Non-AP Calculus 2.2-2.4 Derivatives Quiz Review WS #1

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[4]{5x^3 - 2x + 9\pi}$

3. Find g'(x) for $g(x) = x^3(x^2-7)^5$ (simplify fully in factored form)

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{x^3}{1+x^2}\right)^5$ (simplify fully in factored form)

5. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

6. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 9t^2 + 15t - 7$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

b) What is its velocity at $t = 2$ seconds?

c) What is its acceleration at t = 4 seconds?

d) Find the average velocity of particle in [3, 8]

e) When is the particle at rest?

f) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification.

g) What is displacement of particle from t = 2 to t = 6? Show work.

h) What is the total distance of particle from t = 2 to t = 6? Show work.

i) Is the speed increasing or decreasing at t = 4? Justify.

j) Is velocity increasing or decreasing at t = 2? Justify.

Solution Key

Non-AP Calculus 2.2-2.4 Quiz Review WS #1

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

$y = 7x^4 - 7x^3 - \frac{3}{11}x^2 + 4\pi x - 5\pi^4 + x^{4/5} + 5x^{-7/2}$

$y' = 28x^3 - 21x^2 - \frac{3}{11}\cdot 2x + 4\pi - 0 + \frac{4}{5}x^{-1/5} + 5\cdot -\frac{7}{2}x^{-9/2}$

$$\boxed{\frac{dy}{dx} = 28x^3 - 21x^2 - \frac{6}{11}x + 4\pi + \frac{4}{5x^{1/5}} - \frac{35}{2x^{9/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[4]{5x^3 - 2x + 9\pi}$

* chain rule

outside: $(\ )^{1/4}$

inside: $5x^3 - 2x + 9\pi$

$y = (5x^3 - 2x + 9\pi)^{1/4}$

$y' = \frac{1}{4}(5x^3 - 2x + 9\pi)^{-3/4} \cdot (15x^2 - 2)$

$$\boxed{\frac{dy}{dx} = \frac{15x^2 - 2}{4(5x^3 - 2x + 9\pi)^{3/4}}}$$

3. Find g'(x) for $g(x) = x^3(x^2-7)^5$

* 1) product rule $f'g + fg'$
* 2) chain rule → outside: $(\ )^5$, inside: $x^2 - 7$

$g'(x) = \underbrace{3x^2\cdot(x^2-7)^5}_{x^2(x^2-7)^4} + \underbrace{x^3\cdot 5(x^2-7)^4(2x)}_{x^2(x^2-7)^4}$

$g'(x) = x^2(x^2-7)^4\left[3(x^2-7) + 10x^2\right]$

$3x^2 - 21 + 10x^2$

$$\boxed{g'(x) = x^2(x^2-7)^4(13x^2-21)}$$

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{x^3}{1+x^2}\right)^5$

* 1) chain rule → outside: $(\ )^5$, inside $\frac{x^3}{1+x^2}$
* 2) quotient rule

$y' = 5\left(\frac{x^3}{1+x^2}\right)^4 \cdot \left[\frac{3x^2(1+x^2) - x^3(2x)}{(1+x^2)^2}\right]$

$y' = 5\left(\frac{x^3}{1+x^2}\right)^4 \left[\frac{3x^2 + 3x^4 - 2x^4}{(1+x^2)^2}\right]$

$y' = \frac{5x^{12}(x^4+3x^2)}{(1+x^2)^6} = \frac{5x^{12}\cdot x^2(x^2+3)}{(1+x^2)^6}$

$$\boxed{\frac{dy}{dx} = \frac{5x^{14}(x^2+3)}{(1+x^2)^6}}$$

5. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

* quotient rule

$$f'(x) = \frac{(1)(x^2-2) - (x+4)(2x)}{(x^2-2)^2} \quad -2x(x+4)$$

$$f'(x) = \frac{x^2-2-2x^2-8x}{(x^2-2)^2}$$

$$f'(x) = \frac{-x^2-8x-2}{(x^2-2)^2}$$

point: $f(1) = \frac{1+4}{1^2-2} = \frac{5}{-1} = -5$

slope: $f'(1) = \frac{-1^2-8(1)-2}{(1^2-2)^2} = -11$

point: $(1, -5)$

slope: $m = -11$

$y+5 = -11(x-1)$

6. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 9t^2 + 15t - 7$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

$v(t) = 3t^2 - 18t + 15$

$a(t) = 6t - 18$

b) What is its velocity at $t = 2$ seconds?

$v(2) = 3(2)^2 - 18(2) + 15 = \boxed{-9 \text{ ft/s}}$

c) What is its acceleration at t = 4 seconds?

$a(4) = 6(4) - 18 = \boxed{6 \text{ ft/s}^2}$

d) Find the average velocity of particle in [3, 8]

avg. velocity = $\frac{\text{change in position}}{\text{change in time}} = \frac{x(8)-x(3)}{8-3}$

$x(8) = 49$

$x(3) = -16$

Avg. velocity = $\frac{49-(-16)}{8-3} = \frac{65}{5} = \boxed{13 \text{ ft/s}}$

e) When is the particle at rest? * set $v(t) = 0$

$v(t) = 3t^2 - 18t + 15$

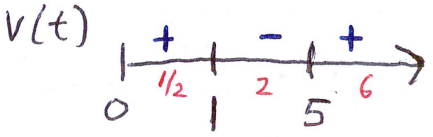
$0 = 3(t^2 - 6t + 5)$

$0 = 3(t-5)(t-1)$

$\boxed{t = 1, 5 \text{ seconds}}$

b/c $v(t) = 0$

f) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification. $v(t) = 3(t-5)(t-1)$



particle moving right $[0, 1) \cup (5, \infty)$ b/c $v(t) > 0$

particle change directions at $t = 1, 5$ seconds b/c $v(t)$ change signs.

g) What is displacement of particle from t = 2 to t = 6? Show work.

* displacement = final position − initial position

$= x(6) - x(2)$

$x(6) = -25$

$x(2) = -5$

$= -25 - (-5) = \boxed{-20 \text{ ft}}$

h) What is the total distance of particle from t = 2 to t = 6? Show work. * changes direction at 5

$x(2) = -5$ → 27

$x(5) = -32$ → 7

$x(6) = -25$

$27 + 7 = \boxed{34 \text{ ft}}$

i) Is the speed increasing or decreasing at t = 4? Justify.

$v(4) = -9 \text{ ft/s}$

$a(4) = 6 \text{ ft/s}^2$

speed decreasing at $t = 4$ because $v(t)$ and $a(t)$ have opposite signs.

j) Is velocity increasing or decreasing at t = 2? Justify.

Since $a(2) = -6 \text{ ft/s}^2$

velocity is decreasing at $t = 2$ because $a(t) < 0$