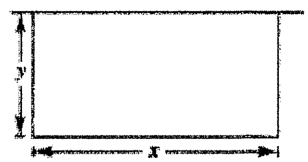
Non-AP Calculus 3.7 Optimization Quiz Review Worksheet #2

1)

Minimum Length A farmer plans to fence a rectangular pasture adjacent to a river (see figure). The pasture must contain 245,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?



2) A shipping crate with a square base is being designed that must contain a volume of 16 ft^3. The material that is used for the base and the lid costs 3 dollars/ft^2, while the material used for the sides costs 2 dollars/ft^2. What are the most cost-effective dimensions of such a crate?

3) A baseball team plays in a stadium that hold 55, 000 spectators. With ticket prices at \$10, the average attendance had been 27, 000. A market survey showed that for each \$0.10 decrease in the ticket prices, on the average, the attendance will increase by 300. How should ticket prices be set to maximize revenue?

4) An open box with a rectangular base is to be constructed from a 16" by 21" piece of cardboard by cutting out squares from each corner and bending up the sides. Find the dimensions of the box that will have the largest volume

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Key

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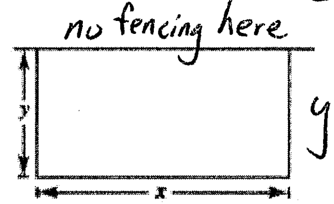


optimize fencing (perimeter)

$*P = x + 2y$

$\text{Area} = xy$

$245{,}000 = xy$

$\frac{245000}{x} = y$

$P = x + 2\left[\frac{245000}{x}\right]$

$P = x + \frac{490000}{x}$

$P = x + 490000x^{-1}$

$P'(x) = 1 - 490000x^{-2}$

$0 = 1 - \frac{490000}{x^2}$

$\frac{490000}{x^2} = 1$

$x^2 = 490000$

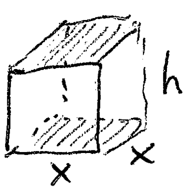
$x = \pm\sqrt{490000}$

$x = 700$ meters

$y = \frac{245000}{700} = 350$ meters

$\boxed{\text{Dimensions: } 700\,\text{m by } 350\,\text{m}}$

2) A shipping crate with a square base is being designed that must contain a volume of 16 ft^3 . The material that is used for the base and the lid costs 3 dollars/ft^2 , while the material used for the sides costs 2 dollars/ft^2 . What are the most cost-effective dimensions of such a crate?



$V = x^2h \rightarrow 16 = x^2h$ $\boxed{\frac{16}{x^2} = h}$

optimize surface Area/cost

$* \text{Surface Area (cost)} = \underbrace{x^2 + x^2}_{\text{base is \$3}} + \underbrace{xh + xh + xh + xh}_{\text{sides are \$2}}$

$C(x) = 3(2x^2) + 2(4xh)$

$C(x) = 6x^2 + 8xh$

$C(x) = 6x^2 + 8x\left(\frac{16}{x^2}\right)$

$C(x) = 6x^2 + \frac{128}{x}$

$C(x) = 6x^2 + 128x^{-1}$

$C'(x) = 12x - 128x^{-2}$

$0 = 12x - \frac{128}{x^2}$

$\frac{128}{x^2} = \frac{12x}{1}$

$12x^3 = 128$

$x^3 = \frac{128}{12}$

$x = \sqrt[3]{\frac{128}{12}}$

$x = 2.20$

$C(2.20) = 6(2.2)^2 + \frac{128}{2.2}$

$= \boxed{\$87.22}$

3) A baseball team plays in a stadium that hold 55, 000 spectators. With ticket prices at \$10, the average attendance had been 27, 000. A market survey showed that for each \$0.10 decrease in the ticket prices, on the average, the attendance will increase by 300. How should ticket prices be set to maximize revenue?

$x =$ each price change

$R(x) = (\text{change in attendance}) \times (\text{change in prices})$

$R(x) = (27{,}000 + 300x)(10 - 0.10x)$

$R(x) = 270000 - 2700x + 3000x - 30x^2$

$R(x) = -30x^2 + 300x + 270000$

$R'(x) = -60x + 300$

$0 = -60x + 300$

$60x = 300$

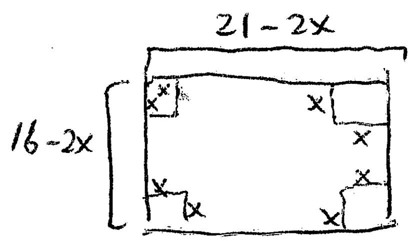
$x = 5$

optimal ticket price at the 5th price decrease

$\$10 - 0.10(5)$

$= \boxed{\$9.50}$

4) An open box with a rectangular base is to be constructed from a 16" by 21" piece of cardboard by cutting out squares from each corner and bending up the sides. Find the dimensions of the box that will have the largest volume



$V(x) = x(16-2x)(21-2x)$

$V(x) = (16x - 2x^2)(21 - 2x)$

$V(x) = 336x - 32x^2 - 42x^2 + 4x^3$

$V(x) = 4x^3 - 74x^2 + 336x$

$V'(x) = 12x^2 - 148x + 336$

$V'(x) = 4(3x^2 - 37x + 84)$

$0 = 3x^2 - 37x + 84$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{37 \pm \sqrt{37^2 - 4(3)(84)}}{2(3)}$

$x = \dfrac{37 \pm \sqrt{361}}{6}$

$x = \dfrac{37 + \sqrt{361}}{6}, \dfrac{37 - \sqrt{361}}{6}$

$x = \cancel{9.33}, x = 3$

$x = 3$ in.

$16 - 2(3) = 10$ in

$21 - 2(3) = 15$ in

Dimensions:

3 in. x 10 in. x 15 in.