this region created by  $f(x)$  and the x-axis and rotate this function  $360^\circ$  around the x-axis. What shapes do you see if we were to separate the resulting object into thin slices? circular discs

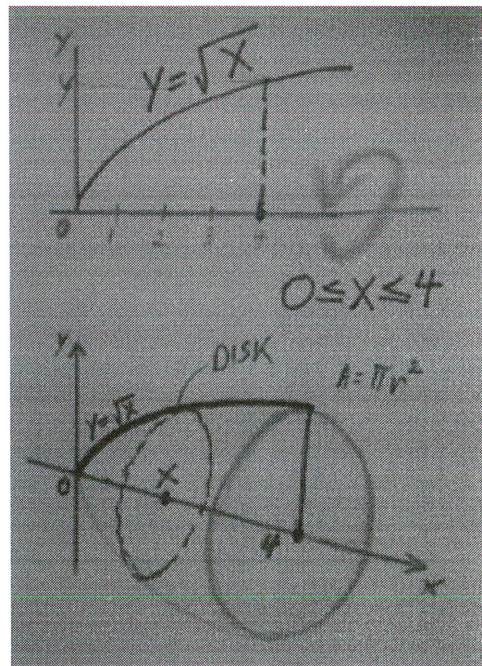


Area of Circle is  $\pi r^2$



$$\pi R(x)^2 \rightarrow \underbrace{\pi R(x)^2}_{\text{Area}} \underbrace{dx}_{\text{width}}$$

Volume (Disc Method):  $V = \int_a^b \pi R(x)^2 dx$



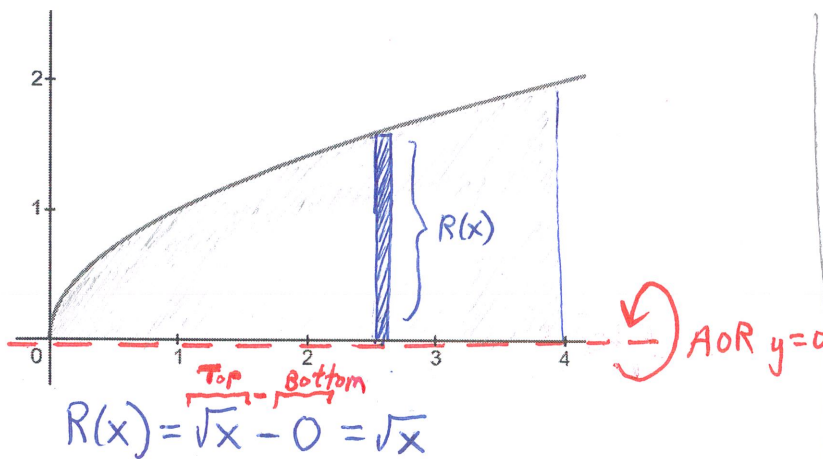
Integral Calculus: Accumulating and stacking circular discs to form the full Volume of curved object

**Volume (Disc Method):**  $V = \pi \int_a^b [R(x)^2] dx$

**Disc Method Steps:**

- Draw dotted line across the x-axis to indicate location of Axis of Revolution (**AOR**)
- Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region  **$[R(x) = \text{Top graph} - \text{bottom}]$**
- Identify the left and right bounds ( a and b). If needed, set the equations equal to find bounds.
- Enter expressions for R(x) and bounds into Disc Method Integral Notation.
- Enter into calculator to find Volume. (TI-84: Math 9 → FnInt or TI-36X Pro: 2<sup>nd</sup> → e)

**Example 1:** Find the volume of the solid formed by rotating the curve  $y = \sqrt{x}$  around the x-axis between [0, 4]



$$V = \pi \int_a^b [R(x)]^2 dx$$

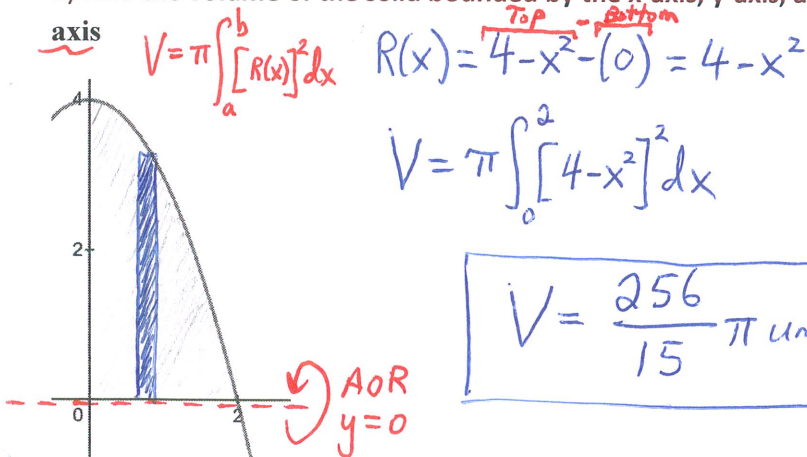
$$V = \pi \int_0^4 [\sqrt{x}]^2 dx$$

*\* or plug in calculator*

$$V = \pi \int_0^4 x dx \rightarrow \pi \left[ \frac{x^2}{2} \right]_0^4$$

$$V = \pi \left( \frac{16}{2} - 0 \right) = 8\pi \text{ units}^3$$

**2)** Find the volume of the solid bounded by the x-axis, y-axis, and the curve  $y = 4 - x^2$  rotated about the x-axis



$$V = \pi \int_0^2 [4 - x^2]^2 dx$$

$$V = \frac{256}{15} \pi \text{ units}^3$$

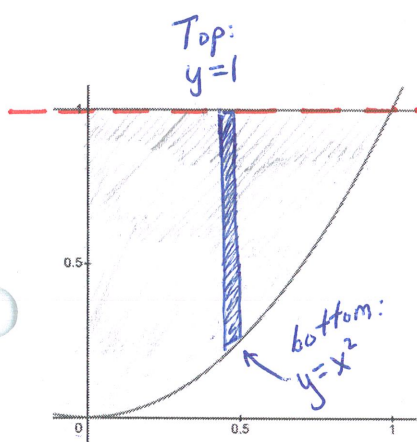


**Volume (Disc Method):**  $V = \pi \int_a^b [R(x)^2] dx$

**Disc Method Steps:**

- Draw dotted line across the <sup>line of rotation</sup> to indicate location of Axis of Revolution (AOR)
- Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region  $[R(x) = \text{Top} - \text{bottom}]$
- Identify the left and right bounds ( a and b). If needed, set the equations equal to find bounds.
- Enter expressions for R(x) and bounds into Disc Method Integral Notation.
- Enter into calculator to find Volume. (TI-84: Math 9 → FnInt or TI-36X Pro: 2<sup>nd</sup> → e)

3) Find the volume of the solid bounded by the  $y = 1$ , y-axis, and the graph  $y = x^2$  rotated about the line  $y = 1$

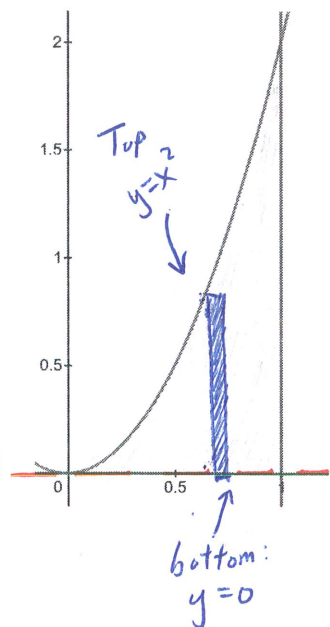


$\downarrow$  AOR  $y=1$   
 $R(x) = \overbrace{1}^{\text{Top}} - \overbrace{(x^2)}^{\text{Bottom}}$   
 $R(x) = 1 - x^2$

$$V = \pi \int_0^1 [1 - x^2]^2 dx$$

$$V = \frac{8}{15} \pi \text{ units}^3$$

4) Find the volume of the solid bounded by the  $y = 0$ ,  $x = 1$ , and the graph  $y = x^2$  rotated about the x-axis



$R(x) = \overbrace{x^2}^{\text{Top}} - \overbrace{0}^{\text{bottom}}$

$R(x) = x^2$

$V = \pi \int_a^b [R(x)]^2 dx$

$$V = \pi \int_0^1 [x^2]^2 dx$$

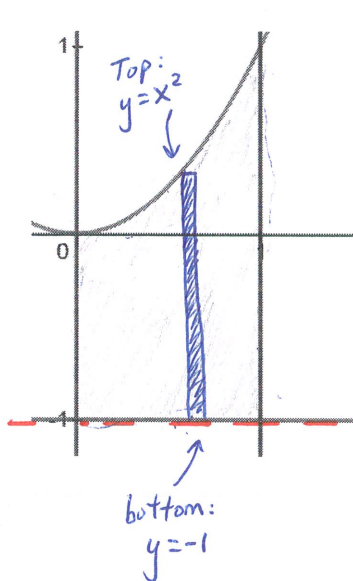
$$V = 0.2 \pi \text{ or } \frac{1}{5} \pi \text{ units}^3$$

$\downarrow$  AOR:  $y=0$

**Disc Method Steps: Volume (Disc Method):**  $V = \pi \int_a^b [R(x)^2] dx$

- Draw dotted line across the *line of rotation* to indicate location of Axis of Revolution (AOR)
- Draw the length of **Radius R(x)**: Place pen/pencil **first** on the dotted line (AOR) and extend to outer boundary of shaded region
- Identify the left and right bounds ( a and b). If needed, set the equations equal to find bounds.
- Enter expressions for R(x) and bounds into Disc Method Integral Notation.
- Enter into calculator to find Volume. (TI-84: Math 9 → FnInt or TI-36X Pro: 2<sup>nd</sup> → e)

5) Find the volume of the solid bounded by  $x = 1$ ,  $y = -1$ ,  $y$ -axis, and the graph  $y = x^2$  rotated about the line  $y = -1$



$$R(x) = \overset{\text{Top}}{x^2} - \overset{\text{Bottom}}{(-1)}$$

$$R(x) = x^2 + 1$$

$$V = \pi \int_a^b [R(x)]^2 dx$$

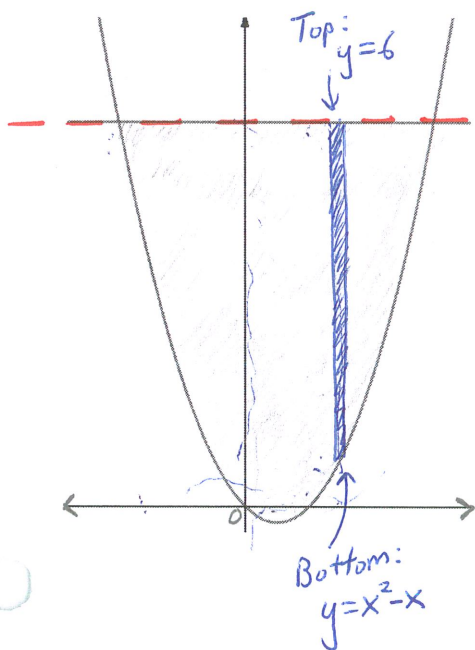
AOR:  $y = -1$

$$V = \pi \int_0^1 [x^2 + 1]^2 dx$$

$$V = \frac{28}{15} \pi \text{ units}^3$$

AOR:  $y = -1$

6) Find the volume of the solid bounded by equations  $y = x^2 - x$  and  $y = 6$  rotated about the line  $y = 6$



AOR:  $y = 6$

$$R(x) = \overset{\text{Top}}{6} - \overset{\text{Bottom}}{(x^2 - x)}$$

$$R(x) = 6 - x^2 + x$$

$$V = \pi \int_a^b [R(x)]^2 dx$$

\* set equations equal to find bounds:

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

$$V = \pi \int_{-2}^3 [6 - x^2 + x]^2 dx$$

$$V = \frac{625}{6} \pi \text{ units}^3$$

AOR:  $y = 6$



## Instructions to use Online Calculator for Definite Integrals

- 1) Google "online definite integral calculator" to take you to website <https://www.integral-calculator.com/>
- 2) Use the Calculator Interface to enter in the Integrand Bar
- 3) Enter in the Bounds of Integration
- 4) and Click on "Go!"
- 5) Scroll to the bottom of page see Value of Definite Integral

integral-calculator.com

Also check the [Derivative Calculator!](#)  
Calculadora de Integrales en español  
Integralrechner auf Deutsch  
Калькулятор Интегралов на Русском

# Integral Calculator

Calculate integrals online – with steps and graphing!

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Squarespace Website Builder  
Make and manage your own professional website with Squarespace's all-in-one platform. **OPEN**

Enter Expression here

Calculate the Integral of ...

$(2x^2 - 4x) - (x + 1)$  **Go!**

CLR + - \* / ^ √ ( )

This will be calculated:

Bounds of Integration

$$\int_1^3 (2x^2 - 4x - (x + 1)) dx$$

Not what you mean? Use parentheses! Set integration variable and bounds in "Options".

About Help Examples Options

Configure the Integral Calculator:

Variable of integration:

Upper bound (to):

Lower bound (from):

Integrate numerically only?

Simplify expressions?

Simplify all roots?

( $\sqrt{x^2}$  becomes x, not |x|)

Use complex domain (C)?

Keep decimals?

Advertisement

DEFINITE INTEGRAL:

$$\int_1^3 f(x) dx =$$
$$-\frac{14}{3}$$

Approximation:

$$-4.666666666666667$$

**Simplify**

After clicking "Go" scroll to bottom of page to find value