<u>Ch. 1 Limits</u> <u>Steps for Important Concepts</u>

I. Algebraic Steps Evaluating Limits Approaching a Real Number $\lim_{x\to c} f(x)$

1. Plug in argument x-value (Ignore one-sided limit for now)
2. Find the Limit (plug in/ reduce if $\frac{0}{0}$, re-evaluate)
3. If Limit DNE (does not exist), then evaluate further ONLY IF one-sided limit)
4. Choose between $+\infty$ and $-\infty$
5. Plug in the appropriate decimal value to determine $+\infty$ or $-\infty$

II. Algebraic Steps Evaluating Limits Approaching Infinity $\lim_{x\to\infty} f(x)$ *or* $\lim_{x\to-\infty} f(x)$

1. Compare degrees between numerator vs. denominator
   a. If Numerator < Denominator $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$
   b. If Numerator = Denominator $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) =$ ratio of coefficients
   c. If Numerator > Denominator, then $\lim_{x\to\infty} f(x) = \pm\infty$ *or* $\lim_{x\to-\infty} f(x) = \pm\infty$

   (Plug in a large positive or large negative value to help you determine the sign at infinity)

III. Continuity Conditions

1. f(c) is defined (point exists on the graph)
2. The $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)\right]$
3. f(c) $= \lim_{x\to c} f(x)$

- If function passes all 3 conditions , the function has continuity at x = c
- If condition #2 FAILS, the function has **nonremovable** discontinuity at x = c
- If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at x = c

IV. Intermediate Value Theorem (IVT) Steps

1. Test and determine continuity on closed interval [a, b]
2. Find the y-value at the endpoints , f(a) and f(b)
3. Confirm that f(c) is between f(a) and f(b) [example: f(a) < f(c) < f(b)]
4. Find the c-value (find the x-value by plugging the y-value given at the start of problem into the function)

   *Make sure c-value(s) are inside the interval [a,b]. c-values that are outside the interval [a,b] are excluded.

DERIVATIVES

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

Calculus Chapter 3 Rules and Process Sheet

Derivative Rules

Power Rule:
If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Quotient Rule:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - fg'}{g^2}$$

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

Chain Rule:
$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

(multiply outer function's derivative with the inner function's derivative)

Extreme Value Theorem

Steps: * Confirm continuous function on closed interval

1. Find critical points
   a. Set f'(x) = 0
   b. Find where f'(x) is undefined (Set denominator of f'(x) = 0)
2. Plug all critical points and endpoints into f(x)
3. Compare y-values to determine absolute maximum(s) and absolute minimum(s)

Mean Value Theorem

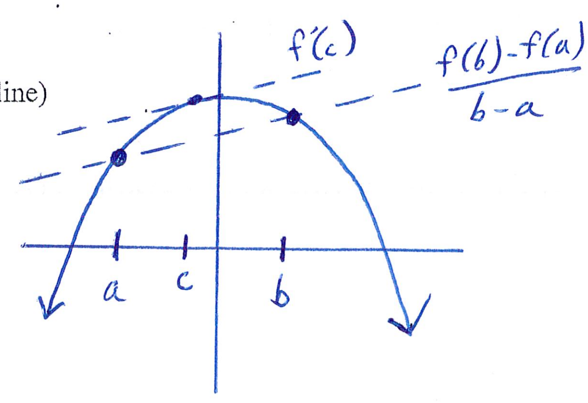
i) continuous on [a, b]
ii) differentiable on (a, b)

1. Check Continuity (no breaks between endpoints)
   a. Does f(x) have variables in the denominator? (V.A. or holes)
   b. If so, then look to see if the x-value lies in the **closed** interval [a, b]
   c. If the x lies between the interval, then function is not continuous on the interval, MVT fails
2. Check Differentiability (smooth curve between endpoints)
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If yes, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails

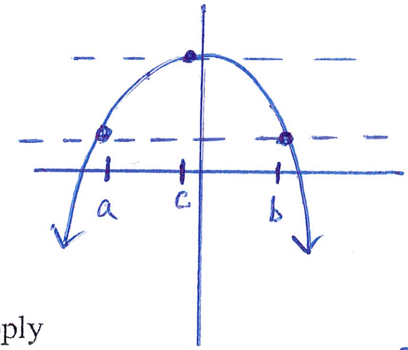
Note, all polynomials are continuous and differentiable everywhere

3. Find m_{avg}. (This is the slope between your endpoints, slope of secant line)
4. Set f'(x) = m_{avg} and solve for x

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Rolle's Theorem



1. Check Continuity (no breaks between endpoints)
2. Check Differentiability (smooth curve between endpoints)

Note, all polynomials are continuous and differentiable everywhere

3. Test endpoints. Does f(a) = f(b)? If not ,then Rolle's fails / does not apply
4. If yes, then set f'(x) = 0 and solve for x

i) continuous on $[a,b]$
ii) differentiable on (a,b)
iii) $f(a) = f(b)$

First Derivative Test Steps (Finds inc/dec and rel max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
   b. Remember, critical points also exist where function is not differentiable (sharp point)
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
      ii. Negative(–) means decreasing slope
4. Write Because Statements
   a. f(x) increasing in interval (a,b) b/c f '(x) > 0
   b. f(x) decreasing in interval (a,b) b/c f '(x) < 0
   c. Relative max at (**a, f(a)**) b/c f '(x) changes from + to –
   d. Relative min at (**a, f(a)**) b/c f '(x) changes from – to +

"Concavity Test" Steps (Finds interval Concave Up/Down and POI)

1. Find $f''(x)$, set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into $f''(x)$ to determine concavity
      i. Positive (+) means concave up
      ii. Negative(–) means concave down
4. Write Because Statements
   a. Concave up in interval (a,b) b/c f ''(x) > 0
   b. Concave down in interval (a,b) b/c f ''(x) < 0
5. Point of Inflection at (**a, f(a)**) b/c $f''(x)$ changes signs

*Note: POI (Point of Inflection) may exist on graph where f''(x) does not exist (sharp point). POI exists as long as the graph is continuous and f''(x) changes concavity (change in signs)