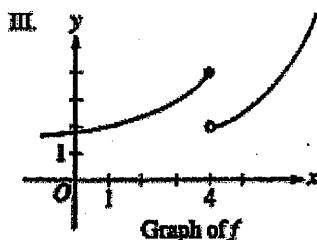
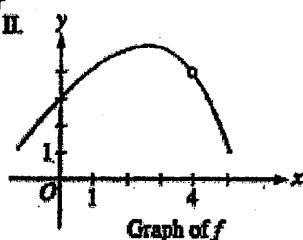
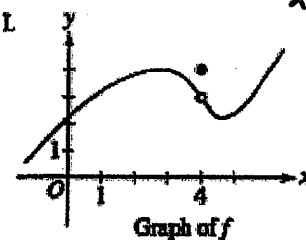


Key

Non-AP Calculus Fall Final Review Packet

Chapter 1 Limits:

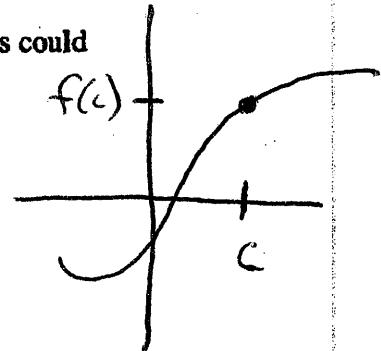
- 1) For which of the following does $\lim_{x \rightarrow 4} f(x)$ exist?



- A) I only
B) II only
C) III only
D) I and II only
E) I and III only

- 2) The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

- ✓ (A) $\lim_{x \rightarrow c} f(x)$ exists ✓ (B) $\lim_{x \rightarrow c} f(x) = f(c)$ ✓ (C) $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$
 ✓ (D) $f(c)$ is defined (E) $f'(c)$ exists *not necessarily*



$$3) \lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3} = \frac{0}{0}$$

- a. 0 b. 1

$$\text{c. } \frac{1}{4}$$

- d. ∞ e. none of these

$$\lim_{x \rightarrow 3} \frac{(x-3)}{(x-3)(x+1)}$$

$$\lim_{x \rightarrow 3} \frac{1}{x+1} = \frac{1}{4} = \boxed{\frac{1}{4}}$$

$$4) \lim_{x \rightarrow \infty} \frac{4-x^2}{4x^2-x-2} =$$

- a. -2

$$\boxed{\text{b. } -\frac{1}{4}}$$

- c. 1

- d. 2

- e. nonexistent

*compare degrees

$$\lim_{x \rightarrow \infty} \frac{4-x^2}{4x^2-x-2} = \boxed{-\frac{1}{4}}$$

$$i) f(1) = 4$$

$$iii) f(1) \neq \lim_{x \rightarrow 1} f(x)$$

$$ii) \lim_{x \rightarrow 1} x^2 - 1 = 0$$

5) Let $f(x) = \begin{cases} x^2 - 1, & x \neq 1 \\ 4, & x = 1 \end{cases}$

Which of the following statements are true?

✓ I. $\lim_{x \rightarrow 1} f(x)$ exists

✓ II. $f(1)$ exists

X III. f is continuous at $x = 1$

a. I only b. II only

c. I and II

d. I, II and III

e. none of them

*ignore one-sided limit

$$6) \lim_{x \rightarrow 4^+} \frac{4x^2 - 14x - 8}{x-4} = \frac{0}{0} \quad \lim_{x \rightarrow 4^+} \frac{2(2x^2 - 7x - 4)}{(x-4)}$$

$$\frac{-8 - 32/4}{2 - 1^2} = \frac{1}{2} \quad (x-4)(x+1/2)$$

A) 18

B) 9

C) ∞

D) $-\infty$

E) Does Not Exist

$$\lim_{x \rightarrow 4^+} \frac{2(x-4)(2x+1)}{(x-4)} = 2(2(4)+1) = 2(9) = \boxed{18}$$

7)

$$\lim_{x \rightarrow 1} \left(\frac{\sqrt{x+3} - 2}{1-x} \right) = \lim_{x \rightarrow 1} \frac{\sqrt{1+3} - 2}{1-1} = \frac{0}{0}$$

(A) 0.5

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x+3} - 2)(\sqrt{x+3} + 2)}{(1-x)(\sqrt{x+3} + 2)}$$

(C) 0

(D) -0.25

$$\lim_{x \rightarrow 1} \frac{x+3-4}{(1-x)(\sqrt{x+3} + 2)}$$

(E) -0.5

$$\lim_{x \rightarrow 1} \frac{(x-1)}{(1-x)(\sqrt{x+3} + 2)}$$

$$\lim_{x \rightarrow 1} \frac{-1}{\sqrt{x+3} + 2} = \frac{-1}{\sqrt{4} + 2}$$

$$= \frac{-1}{2+2} = \boxed{-\frac{1}{4}}$$

$$= -0.25$$

$$8) \lim_{x \rightarrow 2^-} \frac{(x+2)^2}{x^2 - 4} = \frac{4^2}{0} = \frac{16}{0} \rightarrow +\infty$$

A) 4

B) 1

C) ∞

D) $-\infty$

E) Does Not Exist

$$\lim_{x \rightarrow 2^-} \frac{(1.9+2)^2}{(1.9^2 - 4)} = \frac{+}{-} = \boxed{-\infty}$$

test 1.9

$$9) \lim_{x \rightarrow -11^-} \frac{\sqrt{x+19}-\sqrt{8}}{x+11} = \frac{0}{0}$$

A) $\frac{1}{2\sqrt{8}}$

B) $-\frac{1}{2\sqrt{8}}$

C) ∞

D) $-\infty$

E) Does Not Exist

$$\lim_{x \rightarrow -11^-} \frac{\sqrt{x+19}-\sqrt{8}}{x+11} \cdot \frac{\sqrt{x+19}+\sqrt{8}}{\sqrt{x+19}+\sqrt{8}}$$

$$\lim_{x \rightarrow -11^-} \frac{x+19-8}{(x+11)(\sqrt{x+19}+\sqrt{8})} = \lim_{x \rightarrow -11^-} \frac{x+11}{(x+11)(\sqrt{x+19}+\sqrt{8})} = \frac{1}{\sqrt{8}+\sqrt{8}} = \boxed{\frac{1}{2\sqrt{8}}}$$

$$10) \lim_{x \rightarrow -\infty} \frac{5x^4-5x-5}{(2x^2-3)^2} = \lim_{x \rightarrow -\infty} \frac{5x^4-5x-5}{(2x^2-3)(2x^2-3)}$$

A) $\frac{5}{2}$

B) $\frac{5}{4}$

C) ∞

D) $-\infty$

E) Does Not Exist

* Compare degrees $\lim_{x \rightarrow \infty} \frac{5x^4}{4x^4} \dots$

$$\boxed{\frac{5}{4}}$$

$$11) \lim_{x \rightarrow 1} \frac{(x+1)^2}{x^2-1} = \frac{2^2}{0} = \boxed{\frac{4}{0}}$$

A) 1

B) 2

C) ∞

D) $-\infty$

E) Does Not Exist

$$12) \lim_{x \rightarrow -\infty} \frac{4-x^3}{2-x^2} = \begin{cases} +\infty \\ -\infty \end{cases}$$

A) 4

B) 2

C) ∞

D) $-\infty$

E) -2

* compare degrees

test $x = -100$

$$\frac{4-(-100)^3}{2-(-100)^2} =$$

13)

OMIT

Evaluate the limits algebraically or state that the limit does not exist.

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^3 - 4x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - 2}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$$

$$\lim_{x \rightarrow 0} \frac{(3a+x)^2 - 9a^2}{x}$$

$$\lim_{x \rightarrow 8} \frac{x^2 - 64}{x - 9}$$

14)

What is $\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{(3-x)(3+x)}$?

(A) -9

(B) -3

(C) 1

(D) 3

(E) The limit does not exist.

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{9-x^2} = \frac{-3}{1} = \boxed{-3}$$

Limits Review

Refer to the graph of $g(x)$ below in order to answer the following questions. If a limit doesn't exist, explain why.

1. $\lim_{x \rightarrow \infty} g(x) =$

2. $\lim_{x \rightarrow -\infty} g(x) =$

3. $\lim_{x \rightarrow a^+} g(x) =$

4. $\lim_{x \rightarrow a^-} g(x) =$

5. $\lim_{x \rightarrow a} g(x) =$

6. $\lim_{x \rightarrow 0} g(x) =$

7. $\lim_{x \rightarrow b^+} g(x) =$

8. $\lim_{x \rightarrow b^-} g(x) =$

9. $\lim_{x \rightarrow b} g(x) =$

10. $\lim_{x \rightarrow c} g(x) =$

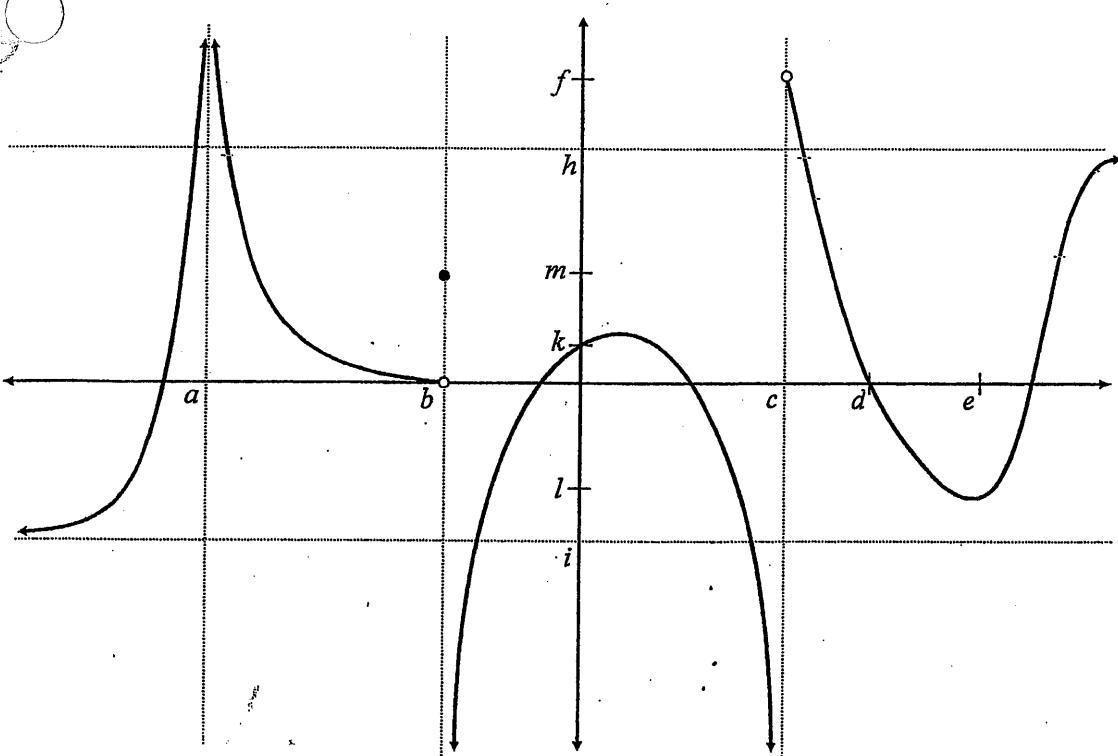
11. $\lim_{x \rightarrow d} g(x) =$

12. $\lim_{x \rightarrow e} g(x) =$

13. $g(e) =$

14. $g(0) =$

15. $g(b) =$



A.P. Calculus AB

Limits Review

Refer to the graph of $g(x)$ below in order to answer the following questions. If a limit doesn't exist, explain why.

key

1. $\lim_{x \rightarrow \infty} g(x) = h$

2. $\lim_{x \rightarrow -\infty} g(x) = l$

3. $\lim_{x \rightarrow a^+} g(x) = +\infty$

4. $\lim_{x \rightarrow a^-} g(x) = +\infty$

5. $\lim_{x \rightarrow a} g(x) = +\infty$

6. $\lim_{x \rightarrow 0} g(x) = k$

7. $\lim_{x \rightarrow b^+} g(x) = -\infty$

8. $\lim_{x \rightarrow b^-} g(x) = o$

9. $\lim_{x \rightarrow b} g(x) = \text{DNE}$, nonremovable discontinuity
 $\lim_{x \rightarrow b^-} g(x) \neq \lim_{x \rightarrow b^+} g(x)$

10. $\lim_{x \rightarrow c} g(x) = \text{DNE}$ (nonremovable discontinuity) $\lim_{x \rightarrow c^-} g(x) \neq \lim_{x \rightarrow c^+} g(x)$

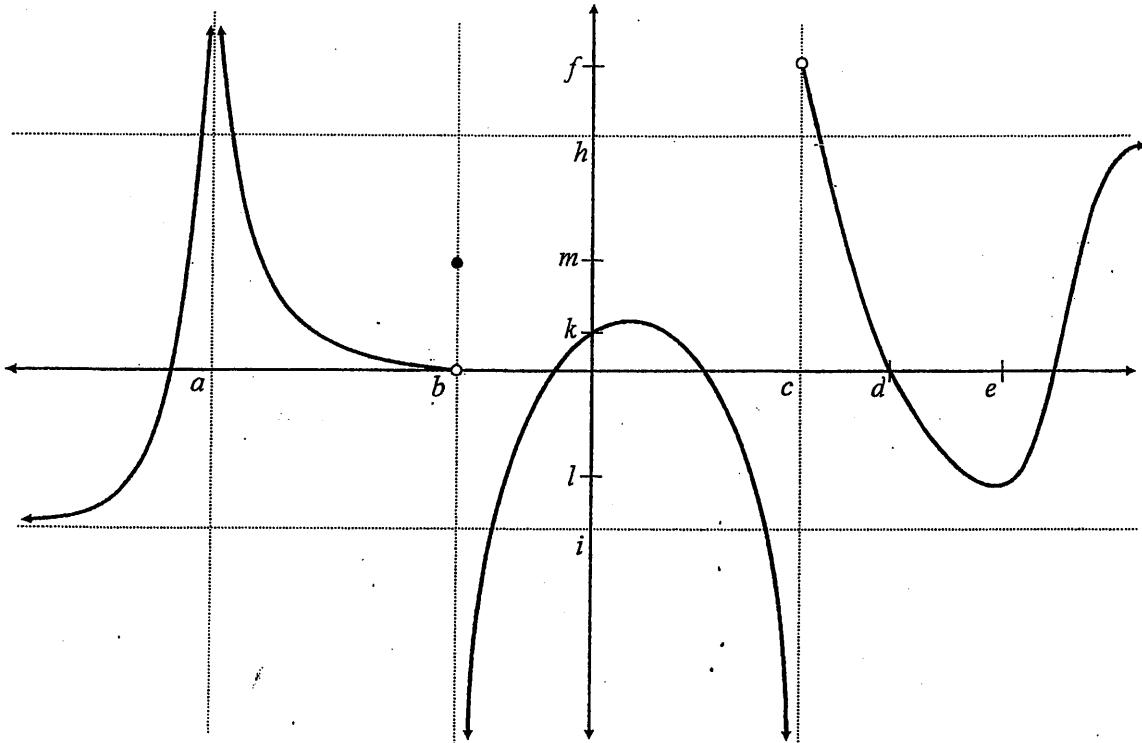
11. $\lim_{x \rightarrow d} g(x) = \text{o}$

12. $\lim_{x \rightarrow e} g(x) = l$

13. $g(e) = l$

14. $g(0) = k$

15. $g(b) = m$



Chapter 2 Derivatives

16) Find the point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3) at which the tangent to the graph has the same slope as the line through (1, 1) and (9, 3).

(A) (1, 1)

(B) (2, $\sqrt{2}$)

(C) (3, $\sqrt{3}$)

(D) (4, 2)

(E) none of the above

$$*MVT: \text{set } f'(c) = \frac{f(9)-f(1)}{9-1} = \frac{f(9)-f(1)}{8}$$

$$\frac{f(9)-f(1)}{9-1} = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\begin{array}{l|l} \frac{1}{2\sqrt{x}} = \frac{1}{4} & \sqrt{x} = 2 \\ 2\sqrt{x} = 4 & x = 4 \end{array}$$

(4, 2)

17) If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$

(B) $2(x^3 + 1)$

(C) $2(3x^2 + 1)$

(D) $3x^2(x^3 + 1)$

(E) $6x^2(x^3 + 1)$

$$y' = 2(x^3 + 1)'(3x^2) = \boxed{6x^2(x^3 + 1)}$$

18) If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$

(B) $\frac{12x-13}{(3x+2)^2}$

(C) $\frac{5}{(3x+2)^2}$

(D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

$$y' = \frac{(2)(3x+2) - (2x+3)(3)}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2}$$

$\frac{-5}{(3x+2)^2}$

19) If $x^3 - xy + y^3 = 1$, find $\frac{dy}{dx}$

a. $\frac{3x^2}{x-3y^2}$

b. $\frac{3x^2-1}{1-3y^2}$

c. $\frac{y-3x^2}{3y^2-x}$

d. $\frac{3x^2+3y^2-y}{x}$

e. $\frac{3x^2+3y^2}{x}$

*implicit

*product rule

$$\begin{aligned} x^3 + \cancel{-xy} + y^3 &= 1 \\ 3x^2 + \cancel{-1 \cdot y} + -x \cdot \frac{dy}{dx} + 3y^2 \left(\frac{dy}{dx}\right) &= 0 \\ 3x^2 - y - x \left(\frac{dy}{dx}\right) + 3y^2 \left(\frac{dy}{dx}\right) &= 0 \end{aligned}$$

$$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$$

$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$

20) If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{3}{2x}$

$$y = 3(4+x^2)^{-1}$$

(B) $\frac{3x}{(1+x^2)^2}$

$$y' = -3(4+x^2)^{-2}(2x)$$

(C) $\frac{6x}{(4+x^2)^2}$

$$y' = -6x(4+x^2)^{-2}$$

(D) $\frac{-6x}{(4+x^2)^2}$

$$y' = \frac{-6x}{(4+x^2)^2}$$

(E) $\frac{-3}{(4+x^2)^2}$

21) If $xy + x^2 = 6$, then the value of $\frac{dy}{dx}$ at $x = -1$ is

$$xy + x^2 = 6$$

$$(-1)y + (-1)^2 = 6$$

$$-y + 1 = 6$$

$$-y = 5$$

$$y = -5$$

(E) 3 point: $(-1, -5)$

(A) -7

(B) -2

(C) 0

(D) 1

$$\frac{f'g}{f} + \frac{fg'}{f}$$

$$1 \cdot y + x \cdot \frac{dy}{dx} + 2x = 0$$

$$x \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x}$$

$$\left. \frac{dy}{dx} \right|_{(-1, -5)} = \frac{-2(-1) - (-5)}{-1} = \frac{2+5}{-1} = -7$$

22) Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point $(2, 1)$ is

(A) $\frac{2}{3}$

* product rule

(B) $\frac{1}{3}$

* implicit diff

(C) $-\frac{1}{3}$

$$1 + \frac{f'g}{f} + \frac{fg'}{f}$$

(D) $-\frac{1}{5}$

$$1 + y + x \cdot \frac{dy}{dx} + 4y \left(\frac{dy}{dx} \right) = 0$$

(E) $-\frac{3}{4}$

$$1 + y + x \left(\frac{dy}{dx} \right) + 4y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} (x+4y) = -1-y$$

$$\frac{dy}{dx} = \frac{-1-y}{x+4y}$$

$$\left. \frac{dy}{dx} \right|_{(2, 1)} = \frac{-1-1}{2+4(1)}$$

$$= \frac{-2}{6}$$

$$= \boxed{-\frac{1}{3}}$$

Chapter 2.6 Related Rates

OMIT

- 23) A balloon is being filled with helium at the rate of $4 \frac{\text{ft}^3}{\text{min}}$. Find the rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3} \text{ ft}^3$. Note, the volume of a sphere is $\frac{4}{3}\pi r^3$ and the surface area of a sphere is $4\pi r^2$.
- a. 4π b. 2 c. 4 d. 1 e. 2π

- 24) Two cars are traveling along perpendicular roads, car A at 40 mi/hour, car B at 60 mi/hour. At noon, when car A reaches the intersection, car B is 90 miles away and moving toward the intersection. At 1 pm, the distance between the cars is changing, in miles per hour, at a rate of
- a. -40 b. 68 c. 4 d. -4 e. 40

- 25) The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube? ($S = 6x^2$ $V = x^3$)

- (A) 5 in³/sec
(B) 10 in³/sec
(C) 15 in³/sec
(D) 20 in³/sec
(E) 25 in³/sec

Chapter 3 Curve Sketching $f'(x)$

26) Find the maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval $[0, 2]$.

(A) -3

*EVT

1) test endpoints

(B) 2

2) test critical pts

(C) 4

(D) 8

$$f'(x) = 6x^2 + 6x - 12$$

(E) 24

$$f'(x) = 6(x^2 + x - 2)$$

$$f'(x) = 6(x+2)(x-1)$$

$$0 = 6(x+2)(x-1)$$

$$x = -2, 1$$

$$f(0) = 0 + 0 - 0 + 4 = 4$$

$$f(1) = 2 + 3 - 12 + 4 = -3$$

$$f(2) = 2(2)^3 + 3(2)^2 - 12(2) + 4 = 8 \quad \text{Abs max}$$

$f(-2)$ = outside interval

27) If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

(A) $\left(0, \frac{1}{3}\right)$

(B) $\left(\frac{1}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{5}{3}\right)$

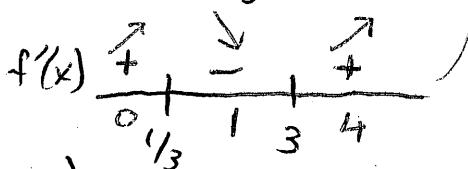
(D) $\left(\frac{5}{3}, 3\right)$

(E) $(3, \infty)$

$$f'(x) = 3x^2 - 10x + 3$$

$$0 = (3x-1)(x-3)$$

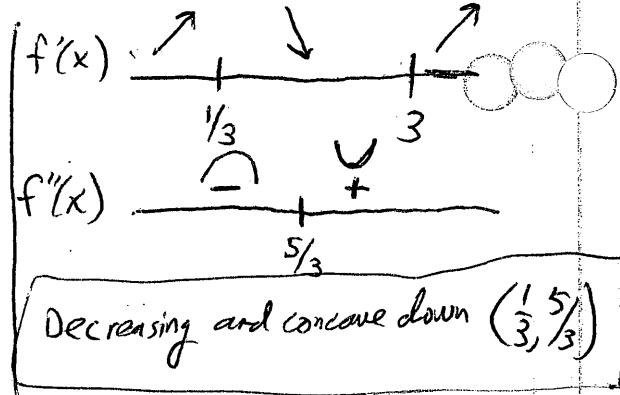
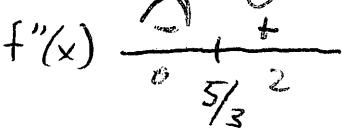
$$x = \frac{1}{3}, 3$$



$$f''(x) = 6x - 10$$

$$0 = 2(3x-5)$$

$$x = \frac{5}{3}$$



28) The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) $(0, 0)$ and $(2, -64)$

(B) $(0, 0)$ and $(3, -81)$

(C) $(0, 0)$ only

(D) $(-3, 81)$ only

(E) $(2, -64)$ only

$$y' = 15x^4 - 40x^3$$

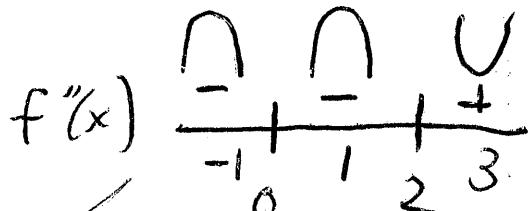
$$y'' = 60x^3 - 120x^2$$

$$0 = 60x^2(x-2)$$

$$60x^2 = 0 \quad x-2 = 0$$

$$x=0$$

$$x=2$$



POI at $x=2$ b/c

$f''(x)$ change signs

$$y(2) = 3(2)^5 - 10(2)^4 = -64$$

POI: $(2, -64)$

The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

$$f'(x) = 4x^3 - 36x$$

(B) 0 and -3 only

$$f'(x) = 4x(x^2 - 9)$$

(C) -3 and 3 only

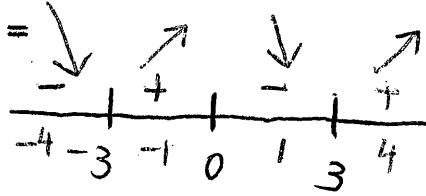
$$f'(x) = 4x(x+3)(x-3)$$

(D) 0 only

$$0 = 4x(x+3)(x-3)$$

(E) -3, 0, 3

$$x=0, 3, -3$$



Rel. min at $x=3, -3$

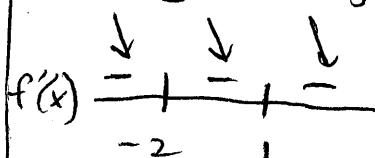
30) Find all open intervals for which the function $f(x) = \frac{x}{x^2+x-2}$ is decreasing.

- a. $(-\infty, \infty)$ b. $(-\infty, 0)$ c. $(-\infty, -2)$ and $(1, \infty)$ d. $(-\infty, -2), (-2, 1)$ and $(1, \infty)$ e. none of these

$$f'(x) = \frac{1 \cdot (x^2+x-2) - x \cdot (2x+1)}{(x^2+x-2)^2}$$

$$= \frac{x^2+x-2 - 2x^2-x}{(x^2+x-2)^2} = \frac{-x^2-2}{[(x+2)(x-1)]^2}$$

$$f'(x) = \frac{-(x^2+2)}{[(x+2)(x-1)]^2}$$



$x \neq -2, 1$

Decreasing in intervals
 $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

31) Find all intervals on which the graph of the function $f(x) = \frac{x-1}{x+3}$ is concave upward.

- a. $(-\infty, \infty)$ b. $(-\infty, -3)$ c. $(1, \infty)$ d. $(-3, \infty)$ e. none of these

*use quotient rule

$$f'(x) = \frac{(1)(x+3) - (x-1)(1)}{(x+3)^2}$$

$$f'(x) = \frac{x+3-x+1}{(x+3)^2}$$

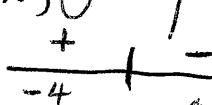
$$f'(x) = \frac{4}{(x+3)^2} = 4(x+3)^{-2}$$

$x \neq -3$

$$f''(x) = -8(x+3)^{-3}(1)$$

$$= \frac{-8}{(x+3)^3}$$

$$f''(x) > 0$$



32) OMIT

On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

(A) $(0, 1)$

(B) $\left(0, \frac{1}{2}\right)$

(C) $\left(0, \frac{1}{4}\right)$

(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(E) $\left(\frac{1}{4}, 1\right)$

33. If c is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval $[0, \sqrt{3}]$, c is

a. 1

b. -1

c. $\sqrt{2}$

d. 0

e. $\sqrt{3}$

* Rolle's Theorem: i) $f(x)$ continuous $[a, b]$

ii) $f(x)$ differentiable (a, b)

iii) $f(a) = f(b)$

34) If $f(x) = \cos(3x)$, then $f'(\frac{\pi}{9}) =$

- (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

$$f'(x) = -\sin(3x) \cdot 3 \quad \left| \begin{array}{l} f'(\frac{\pi}{9}) = -3 \cdot (\sin(\frac{\pi}{3})) \\ = -3 \cdot \frac{\sqrt{3}}{2} \end{array} \right.$$

$$f'(x) = -3 \sin(3x)$$

$$f'(\frac{\pi}{9}) = -3 \sin(3 \cdot \frac{\pi}{9})$$

Free Response Practice Problems:

I. Limit problem/Continuity/Discontinuities

1) If $f(x) = \begin{cases} 4 - x^2, & x < 3 \\ 2, & x = 3 \\ 5 - 4x, & x > 3 \end{cases}$, then find the following

a) $\lim_{x \rightarrow 3} f(x) = \boxed{DNE}$

b) $f(3) = \boxed{2}$

c) Use continuity condition to determine if function is continuous or not continuous at $x = 3$ (If not continuous, determine type of discontinuity)

i) $f(3) = 2$

ii) $\lim_{x \rightarrow 3^-} 4 - x^2 = \boxed{-5}$ $\lim_{x \rightarrow 3^+} 5 - 4x = 5 - 12 = \boxed{-7}$

$\lim_{x \rightarrow 3^-} f(x)$ does not exist, so $f(x)$ not continuous, and is a nonremovable discontinuity.

$$\boxed{f'(x) = 6x^2 - 6}$$

$$\boxed{f(0) = 2(0)^3 - 6(0) = 0}$$

$$\boxed{f(\sqrt{3}) = 2(\sqrt{3})^3 - 6\sqrt{3} = 0}$$

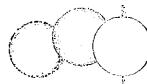
$$\boxed{6x^2 - 6 = 0}$$

$$\boxed{6(x^2 - 1) = 0}$$

$$\boxed{x^2 - 1 = 0}$$

$$x = \pm 1$$

$$\boxed{c=1} \quad c=-1$$



II. Find Derivative using Graph

The graphs of f and g are shown. Let $h(x) = g(f(x))$. Let $p(x) = \frac{g(x)}{f(x)}$. Let $q(x) = f(x)g(x)$

a) Find $q'(3)$

*product rule

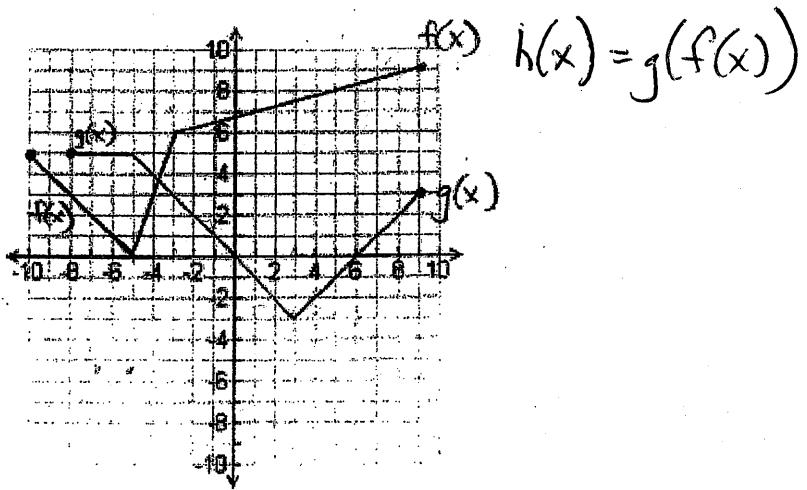
$$g'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$g'(3) = f'(3) \cdot g(3) + f(3) \cdot g'(3)$$

$$\left(\frac{1}{4}\right) \cdot (-3) + (7.5)(\text{undefined}) = \text{undefined}$$

b) Find $p'(-1)$

c) Find $h'(-6)$



$$b) p'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{[f(x)]^2} \quad \left| \begin{array}{l} p'(-1) = \frac{g'(-1)f(-1) - g(-1)f'(-1)}{[f(-1)]^2} \\ p'(-1) = \frac{(-1)(6.5) - (1)(\frac{1}{4})}{[6.5]^2} \end{array} \right.$$

$$p'(-1) = \frac{(-1)(6.5) - (1)(\frac{1}{4})}{[6.5]^2} = \boxed{-0.1597}$$

$$h(x) = g(f(x))$$

$$c) h'(x) = g'(f(x)) \cdot f'(x)$$

*chain rule $h'(-6) = g'[f(-6)] \cdot f'(-6)$

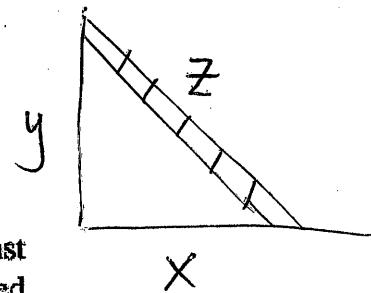
$$= g'[1] \cdot f'(-6)$$

$$= g'(1) \cdot f'(-6)$$

$$= (-1) \cdot (-1)$$

$$= \boxed{1}$$

III. Related Rates



Moving Ladder A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

- (a) How fast is the top of the ladder moving down the wall when its base is 7 feet from the wall?

$$x^2 + y^2 = z^2$$

$$x = 7 \quad \frac{dx}{dt} = 2$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$$

$$y = 24 \quad \frac{dy}{dt} = -$$

$$z = 25 \quad \frac{dz}{dt} = 0$$

$$z^2 + y^2 = 25^2$$

$$49 + y^2 = 625$$

$$y = 24$$

- (b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right) \cdot y + \frac{1}{2}x \cdot \frac{dy}{dt}$$

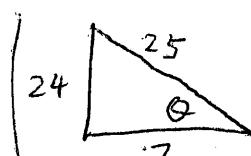
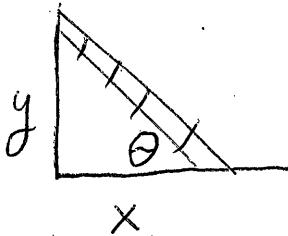
$$\frac{dA}{dt} = \frac{1}{2}(2)(24) + \frac{1}{2}(7)\left(-\frac{7}{12}\right)$$

$$= 24 - \frac{49}{24}$$

$$= \frac{527}{24} \text{ or}$$

$$21.958 \text{ ft}^2/\text{s}$$

- (c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.



$$\cos \theta = \frac{7}{25}$$

$$\frac{d\theta}{dt} = \frac{\left(-\frac{7}{12}\right)(7) - (24)(2)}{7^2} \cdot \left(\frac{7}{25}\right)^2$$

$$\frac{d\theta}{dt} = -\frac{1}{12} \text{ rad/sec}$$

$$\tan \theta = \frac{y}{x}$$

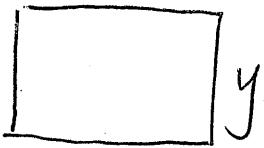
$$\sec^2 \theta \left(\frac{d\theta}{dt}\right) = \frac{\left(\frac{dy}{dt}\right)(x) - (y)\left(\frac{dx}{dt}\right)}{x^2}$$

$$\frac{d\theta}{dt} = \frac{\frac{dy}{dt}(x) - (y)\left(\frac{dx}{dt}\right)}{x^2} \cdot (\cos \theta)^2$$

optimize Area

IV. Optimization

A rectangle has a perimeter of 80 cm. If its width is x , express its length and area in terms of x , and find the maximum area.



$$A = xy$$

$$P = 2x + 2y$$

$$80 = 2x + 2y$$

$$\frac{80 - 2x}{2} = \frac{2y}{2}$$

$$40 - x = y$$

$$A = x(40 - x)$$

$$A = 40x - x^2$$

$$A'(x) = 40 - 2x$$

$$0 = 40 - 2x$$

$$2x = 40$$

$$x = 20$$

$$y = 40 - x$$

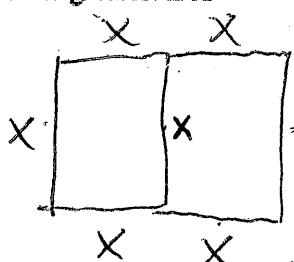
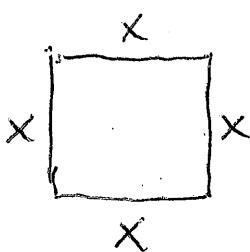
$$y = 40 - 20 \boxed{= 20}$$

Find max. Area

$$A = (20)(20)$$

$$A = 400 \text{ cm}^2$$

Suppose you had to use exactly 200 m of fencing to make either one square enclosure or two separate square enclosures of any size you wished. What plan would give you the least area? What plan would give you the greatest area?



$$A = (2x)(x) = 2x^2$$

$$P = 7x$$

$$200 = 7x$$

$$A = x^2$$

$$P = 4x$$

$$200 = 4x$$

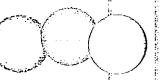
$$50 = x$$

$$\cancel{A(x)} = x^2$$

$$X = 50 \text{ m}$$

$$\frac{200}{7} = x$$

$$X = \frac{200}{7} \text{ m}$$



Ch. 1 Limits

Steps for Important Concepts

Algebraic Steps Evaluating Limits Approaching a Real Number $\lim_{x \rightarrow c} f(x)$

1. Plug in argument x-value (Ignore one-sided limit for now)
2. Find the Limit (plug in/ reduce if $\frac{0}{0}$, re-evaluate)
3. If Limit DNE (does not exist), then evaluate further ONLY IF one-sided limit)
4. Choose between $+\infty$ and $-\infty$
5. Plug in the appropriate decimal value to determine $+\infty$ or $-\infty$

II. Algebraic Steps Evaluating Limits Approaching Infinity $\lim_{x \rightarrow \infty} f(x)$ or $\lim_{x \rightarrow -\infty} f(x)$

1. Compare degrees between numerator vs. denominator
 - a. If Numerator < Denominator $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$
 - b. If Numerator = Denominator $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \text{ratio of coefficients}$
 - c. If Numerator > Denominator, then $\lim_{x \rightarrow \infty} f(x) = \pm\infty$ or $\lim_{x \rightarrow -\infty} f(x) = \pm\infty$

(Plug in a large positive or large negative value to help you determine the sign at infinity)

Continuity Conditions

1. $f(c)$ is defined (point exists on the graph)
 2. The $\lim_{x \rightarrow c} f(x)$ exists $\left[\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x) \right]$
 3. $f(c) = \lim_{x \rightarrow c} f(x)$
- If function passes all 3 conditions , the function has continuity at $x = c$
 - If condition #2 FAILS, the function has **nonremovable** discontinuity at $x = c$
 - If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at $x = c$

IV. Intermediate Value Theorem (IVT) Steps

1. Test and determine continuity on closed interval $[a, b]$
2. Find the y-value at the endpoints , $f(a)$ and $f(b)$
3. Confirm that $f(c)$ is between $f(a)$ and $f(b)$ [example: $f(a) < f(c) < f(b)$]
4. Find the c-value (find the x-value by plugging the y-value given at the start of problem into the function)

*Make sure c-value(s) are inside the interval $[a,b]$. c-values that are outside the interval $[a,b]$ are excluded.

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