Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #1

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3\left(\sqrt{x} - 1\right) - \frac{3}{7x^2} + 4\pi x - 5\pi^4 + \sqrt[5]{x} + \frac{5}{\sqrt{x^9}}$

2. Find $\frac{dy}{dx}$ for $y = \frac{7}{\sqrt[4]{(5x^3 - 2x + 9\pi)^3}}$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-x}{3+x^2}\right)^4$ (simplify fully in factored form)

4. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

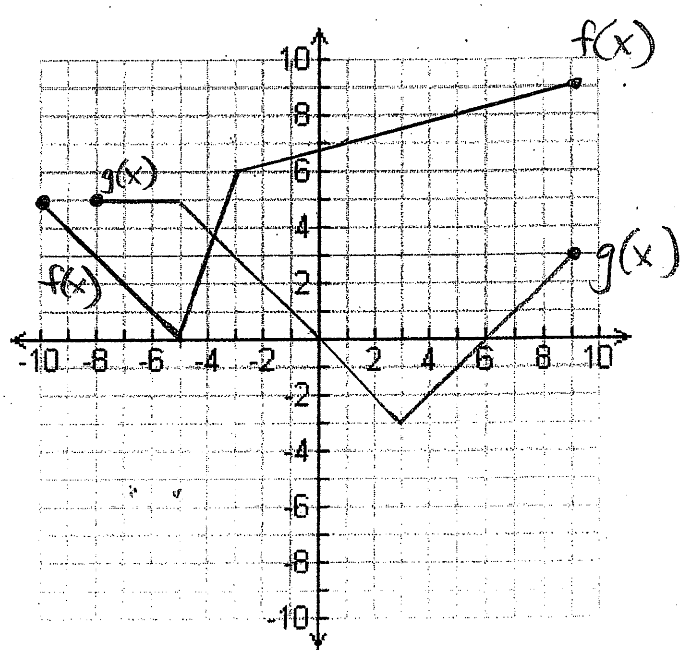
5) Find $\frac{dy}{dx}$ $for\ 4y^3x^2 - 4y + 5 = x$

6) $The\ graphs\ of\ f\ and\ g\ are\ shown.\ Let\ h(x) = g(f(x)).\ Let\ p(x) = \frac{g(x)}{f(x)}.\ Let\ q(x) = f(x)g(x)$

a) Find q'(3)

b) Find p'(-1)

c) Find h'(-6)



Solution Key

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No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(\sqrt{x} - 1) - \frac{3}{7x^2} + 4\pi x - 5\pi^4 + \sqrt[5]{x} + \frac{5}{\sqrt{x^9}}$ $\quad \frac{5}{x^{9/2}}$

$$y = 7x^3(x^{1/2} - 1) - \frac{3}{7}x^{-2} + 4\pi x - 5\pi^4 + x^{1/5} + 5x^{-9/2}$$

$$y = 7x^{7/2} - 7x^3 - \frac{3}{7}x^{-2} + 4\pi x - 5\pi^4 + x^{1/5} + 5x^{-9/2}$$

$$y' = 7 \cdot \frac{7}{2}x^{5/2} - 21x^2 + \frac{6}{7}x^{-3} + 4\pi - 0 + \frac{1}{5}x^{-4/5} - \frac{45}{2}x^{-11/2}$$

$$\boxed{y' = \frac{49}{2}x^{5/2} - 21x^2 + \frac{6}{7x^3} + 4\pi + \frac{1}{5x^{4/5}} - \frac{45}{2x^{11/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \frac{7}{\sqrt[4]{(5x^3 - 2x + 9\pi)^3}}$

$$y = \frac{7}{(5x^3 - 2x + 9\pi)^{3/4}} = 7(5x^3 - 2x + 9\pi)^{-3/4}$$

$$y' = \frac{-21}{4}(5x^3 - 2x + 9\pi)^{-7/4} \cdot (15x^2 - 2)$$

$$\boxed{y' = \frac{-21(15x^2 - 2)}{4(5x^3 - 2x + 9\pi)^{7/4}}}$$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-x}{3+x^2}\right)^4$ (simplify fully in factored form)

*chain rule:
outside: $5(\ )^4$
inside: $\frac{1-x}{3+x^2}$

$$y' = 5 \cdot 4\left(\frac{1-x}{3+x^2}\right)^3 \cdot \left[\frac{\overbrace{(-1)(3+x^2) - (1-x)(2x)}^{-3-x^2-2x+2x^2}}{(3+x^2)^2}\right]$$

$$y' = 20\left(\frac{1-x}{3+x^2}\right)^3 \left[\frac{x^2 - 2x - 3}{(3+x^2)^2}\right]$$

$$\boxed{y' = \frac{20(1-x)^3(x^2 - 2x - 3)}{(3+x^2)^5}}$$

4. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$$f'(x) = \frac{(1)(x^2 - 2) - (x+4)(2x)}{(x^2 - 2)^2}$$

$$f'(x) = \frac{x^2 - 2 - 2x^2 - 8x}{(x^2 - 2)^2}$$

$$f'(x) = \frac{-x^2 - 8x - 2}{(x^2 - 2)^2}$$

$$f(1) = \frac{1+4}{1^2 - 2} = -5$$

$$f'(1) = \frac{-1 - 8 - 2}{(-1)^2} = -11$$

point: $(1, -5)$

Slope: $m = -11$

$$\boxed{y + 5 = -11(x - 1)}$$

5) Find $\frac{dy}{dx}$ for $\underbrace{4y^3}_{f}\underbrace{x^2}_{g} - 4y + 5 = x$

$$\overbrace{12y^2\left(\tfrac{dy}{dx}\right)}^{f'}\cdot\overbrace{x^2}^{g} + \overbrace{4y^3}^{f}\cdot\overbrace{2x}^{g'} - 4\left(\frac{dy}{dx}\right) = 1$$

$$12x^2y^2\left(\frac{dy}{dx}\right) + 8xy^3 - 4\left(\frac{dy}{dx}\right) = 1$$

$$12x^2y^2\left(\frac{dy}{dx}\right) - 4\left(\frac{dy}{dx}\right) = 1 - 8xy^3$$

$$\frac{dy}{dx}\left(12x^2y^2 - 4\right) = 1 - 8xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{1-8x^3y}{12x^2y^2-4}}$$

6) *The graphs of f and g are shown. Let* $h(x) = g(f(x))$. *Let* $p(x) = \frac{g(x)}{f(x)}$. *Let* $q(x) = f(x)g(x)$

a) Find q'(3)

product rule

$$q(x) = f(x)g(x)$$

$$q'(x) = f'(x)g(x) + f(x)g'(x)$$

$$q'(3) = f'(3)\cdot g(3) + f(3)\cdot g'(3)$$

$$q'(3) = \left(\frac{1}{4}\right)(-3) + (7.5)(\text{undefined})$$

$$\boxed{q'(3) = \text{undefined}}$$

b) Find p'(-1)

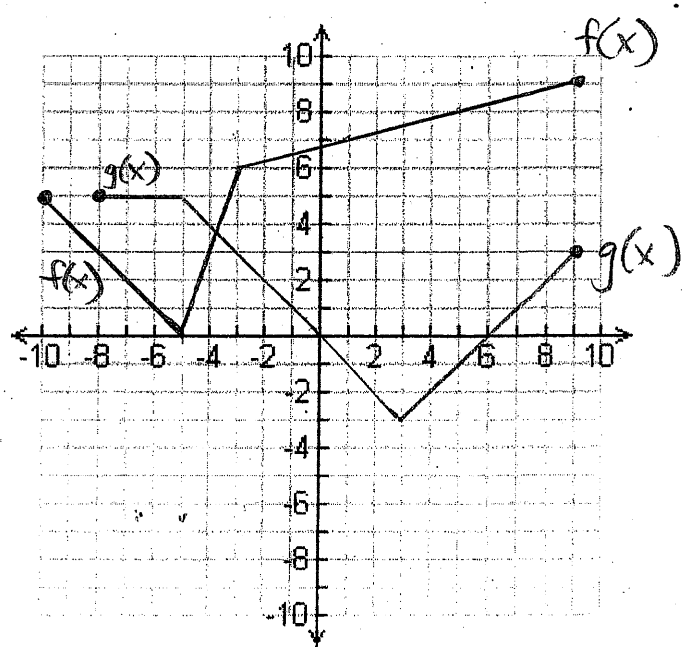
$$p(x) = \frac{g(x)}{f(x)}$$

$$p'(x) = \frac{g'(x)f(x) - g(x)\cdot f'(x)}{[f(x)]^2}$$

$$p'(-1) = \frac{g'(-1)f(-1) - g(-1)f'(-1)}{[f(-1)]^2} = \frac{(-1)(6.5)-(+1)\left(\frac{1}{4}\right)}{[6.5]^2}$$

$$= \frac{-27}{169}$$

$$\approx \boxed{-0.159}$$



c) Find h'(-6)

chain rule

$$h(x) = g(f(x))$$

$$h'(x) = g'[f(x)]\cdot f'(x)$$

$$h'(-6) = g'[f(-6)]\cdot f'(-6)$$

$$= g'[1]\cdot f'(-6)$$

$$= (-1)\cdot(-1)$$

$$\boxed{h'(-6) = 1}$$