Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #2

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5x^3(3 - 2\sqrt{x}) - \frac{3}{\pi x^4} + 4\pi x^3 - 4\pi^4 + \frac{\sqrt[6]{x}}{2} + \frac{2}{\sqrt{x^9}}$

2. Find $\frac{dy}{dx}$ for $y = \frac{6}{\sqrt[5]{(3x^3 - 5x + 2\pi)^4}}$

3. Find $\frac{dy}{dx}$ for $y = 2\left(\frac{1-x}{3-x^2}\right)^5$ (simplify fully in factored form)

4. If $f(x) = \frac{7-x}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

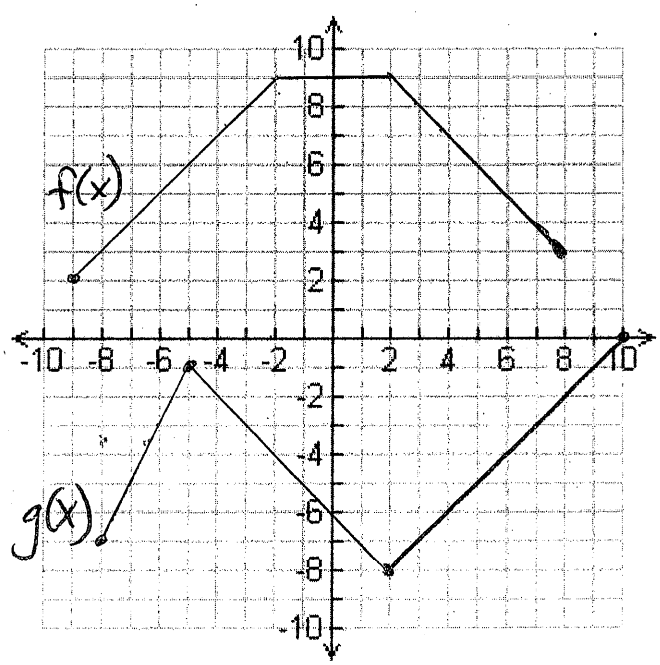
5) Find $\frac{dy}{dx}$ $for\ \ 2y^2x^5 - y + 5\pi = x$

6) *The graphs of f and g are shown. Let* $h(x) = f(g(x))$. *Let* $p(x) = \frac{g(x)}{f(x)}$. *Let* $q(x) = f(x)g(x)$

a) Find q'(3)

b) Find p'(1)

c) Find h'(-7)



Key

Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #2

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5x^3(3 - 2\sqrt{x}) - \frac{3}{\pi x^4} + 4\pi x^3 - 4\pi^4 + \frac{\sqrt[6]{x}}{2} + \frac{2}{\sqrt{x^9}}$

$2x^{1/2}$

$y = 15x^3 - 10x^{7/2} - \frac{3}{\pi}x^{-4} + 4\pi x^3 - 4\pi^4 + \frac{1}{2}x^{1/6} + 2x^{-9/2}$

$y' = 45x^2 - \frac{70}{2}x^{5/2} + \frac{12}{\pi}x^{-5} + 12\pi x^2 - 0 + \frac{1}{12}x^{-5/6} - \frac{18}{2}x^{-11/2}$

$$\boxed{\frac{dy}{dx} = 45x^2 - 35x^{5/2} + \frac{12}{\pi x^5} + 12\pi x^2 + \frac{1}{12x^{5/6}} - \frac{9}{x^{11/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \frac{6}{\sqrt[5]{(3x^3-5x+2\pi)^4}}$

$y = \frac{6}{(3x^3-5x+2\pi)^{4/5}}$

$y = 6(3x^3-5x+2\pi)^{-4/5}$

$y' = 6 \cdot \frac{-4}{5}(3x^3-5x+2\pi)^{-9/5} \cdot (9x^2-5)$

$$\boxed{\frac{dy}{dx} = \frac{-24(9x^2-5)}{5(3x^3-5x+2\pi)^{9/5}}}$$

*chain rule:

outside: $6(\ )^{-4/5}$

inside: $3x^3-5x+2\pi$

3. Find $\frac{dy}{dx}$ for $y = 2\left(\frac{1-x}{3-x^2}\right)^5$ (simplify fully in factored form)

1) chain:

outside: $2(\ )^5$

inside: $\frac{1-x}{3-x^2}$ ← quotient

$$\frac{dy}{dx} = 2 \cdot 5\left(\frac{1-x}{3-x^2}\right)^4 \cdot \left[\frac{(-1)(3-x^2)-(1-x)(-2x)}{(3-x^2)^2}\right]$$

$$= 10\frac{(1-x)^4}{(3-x^2)^4} \cdot \frac{-3+x^2+2x-2x^2}{(3-x^2)^2} = \boxed{\frac{10(1-x)^4(-x^2+2x-3)}{(3-x^2)^6}}$$

4. If $f(x) = \frac{7-x}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$f(1) = \frac{7-1}{1-2} = \frac{6}{-1} = -6$

$f'(x) = \frac{(-1)(x^2-2)-(7-x)(2x)}{(x^2-2)^2}$

$f'(x) = \frac{-x^2+2-14x+2x^2}{(x^2-2)^2} = \frac{x^2-14x+2}{(x^2-2)^2}$

$f'(1) = \frac{1-14+2}{(1-2)^2} = -11$

point: $(1,-6)$

slope: $m = -11$

$y - y_1 = m(x - x_1)$

$\boxed{y+6 = -11(x-1)}$

5) Find $\frac{dy}{dx}$ *for* $\overbrace{2y^2}^{f}\overbrace{x^5}^{g} - y + 5\pi = x$

$$\overbrace{4y\left(\frac{dy}{dx}\right)}^{f'} \cdot \overbrace{x^5}^{g} + \overbrace{2y^2}^{f} \cdot \overbrace{5x^4}^{g'} - 1\left(\frac{dy}{dx}\right) + 0 = 1$$

$$4x^5y\left(\frac{dy}{dx}\right) + 10x^4y^2 - 1\left(\frac{dy}{dx}\right) = 1$$

$$4x^5y\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 1 - 10x^4y^2$$

$$\frac{dy}{dx}\left(4x^5y - 1\right) = 1 - 10x^4y^2$$

$$\boxed{\frac{dy}{dx} = \frac{1-10x^4y^2}{4x^5y-1}}$$

6) *The graphs of f and g are shown. Let* $h(x) = f(g(x))$. *Let* $p(x) = \frac{g(x)}{f(x)}$. *Let* $q(x) = f(x)g(x)$

a) Find q'(3)

$f(3) = 8$ $\quad g(3) = -7$

$f'(3) = -1$ $\quad g'(3) = 1$

$$q'(x) = f'(x)g(x) + f(x)g'(x)$$

$$q'(3) = f'(3)g(3) + f(3)g'(3)$$

$$= (-1)(-7) + (8)(1)$$

$$= 7 + 8 = \boxed{15}$$

$f(1) = 9$ $\quad g(1) = -7$

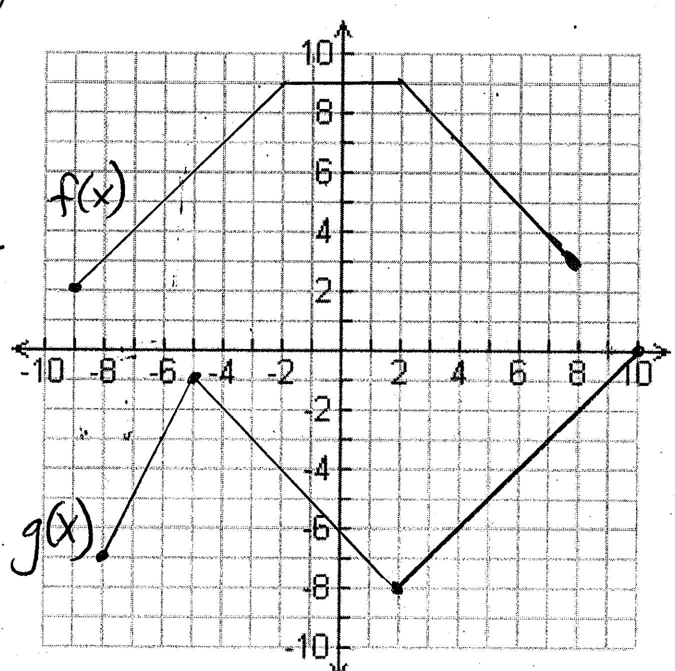
$f'(1) = 0$ $\quad g'(1) = -1$

b) Find p'(1)

$$p'(x) = \frac{g'(x)\cdot f(x) - g(x)f'(x)}{f(x)^2}$$

$$p'(1) = \frac{g'(1)f(1) - g(1)f'(1)}{f(1)^2}$$

$$p'(1) = \frac{(-1)(9) - (-7)(0)}{9^2} = \frac{-9}{81} = \boxed{\frac{-1}{9}}$$



c) Find h'(-7)

$g(-7) = -5$

$g'(-7) = 2$

$f'(-5) = 1$

$$h(x) = f(g(x))$$

$$h'(x) = f'[g(x)] \cdot g'(x)$$

$$h'(-7) = f'[g(-7)] \cdot g'(-7)$$

$$= f'[-5] \cdot g'(-7)$$

$$= (1) \cdot (2) = 2$$

$$\boxed{h'(-7) = 2}$$