Non-AP Calculus Spring Review WS #2 Show all appropriate work

1) Use right – handed rectangles to approximate $\int_{2}^{27} f(x)dx$ using 4 subintervals

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

2) Use trapezoidal sum to approximate $\int_{2}^{30} f(x)dx$ using 4 subintervals

x	2	5	8	12	15	18	22	25	30
f(x)	0	11	6	9	3	6	12	8	12

3) Find $f'(x)$ for $f(x) = \dfrac{2}{\sqrt[4]{(e^{7x} - \ln 6x)^5}}$

4) Find $\frac{dy}{dx}$ for $y = \ln\left(5x\sqrt{(3-6x^2)^7}\right)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{3x}{\sqrt{5y}}\right) - e^{6y} = 5\pi - 5x^3 - 4y^2$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$f(x) = 2e^{10-x^2}$ $\quad at\ point\ (3, 2e)$

7)

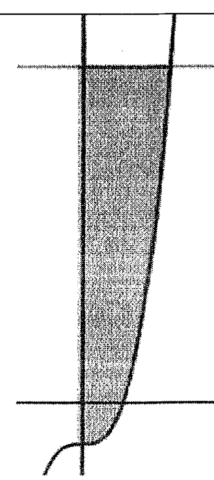
$$\int \frac{5\sqrt{x} - 7x^3}{x^4}\,dx$$

8) $\int \tan(5x) - \csc(3x)dx$

9) $\int \frac{4x^3}{\sqrt[3]{(6-4x^4)^4}}dx$

10)

The diagram shows the shaded region bounded by y = $x^3 - 3$, y = 24, and x = 0. Find the volume of the solid created by revolving the shaded region about the AOR of y = –5.



Non-AP Calculus Spring Review WS #2 Show all appropriate work

Key

1) Use right − handed rectangles to approximate $\int_2^{27} f(x)dx$ using 4 subintervals

5 9 5 6

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

$$\int_2^{27} f(x)dx \approx 5(2)+9(6)+5(12)+6(10) = \boxed{184}$$

2) Use trapezoidal sum to approximate $\int_2^{30} f(x)dx$ using 4 subintervals

$Area = \frac{w}{2}[h_1+h_2]$

6 7 7 8

x	2	5	8	12	15	18	22	25	30
f(x)	0	11	6	9	3	6	12	8	12

$$\int_2^{30} f(x)dx \approx \frac{6}{2}[0+6]+\frac{7}{2}[6+3]+\frac{7}{2}[3+12]+\frac{8}{2}[12+12] = \boxed{198}$$

3) Find $f'(x)$ for $f(x) = \frac{2}{\sqrt[4]{(e^{7x}-\ln 6x)^5}}$

*chain rule: $f(x) = 2(e^{7x}-\ln 6x)^{-5/4}$

outside: $2(\quad)^{-5/4}$

inside: $e^{7x}-\ln 6x$

$f'(x) = -\frac{10}{4}(\quad)^{-9/4}\left(7e^{7x}-\frac{6}{6x}\right)$

$$\boxed{f'(x) = \frac{-5(7e^{7x}-\frac{1}{x})}{2(e^{7x}-\ln 6x)^{9/4}}}$$

4) Find $\frac{dy}{dx}$ for $y = \ln\left(5x\sqrt{(3-6x^2)^7}\right)$

*expand first: $\ln(ab) = \ln a + \ln b$

$y = \ln(5x) + \ln(3-6x^2)^{7/2}$

$y = \ln(5x) + \frac{7}{2}\ln(3-6x^2)$

$y' = \frac{5}{5x} + \frac{7}{\cancel{2}}\cdot\frac{-\cancel{12}^{6}x}{3-6x^2}$

$y' = \frac{1}{x} - \frac{42x}{3-6x^2} = \boxed{\frac{1}{x} - \frac{14}{1-2x^2}}$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{3x}{\sqrt{5y}}\right) - e^{6y} = 5\pi - 5x^3 - 4y^2$

*expand first!

$\ln(3x) - \ln(5y)^{1/2} - e^{6y} = 5\pi - 5x^3 - 4y^2$

$\ln(3x) - \frac{1}{2}\ln(5y) - e^{6y} = 5\pi - 5x^3 - 4y^2$

$\frac{3}{3x} - \frac{1}{2}\cdot\frac{\cancel{5}}{\cancel{5}y}\left(\frac{dy}{dx}\right) - e^{6y}\cdot 6\left(\frac{dy}{dx}\right) = 0 - 15x^2 - 8y\left(\frac{dy}{dx}\right)$

$8y\left(\frac{dy}{dx}\right) - \frac{1}{2y}\left(\frac{dy}{dx}\right) - 6e^{6y}\left(\frac{dy}{dx}\right) = -15x^2 - \frac{1}{x}$

$\frac{dy}{dx}\left(8y - \frac{1}{2y} - 6e^{6y}\right) = -15x^2 - \frac{1}{x}$

$$\boxed{\frac{dy}{dx} = \frac{-15x^2 - \frac{1}{x}}{8y - \frac{1}{2y} - 6e^{6y}}}$$

6. Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$
$f(x) = 2e^{10-x^2}$ at point $(3, 2e)$

$f'(x) = 2e^{10-x^2} \cdot -2x$
$f'(x) = -4xe^{10-x^2}$
$f'(3) = -4(3)e^{10-3^2} = -12e^1 = -12e$
point: $(3, 2e)$
slope: $m = -12e$
$y - y_1 = m(x - x_1)$
$\boxed{y - 2e = -12e(x-3)}$

7) $\int \frac{5\sqrt{x} - 7x^3}{x^4} dx$ *expand

$\int (5x^{1/2} - 7x^3)x^{-4} dx$
$\int 5x^{-7/2} - 7x^{-1} dx$
$5\int x^{-7/2} dx - 7\int \frac{1}{x} dx$

$5\left(\frac{x^{-5/2}}{-5/2}\right) - 7\ln|x| + c$
$-2x^{-5/2} - 7\ln|x| + c$
$\boxed{\frac{-2}{x^{5/2}} - 7\ln|x| + c}$

8) $\int \tan(5x) - \csc(3x) dx$

$\int \tan(5x) dx - \int \csc(3x) dx$

$u = 5x$
$\frac{du}{dx} = 5$
$dx = \frac{du}{5}$
$\int \tan u \cdot \frac{du}{5}$

$u = 3x$
$\frac{du}{dx} = 3$
$dx = \frac{du}{3}$
$\int \csc u \cdot \frac{du}{3}$

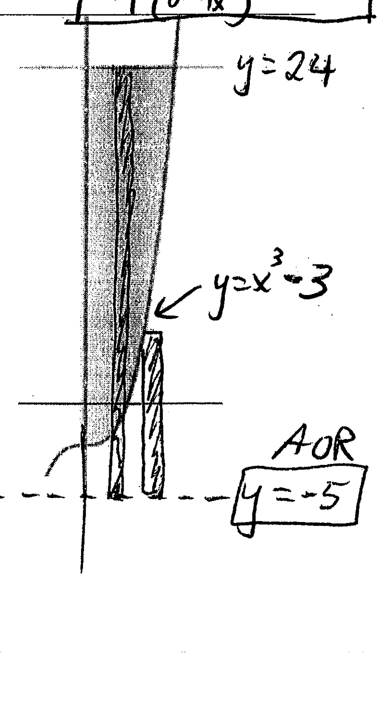
$-\frac{1}{5}\ln|\cos(5x)| - \left(-\frac{1}{3}\ln|\csc 3x + \cot 3x|\right) + C$
$\boxed{-\frac{1}{5}\ln|\cos(5x)| + \frac{1}{3}\ln|\csc 3x + \cot 3x| + C}$

9) $\int \frac{4x^3}{\sqrt[3]{(6-4x^4)^4}} dx$ u-sub

$\int \frac{4x^3}{(6-4x^4)^{4/3}} dx$
$\int 4x^3(6-4x^4)^{-4/3} dx$
$u = 6 - 4x^4$
$\frac{du}{dx} = -16x^3$
$dx = \frac{du}{-16x^3}$

$\int 4x^3 \cdot u^{-4/3} \cdot \frac{du}{-16x^3} = -\frac{1}{4}\int u^{-4/3} du$
$-\frac{1}{4}\left(\frac{u^{-1/3}}{-1/3}\right) \rightarrow \frac{3}{4}u^{-1/3} + c$
$\boxed{\frac{3}{4(6-4x^4)^{1/3}} + c}$

10) The diagram shows the shaded region bounded by y = $x^3 - 3$, y = 24, and x = 0. Find the volume of the solid created by revolving the shaded region about the AOR of y = –5.



* washer method
*intersections
$x^3 - 3 = 24$
$x^3 = 27$
$x = 3$

$R(x) = 24 - (-5) = 29$
$r(x) = x^3 - 3 - (-5) = x^3 + 2$
$V = \pi \int_0^3 [29]^2 - [x^3+2]^2 dx$
$\boxed{V = 2117.571\pi \text{ units}^3}$