Non-AP Calculus 3.7 Optimization Quiz Review Worksheet #1

1) A nursery wants to add a 1000 square foot rectangular area to its greenhouse to sell seedlings. For aesthetic reasons, they have decided to border the area on three sides by cedar siding at a cost of $10 per foot. The remaining side is to be a wall with a brick mosaic that costs $25 per foot. What should the dimensions of the sides be so that the cost of the project will be minimized?

2)

An open box with a rectangular base is to be constructed from a 12" by 18" piece of cardboard by cutting out squares from each corner and bending up the sides. Find the dimensions of the box that will have the largest volume.

3)

If 1200 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

4) A bus company will charter a bus that holds 50 people to groups of 35 or more. If a group contains exactly 35 people, each person pays \$60. In larger groups, everybody's fare is reduced by \$1 for each person in excess of 35. Determine the size of the group for which the bus company's revenue will be the greatest.

Key

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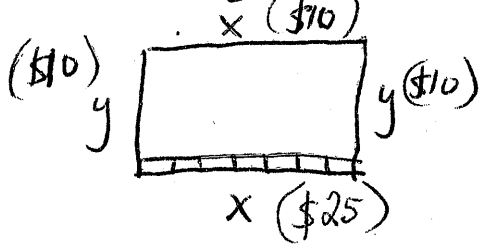
1) A nursery wants to add a 1000 square foot rectangular area to its greenhouse to sell seedlings. For aesthetic reasons, they have decided to border the area on three sides by cedar siding at a cost of $10 per foot. The remaining side is to be a wall with a brick mosaic that costs $25 per foot. What should the dimensions of the sides be so that the cost of the project will be minimized? *optimize Perimeter/cost



Perimeter (cost) $= 10x + 25x + 10y + 10y$

Cost $= 35x + 20y$

Area $= xy$

$1000 = xy$

$\frac{1000}{x} = y$

$C(x) = 35x + 20\left(\frac{1000}{x}\right)$

$C(x) = 35x + \frac{20{,}000}{x}$

$C(x) = 35x + 20{,}000x^{-1}$

$C'(x) = 35 - 20{,}000x^{-2}$

$0 = 35 - \frac{20000}{x^2}$

$\frac{20000}{x^2} = 35$

$35x^2 = 20{,}000$

$x^2 = \frac{20{,}000}{35}$

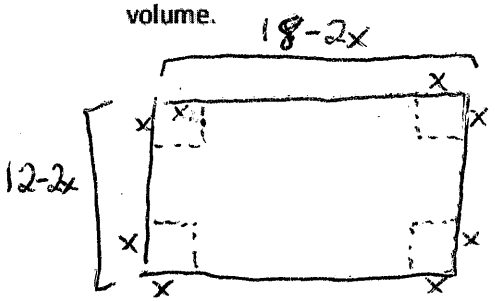
$x = \pm\sqrt{\frac{20{,}000}{35}}$

$x = 23.905$ ft

$y = \frac{1000}{23.905} = 41.833$ ft

Dimensions: 23.905 ft by 41.833 ft

2) An open box with a rectangular base is to be constructed from a 12" by 18" piece of cardboard by cutting out squares from each corner and bending up the sides. Find the dimensions of the box that will have the largest volume.



Volume $= x(12-2x)(18-2x)$

$V = (12x - 2x^2)(18-2x)$

$V = 216x - 24x^2 - 36x^2 + 4x^3$

$V = 4x^3 - 60x^2 + 216x$

$V'(x) = 12x^2 - 120x + 216$

$V'(x) = 12(x^2 - 10x + 18)$

$0 = x^2 - 10x + 18$

$0 = x^2 - 10x + 18$

$x = \frac{10 \pm \sqrt{(10)^2 - 4(1)(18)}}{2(1)}$

$x = \frac{10 \pm \sqrt{28}}{2}$

$x = \frac{10 + \sqrt{28}}{2}, \frac{10 - \sqrt{28}}{2}$

$x = 7.646, \boxed{x = 2.354}$

$x = 2.354$

$18 - 2(2.354) = 13.292$

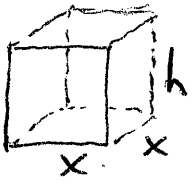
$12 - 2(2.354) = 7.292$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Dimensions: 2.354 in. × 7.292 in. × 13.292 in.

3) surface Area

If 1200 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box. * optimize volume



* $V = x^2h$

Surface Area $= x^2 + xh + xh + xh + xh$

$S = x^2 + 4xh$

$1200 = x^2 + 4xh$

$1200 - x^2 = 4xh$

$\frac{1200 - x^2}{4x} = h$

$V = x^2\left[\frac{1200 - x^2}{4x}\right] = \frac{1200x^2 - x^4}{4x}$

$V = \frac{1200x^2}{4x} - \frac{x^4}{4x}$

$V = 300x - \frac{1}{4}x^3$

$V'(x) = 300 - \frac{3}{4}x^2$

$0 = 300 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 300$

$\frac{4}{3}\left(\frac{3}{4}x^2\right) = (300)\frac{4}{3}$

$x^2 = 400$

$x = \pm\sqrt{400}$

$x = 20$

Max Volume:

$V(20) = 300(20) - \frac{1}{4}(20)^3$

$V = 4000\ m^3$

4) A bus company will charter a bus that holds 50 people to groups of 35 or more. If a group contains exactly 35 people, each person pays $60. In larger groups, everybody's fare is reduced by $1 for each person in excess of 35. Determine the size of the group for which the bus company's revenue will be the greatest.

x = # of fare changes

R(x) = (# of people) x (Amount of fare)

$R(x) = (35 + 1x)(60 - 1x)$

$R(x) = 2100 - 35x + 60x - x^2$

$R(x) = -x^2 + 25x + 2100$

$R'(x) = -2x + 25$

$0 = -2x + 25$

$2x = 25$

$x = 12.5$

At $x = 13$, $R(13) = 2256$

At $x = 12$, $R(12) = 2256$

size of group:

either 35+13 or 35+12

either 48 or 47 people