Chapter 1 Limits:

1) For which of the following does $\lim_{x \to 4} f(x)$ exist ?

I.

II.

III.

Graph of f Graph of f Graph of f
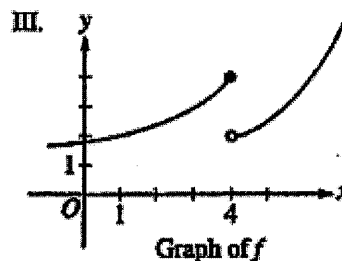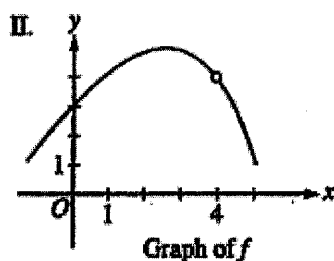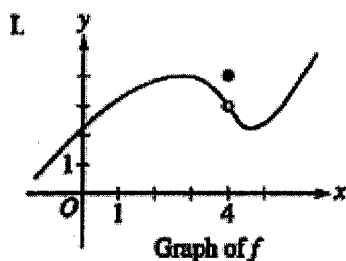
A) I only

B) II only

C) III only

D) I and II only

E) I and III only

2) The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x \to c} f(x)$ exists

(B) $\lim_{x \to c} f(x) = f(c)$

(C) $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

(D) $f(c)$ is defined

(E) $f'(c)$ exists

3) $\lim_{x \to 3} \dfrac{x-3}{x^2-2x-3} =$

a. 0

b. 1

c. $\dfrac{1}{4}$

d. ∞

e. none of these

4) $\lim_{x \to \infty} \dfrac{4-x^2}{4x^2-x-2} =$

a. -2

b. $-\dfrac{1}{4}$

c. 1

d. 2

e. nonexistent

5) Let $f(x) = \begin{cases} x^2 - 1, & x \neq 1 \\ 4, & x = 1 \end{cases}$ **Which of the following statements are true?**

    I. $\lim_{x \to 1} f(x)$ exists   II. $f(1)$ exists   III. f is continuous at $x = 1$

    a. I only   b. II only   c. I and II   d. I, II and III   e. none of them

6) $\lim_{x \to 4^+} \dfrac{4x^2 - 14x - 8}{x - 4} =$

A) 18   B) 9   C) ∞ D) $-\infty$ E) Does Not Exist

7) $\lim_{x \to 1} \left(\dfrac{\sqrt{x+3} - 2}{1 - x} \right)$

(A) 0.5

(B) 0.25

(C) 0

(D) –0.25

(E) –0.5

8) $\lim_{x \to 2^-} \dfrac{(x+2)^2}{x^2 - 4} =$

A) 4   B) 1   C) ∞ D) $-\infty$ E) Does Not Exist

9) $\lim\limits_{x\to-11^-} \dfrac{\sqrt{x+19}-\sqrt{8}}{x+11} =$

A) $\dfrac{1}{2\sqrt{8}}$ B) $-\dfrac{1}{2\sqrt{8}}$ C) ∞ D) $-\infty$ E) Does Not Exist

10) $\lim\limits_{x\to-\infty} \dfrac{5x^4-5x-5}{(2x^2-3)^2} =$

A) $\dfrac{5}{2}$ B) $\dfrac{5}{4}$ C) ∞ D) $-\infty$ E) Does Not Exist

11) $\lim\limits_{x\to1} \dfrac{(x+1)^2}{x^2-1} =$

A) 1   B) 2   C) ∞ D) $-\infty$ E) Does Not Exist

12) $\lim\limits_{x\to-\infty} \dfrac{4-x^3}{2-x^2} =$

A) 4   B) 2   C) ∞ D) $-\infty$ E) -2

13) Evaluate the limits algebraically or state that the limit does not exist.

$$\lim_{x \to 2} \frac{x - 3x + 2}{x - 2}$$

$$\lim_{x \to 2} \frac{x - 2}{x^3 - 4x}$$

$$\lim_{x \to 0} \frac{\sqrt{2 + x} - 2}{x}$$

$$\lim_{x \to 0} \frac{\frac{1}{3 + x} - \frac{1}{3}}{x}$$

$$\lim_{x \to 0} \frac{(3a + x)^2 - 9a^2}{x}$$

$$\lim_{x \to 8} \frac{x^2 - 64}{x - 9}$$

14) What is $\displaystyle \lim_{x \to \infty} \frac{3x^2 + 1}{(3 - x)(3 + x)}$?

(A) –9 (B) –3 (C) 1 (D) 3 (E) The limit does not exist.

15)

Limits Review

Refer to the graph of $g(x)$ below in order to answer the following questions. If a limit doesn't exist, explain why.

1. $\lim\limits_{x \to \infty} g(x) =$

2. $\lim\limits_{x \to -\infty} g(x) =$

3. $\lim\limits_{x \to a^+} g(x) =$

4. $\lim\limits_{x \to a^-} g(x) =$

5. $\lim\limits_{x \to a} g(x) =$

6. $\lim\limits_{x \to 0} g(x) =$

7. $\lim\limits_{x \to b^+} g(x) =$

8. $\lim\limits_{x \to b^-} g(x) =$

9. $\lim\limits_{x \to b} g(x) =$

10. $\lim\limits_{x \to c} g(x) =$

11. $\lim\limits_{x \to d} g(x) =$

12. $\lim\limits_{x \to e} g(x) =$

13. $g(e) =$

14. $g(0) =$

15. $g(b) =$
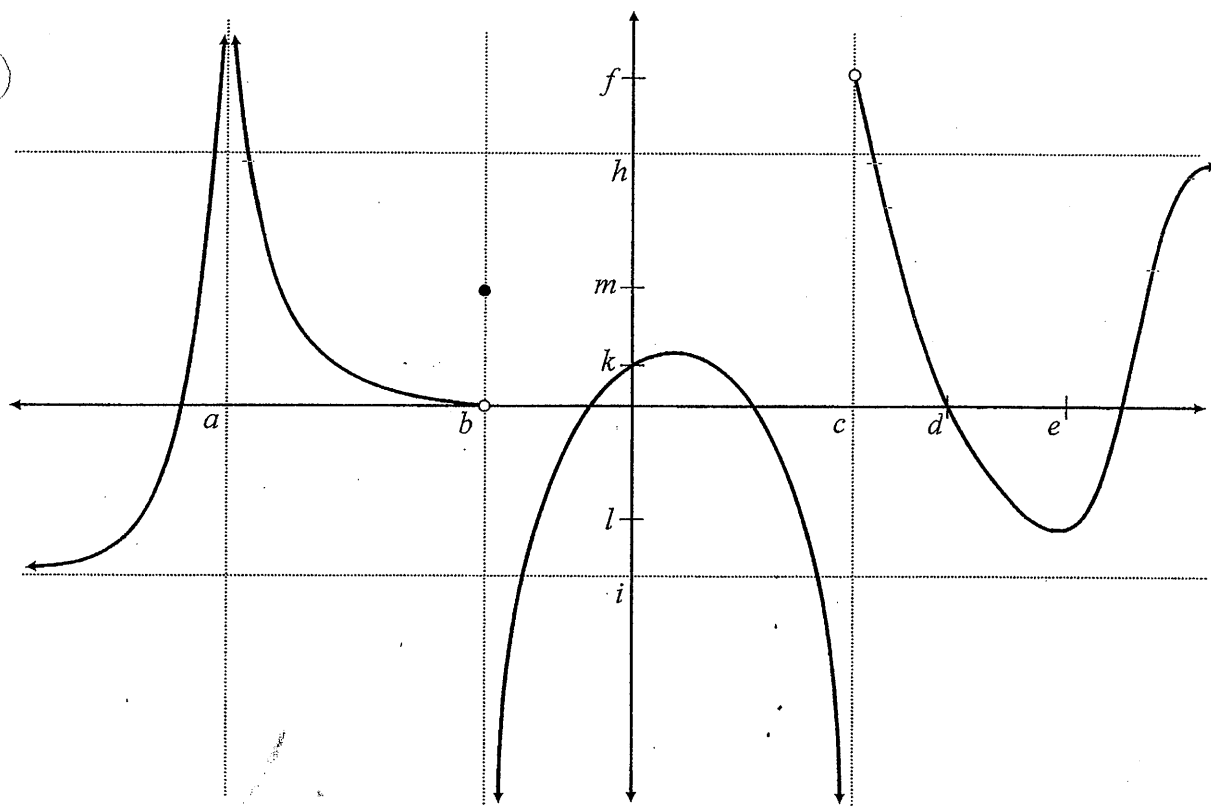
A.P. Calculus AB   Limits Review

Refer to the graph of $g(x)$ below in order to answer the following questions. If a limit doesn't exist, explain why.

1. $\lim\limits_{x\to\infty} g(x) =$ h

2. $\lim\limits_{x\to-\infty} g(x) =$ i

3. $\lim\limits_{x\to a^+} g(x) =$ $+\infty$

4. $\lim\limits_{x\to a^-} g(x) =$ $+\infty$

5. $\lim\limits_{x\to a} g(x) =$ $+\infty$

6. $\lim\limits_{x\to 0} g(x) =$ k

7. $\lim\limits_{x\to b^+} g(x) =$ $-\infty$

8. $\lim\limits_{x\to b^-} g(x) =$ 0

9. $\lim\limits_{x\to b} g(x) =$ DNE, nonremovable discontinuity $\lim\limits_{x\to b^-} g(x) \neq \lim\limits_{x\to b^+} g(x)$

10. $\lim\limits_{x\to c} g(x) =$ DNE $\left(\begin{array}{c}\text{nonremovable}\\\text{discontinuity}\end{array}\right)$ $\lim\limits_{x\to c^-} g(x) \neq \lim\limits_{x\to c^+} g(x)$
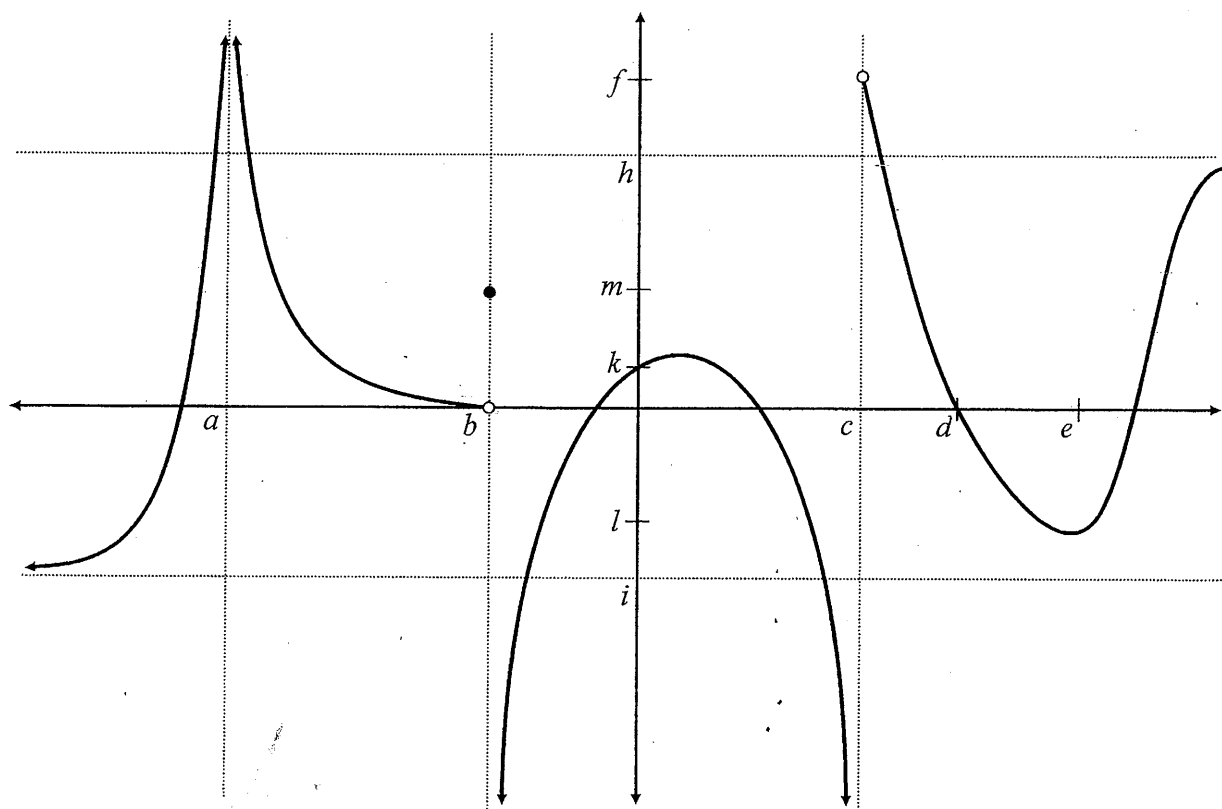
11. $\lim\limits_{x\to d} g(x) =$ 0

12. $\lim\limits_{x\to e} g(x) =$ l

13. $g(e) =$ l

14. $g(0) =$ k

15. $g(b) =$ m

16) Find the point on the graph of $y = \sqrt{x}$ between $(1, 1)$ and $(9, 3)$ at which the tangent to the graph has the same slope as the line through $(1, 1)$ and $(9, 3)$.

(A) $(1, 1)$

(B) $(2, \sqrt{2})$

(C) $(3, \sqrt{3})$

(D) $(4, 2)$

(E) none of the above

17) . If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

18) If $y = \frac{2x + 3}{3x + 2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x + 13}{(3x + 2)^2}$ (B) $\frac{12x - 13}{(3x + 2)^2}$ (C) $\frac{5}{(3x + 2)^2}$ (D) $\frac{-5}{(3x + 2)^2}$ (E) $\frac{2}{3}$

19) . If $x^3 - xy + y^3 = 1$, find $\frac{dy}{dx}$

a. $\frac{3x^2}{x - 3y^2}$ b. $\frac{3x^2 - 1}{1 - 3y^2}$ c. $\frac{y - 3x^2}{3y^2 - x}$ d. $\frac{3x^2 + 3y^2 - y}{x}$ e. $\frac{3x^2 + 3y^2}{x}$

20) If $y = \dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3}{2x}$

(B) $\dfrac{3x}{\left(1+x^2\right)^2}$

(C) $\dfrac{6x}{\left(4+x^2\right)^2}$

(D) $\dfrac{-6x}{\left(4+x^2\right)^2}$

(E) $\dfrac{-3}{\left(4+x^2\right)^2}$

21) If $xy + x^2 = 6$, then the value of $\dfrac{dy}{dx}$ at $x = -1$ is

(A) –7   (B) –2   (C) 0   (D) 1   (E) 3

22) Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point $(2,1)$ is

(A) $\dfrac{2}{3}$

(B) $\dfrac{1}{3}$

(C) $-\dfrac{1}{3}$

(D) $-\dfrac{1}{5}$

(E) $-\dfrac{3}{4}$

23) A balloon is being filled with helium at the rate of $4 \frac{ft^3}{min}$. Find the rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3} ft^3$. Note, the volume of a sphere is $\frac{4}{3}\pi r^3$ and the surface area of a sphere is $4\pi r^2$.

   a. 4π b. 2   c. 4   d. 1   e. 2π

24) Two cars are traveling along perpendicular roads, car A at 40 mi/hour, car B at 60 mi/hour. At noon, when car A reaches the intersection, car B is 90 miles away and moving toward the intersection. At 1 pm, the distance between the cars is changing, in miles per hour, at a rate of

   a. -40   b. 68   c. 4   d. -4   e. 40

25)

The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube? $\left(S = 6x^2 \quad V = x^3\right)$

(A) 5 in³/sec

(B) 10 in³/sec

(C) 15 in³/sec

(D) 20 in³/sec

(E) 25 in³/sec

26) Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval $[0,2]$.

(A) –3

(B) 2

(C) 4

(D) 8

(E) 24

27) If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

(A) $\left(0, \frac{1}{3}\right)$　　　(B) $\left(\frac{1}{3}, \frac{2}{3}\right)$　　　(C) $\left(\frac{1}{3}, \frac{5}{3}\right)$　　　(D) $\left(\frac{5}{3}, 3\right)$　　　(E) $(3, \infty)$

28) The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) $(0, 0)$ and $(2, -64)$

(B) $(0, 0)$ and $(3, -81)$

(C) $(0, 0)$ only

(D) $(-3, 81)$ only

(E) $(2, -64)$ only

29) The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

(B) 0 and –3 only

(C) –3 and 3 only

(D) 0 only

(E) –3, 0, 3

30) Find all open intervals for which the function $f(x) = \frac{x}{x^2+x-2}$ is decreasing.

   a. $(-\infty, \infty)$ b. $(-\infty, 0)$ c. $(-\infty, -2)$ and $(1, \infty)$ d. $(-\infty, -2), (-2, 1)$ and $(1, \infty)$ e. none of these

31) Find all intervals on which the graph of the function $f(x) = \frac{x-1}{x+3}$ is concave upward.

   a. $(-\infty, \infty)$ b. $(-\infty, -3)$ c. $(1, \infty)$ d. $(-3, \infty)$ e. none of these

32) On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

(A) $(0, 1)$

(B) $\left(0, \frac{1}{2}\right)$

(C) $\left(0, \frac{1}{4}\right)$

(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(E) $\left(\frac{1}{4}, 1\right)$

33. If c is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval $[0, \sqrt{3}]$, c is

   a. 1   b. -1   c. $\sqrt{2}$ d. 0   e. $\sqrt{3}$

34) If $f(x) = \cos(3x)$, then $f'\left(\dfrac{\pi}{9}\right) =$

  (A) $\dfrac{3\sqrt{3}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $-\dfrac{\sqrt{3}}{2}$ (D) $-\dfrac{3}{2}$ (E) $-\dfrac{3\sqrt{3}}{2}$

Free Response Practice Problems:

   I.   **Limit problem/Continuity/Discontinuities**

1) If $f(x) = \begin{cases} 4 - x^2, x < 3 \\ 2, \quad\quad x = 3 \\ 5 - 4x, x > 3 \end{cases}$, then find the following

  a) $\lim\limits_{x \to 3} f(x) =$ b) $f(3) =$

  c) Use continuity condition to determine if function is continuous or not continuous at x = 3 (If not continuous, determine type of discontinuity)
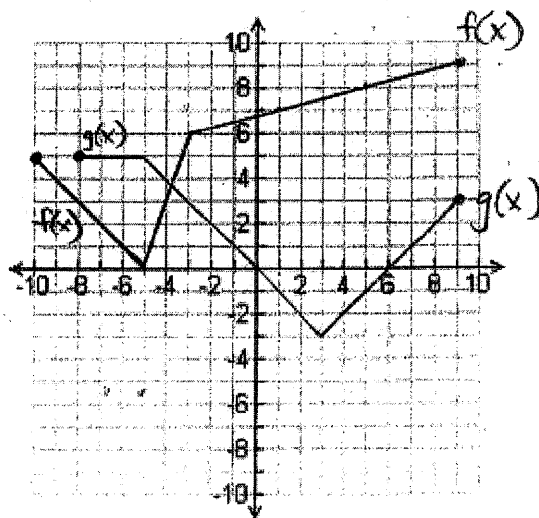
II.   **Find Derivative using Graph**

The graphs of f and g are shown. Let $h(x) = g\big(f(x)\big)$. Let $p(x) = \frac{g(x)}{f(x)}$. Let $q(x) = f(x)g(x)$

a) Find q'(3)

b) Find p'(-1)

c) Find h' $\left(-6\right)$

Moving Ladder   A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

(a) How fast is the top of the ladder moving down the wall when its base is 7 feet, 15 feet, and 24 feet from the wall?

(b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

(c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.

IV. Optimization

A rectangle has a perimeter of 80 cm. If its width is x, express its length and area in terms of x, and find the maximum area.

Suppose you had to use exactly 200 m of fencing to make either one square enclosure or two separate square enclosures of any size you wished. What plan would give you the least area? What plan would give you the greatest area?