Non-AP Calculus Spring Review WS #1 Show all appropriate work

1) Use left – handed rectangles to approximate $\int_{4}^{23} f(x)dx$ *using* 3 *subintervals*

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

2) Use trapezoidal sum to approximate $\int_{2}^{30} f(x)dx$ *using* 4 *subintervals*

x	2	4	7	9	16	17	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

3) $Find\ f'(x)\ for\ \ f(x) = \dfrac{2}{\sqrt{(e^{3x} - \ln 2x)^9}}$

4)$Find\ \frac{dy}{dx}\ for\ y = \ln\left(x\sqrt{(2-3x)^5}\right)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{x}{\sqrt{y}}\right) - e^{2y} = 5\pi - 15x - 3y^2$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$f(x) = 2e^{4-x}$ at point $(4,2)$

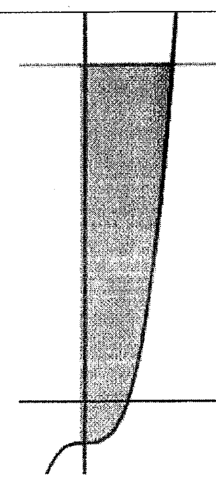
7) $\displaystyle\int \frac{2\sqrt{x} - 3x^2}{x^3}\,dx$

8) $\displaystyle\int \cot(4x) - \sec(2x)dx$

9) $\displaystyle\int \frac{5x^2}{\sqrt[3]{(1 - 3x^3)^4}}\,dx$

10)

The diagram shows the shaded region bounded by y = x^3 – 1, y = 26, and x = 0. Find the volume of the solid created by revolving the shaded region about the AOR of y = –4.



Non-AP Calculus Spring Review WS #1 Show all appropriate work

Key

1) Use left – handed rectangles to approximate $\int_4^{23} f(x)dx$ using 3 subintervals

*Area= w×h

5 8 6

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

$$\int_4^{23} f(x)dx \approx 5(1) + 8(4) + 6(7) = \boxed{79}$$

2) Use trapezoidal sum to approximate $\int_2^{30} f(x)dx$ using 4 subintervals

*Area = $\frac{w}{2}[h_1+h_2]$

5 9 5 9

x	2	4	7	9	16	17	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

$$\int_2^{30} f(x)dx = \frac{5}{2}[6+2] + \frac{9}{2}[2+5] + \frac{5}{2}[5+12] + \frac{9}{2}[12+10] = \boxed{173}$$

3) Find $f'(x)$ for $f(x) = \frac{2}{\sqrt{(e^{3x} - \ln 2x)^9}}$

$f(x) = \frac{2}{(e^{3x}-\ln 2x)^{9/2}}$ $f(x) = 2(e^{3x}-\ln 2x)^{-9/2}$

*chain rule:

outside: $2(\ )^{-9/2}$

inside: $e^{3x}-\ln 2x$

$f'(x) = 2\cdot\frac{-9}{2}(\ )^{-11/2}\cdot\left[e^{3x}\cdot 3 - \frac{2}{2x}\right]$

$f'(x) = -9(e^{3x}-\ln 2x)^{-11/2}\left(3e^{3x} - \frac{1}{x}\right)$

$$\boxed{f'(x) = \frac{-9\left(3e^{3x} - \frac{1}{x}\right)}{(e^{3x}-\ln 2x)^{11/2}}}$$

4) Find $\frac{dy}{dx}$ for $y = \ln\left(x\sqrt{(2-3x)^5}\right)$

$y = \ln\left[x\cdot(2-3x)^{5/2}\right]$

*expand first!

$y = \ln x + \ln(2-3x)^{5/2} = \ln x + \frac{5}{2}\ln(2-3x)$

$y' = \frac{1}{x} + \frac{5}{2}\cdot\frac{-3}{2-3x}$

$$\boxed{y' = \frac{1}{x} - \frac{15}{2(2-3x)}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{x}{\sqrt{y}}\right) - e^{2y} = 5\pi - 15x - 3y^2$

$\ln x - \ln y^{1/2}$

*expand first:

$\ln x - \frac{1}{2}\ln y - e^{2y} = 5\pi - 5x - 3y^2$

$\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{y}\left(\frac{dy}{dx}\right) - e^{2y}\cdot 2\left(\frac{dy}{dx}\right) = 0 - 5 - 6y\left(\frac{dy}{dx}\right)$

$\frac{-1}{2y}\left(\frac{dy}{dx}\right) - 2e^{2y}\left(\frac{dy}{dx}\right) + 6y\left(\frac{dy}{dx}\right) = -5 - \frac{1}{x}$

$\frac{dy}{dx}\left(\frac{-1}{2y} - 2e^{2y} + 6y\right) = -5 - \frac{1}{x}$

$$\boxed{\frac{dy}{dx} = \frac{-5 - \frac{1}{x}}{\frac{-1}{2y} - 2e^{2y} + 6y}}$$

6. Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$
$f(x) = 2e^{4-x}$ at point $(4,2)$

$f'(x) = 2e^{4-x}(-1) = -2e^{4-x}$

$f'(4) = -2e^{4-4} = -2e^0 = -2$

point: $(4,2)$
slope: $m = -2$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -2(x-4)}$

7) $\int \frac{2\sqrt{x} - 3x^2}{x^3} dx$

*expand/power rule:

$\int (2x^{1/2} - 3x^2) x^{-3} dx$

$\int 2x^{-5/2} - 3x^{-1} dx$

natural log rule

$\int 2x^{-5/2} dx - 3\int \frac{1}{x} dx$ (+2/2)

$\frac{2x^{-3/2}}{-3/2} - 3\ln|x| + C$

$-\frac{4}{3}x^{-3/2} - 3\ln|x| + C$

$\boxed{\frac{-4}{3x^{3/2}} - 3\ln|x| + C}$

8) $\int \cot(4x) - \sec(2x) dx$

*u-sub: * $\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \cot(4x)dx - \int \sec(2x)dx$

$u = 4x$ $dx = \frac{du}{4}$
$\frac{du}{dx} = 4$

$u = 2x$ $dx = \frac{du}{2}$
$\frac{du}{dx} = 2$

$\frac{1}{4}\int \cot u\,du$ $\frac{1}{2}\int \sec u\,du$

$\boxed{\frac{1}{4}\ln|\sin(4x)| - \frac{1}{2}\ln|\sec(2x) + \tan(2x)| + C}$

9) $\int \frac{5x^2}{\sqrt[3]{(1-3x^3)^4}} dx$

$\int \frac{5x^2}{(1-3x^3)^{4/3}} dx$

$\int 5x^2 (1-3x^3)^{-4/3} dx$

*u-sub:
$u = 1 - 3x^3$
$\frac{du}{dx} = -9x^2$
$dx = \frac{du}{-9x^2}$

$\int 5x^2 \cdot u^{-4/3} \cdot \frac{du}{-9x^2}$

$-\frac{5}{9}\int u^{-4/3} du$ (+3/3)

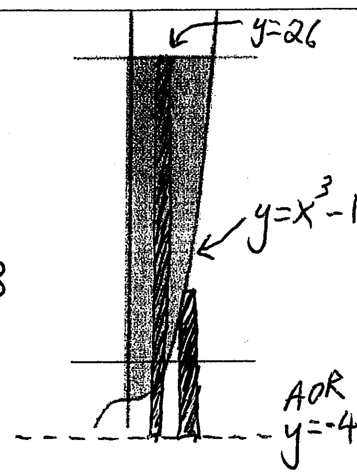
$-\frac{5}{9}\left(\frac{u^{-1/3}}{-1/3}\right) + C$

$-\frac{5}{9} \cdot -\frac{3}{1} u^{-1/3} + C$

$\boxed{\frac{15}{9(1-3x^3)^{1/3}} + C}$

10) The diagram shows the shaded region bounded by $y = x^3 - 1$, $y = 26$, and $x = 0$. Find the volume of the solid created by revolving the shaded region about the AOR of $y = -4$.

*washer method:



*intersection:
$x^3 - 1 = 26$
$x^3 = 27$
$x = \sqrt[3]{27} = 3$

$R(x) = 26 - (-4) = 30$
$r(x) = x^3 - 1 - (-4) = x^3 - 1 + 4 = x^3 + 3$

$V = \pi \int_0^3 [30]^2 - [x^3+3]^2 dx$

$\boxed{V = 2239.071\pi \text{ units}^3}$