Non-AP Calculus Spring Review WS #3 Show all appropriate work

1) Use middle rectangles to approximate $\int_{2}^{27} f(x)dx$ *using 2 subintervals*

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

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2) Use trapezoidal sum to approximate $\int_{2}^{30} f(x)dx$ *using 2 subintervals*

x	2	4	7	9	16	17	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

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3) *Find* $f'(x)$ *for* $f(x) = \dfrac{6}{\sqrt{(\cos(3x) - \ln 2x)^5}}$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln\left(\dfrac{x}{\sqrt{(2-3x)^3}}\right)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln(x\sqrt{y}) - e^{6x} = 3e - 5y - 3x^2$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

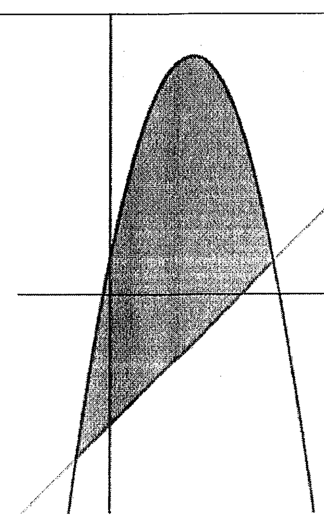
$f(x) = 2e^{3-5x} \quad at\ point\ \left(1, \frac{2}{e^2}\right)$

7) $\int \frac{2x^3 - 3x^2 + \sqrt{x}}{x^2} dx$

8) $\int 3^{5x} - \cot(2x) dx$

9) $\int \frac{5x^2}{\sqrt[5]{(e - 2x^3)^4}} dx$

10) The diagram shows the shaded region bounded by y = -x² + 5x + 1 and y = x – 4. Find the volume of the solid created by revolving the shaded region about the AOR of y = 8.



Non-AP Calculus Spring Review WS #3 Show all appropriate work

Key

1) Use middle rectangles to approximate $\int_2^{27} f(x)dx$ *using 2 subintervals*

14 11

x	2	4	7	9	16	17	21	23	27
f(x)	6	1	2	4	6	7	12	7	10

$$\int_2^{27} f(x)dx \approx 14(2) + 11(12) = \boxed{160}$$

2) Use trapezoidal sum to approximate $\int_2^{30} f(x)dx$ *using 2 subintervals*

Trapezoidal Area $= \frac{w}{2}[h_1+h_2]$

14 14

x	2	4	7	9	16	17	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

$$\int_2^{30} f(x)dx \approx \frac{14}{2}[6+5] + \frac{14}{2}[5+10] = \boxed{182}$$

*chain rule

3) *Find* $f'(x)$ *for* $f(x) = \frac{6}{\sqrt{(\cos(3x) - \ln 2x)^5}}$

$f(x) = \frac{6}{(\cos 3x - \ln 2x)^{5/2}}$ $f(x) = 6(\cos 3x - \ln 2x)^{-5/2}$

outside: $6(\ )^{-5/2}$

inside: $\cos 3x - \ln 2x$

$f'(x) = 6 \cdot \frac{-5}{2}(\ )^{-7/2} \cdot \left(-\sin 3x \cdot 3 - \frac{2}{2x}\right)$

$$\boxed{f'(x) = \frac{-15\left(-3\sin 3x - \frac{1}{x}\right)}{(\cos 3x - \ln 2x)^{7/2}}}$$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln\left(\frac{x}{\sqrt{(2-3x)^3}}\right)$

*Expand : $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$y = \ln x - \ln(2-3x)^{3/2}$

$y = \ln x - \frac{3}{2}\ln(2-3x)$

$y' = \frac{1}{x} - \frac{3}{2} \cdot \frac{-3}{2-3x}$

$$\boxed{y' = \frac{1}{x} + \frac{9}{2(2-3x)}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln(x\sqrt{y}) - e^{6x} = 3e - 5y - 3x^2$

$\ln x + \ln y^{1/2}$

expand first:

$\ln x + \frac{1}{2}\ln y - e^{6x} = 3e - 5y - 3x^2$

$\frac{1}{x} + \frac{1}{2}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - e^{6x} \cdot 6 = 0 - 5\left(\frac{dy}{dx}\right) - 6x$

$\frac{1}{2y}\left(\frac{dy}{dx}\right) + 5\left(\frac{dy}{dx}\right) = 6e^{6x} - \frac{1}{x} - 6x$

$\frac{dy}{dx}\left(\frac{1}{2y} + 5\right) = 6e^{6x} - \frac{1}{x} - 6x$

$$\boxed{\frac{dy}{dx} = \frac{6e^{6x} - \frac{1}{x} - 6x}{\frac{1}{2y} + 5}}$$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$f(x) = 2e^{3-5x}$ at point $\left(1, \frac{2}{e^2}\right)$

$f'(x) = 2e^{3-5x} \cdot (-5)$

$f'(x) = -10e^{3-5x}$

$f'(1) = -10e^{3-5} = -10e^{-2} = \frac{-10}{e^2}$

point: $\left(1, \frac{2}{e^2}\right)$

slope: $m = \frac{-10}{e^2}$

$y - y_1 = m(x - x_1)$

$\boxed{y - \frac{2}{e^2} = \frac{-10}{e^2}(x-1)}$

7)

$\int \frac{2x^3 - 3x^2 + \sqrt{x}}{x^2} dx$

$\int (2x^3 - 3x^2 + x^{1/2}) x^{-2} dx$

$\int 2x - 3 + x^{-3/2} dx$

$\frac{2x^2}{2} - 3x + \frac{x^{-1/2}}{-1/2} + c$

$x^2 - 3x - 2x^{-1/2} + c$

$\boxed{x^2 - 3x - \frac{2}{x^{1/2}} + c}$

8) $\int 3^{5x} - \cot(2x) dx$

$* \int a^u du = \frac{1}{\ln a} \cdot a^u + c$ $\quad * \int \cot u \, du = \ln|\sin u| + c$

$\int 3^{5x} dx - \int \cot(2x) dx$

$u = 5x$, $\frac{du}{dx} = 5$, $dx = \frac{du}{5}$, $\int 3^u \cdot \frac{du}{5}$

$u = 2x$, $\frac{du}{dx} = 2$, $dx = \frac{du}{2}$, $\int \cot u \cdot \frac{du}{2}$

$\frac{1}{5}\int 3^u du - \frac{1}{2}\int \cot u \, du$

$\frac{1}{5} \cdot \frac{1}{\ln 3} \cdot 3^u - \frac{1}{2}\ln|\sin u| + c$

$\boxed{\frac{1}{5\ln 3} \cdot 3^{5x} - \frac{1}{2}\ln|\sin 2x| + c}$

9) $\int \frac{5x^2}{\sqrt[5]{(e - 2x^3)^4}} dx$ $\quad \int 5x^2 (e - 2x^3)^{-4/5} dx$

$u = e - 2x^3$

$\frac{du}{dx} = -6x^2$

$dx = \frac{du}{-6x^2}$

$\int 5\cancel{x^2} \cdot u^{-4/5} \cdot \frac{du}{-6\cancel{x^2}}$

$-\frac{5}{6}\int u^{-4/5} du$

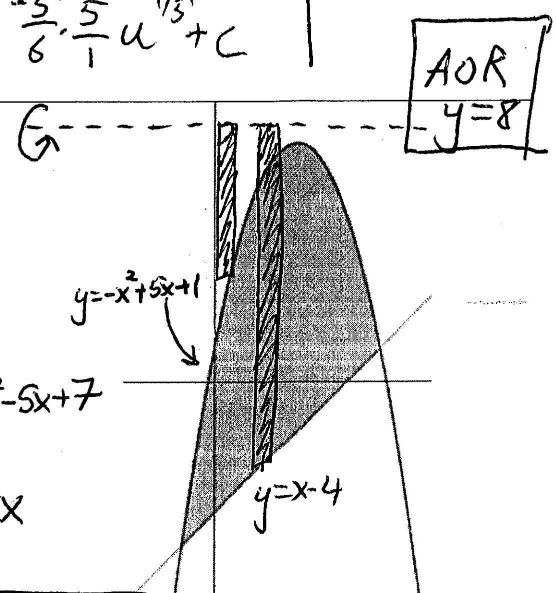
$-\frac{5}{6}\left(\frac{u^{1/5}}{1/5}\right) + c$

$-\frac{5}{6} \cdot \frac{5}{1} u^{1/5} + c$

$\boxed{\frac{-25}{6}(e - 2x^3)^{1/5} + c}$

10)

The diagram shows the shaded region bounded by y = -x² + 5x + 1 and y = x – 4. Find the volume of the solid created by revolving the shaded region about the AOR of y = 8.



* washer method

* intersection:

$x - 4 = -x^2 + 5x + 1$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, x = -1$

$R(x) = 8 - (x-4) = 8 - x + 4 = 12 - x$

$r(x) = 8 - (-x^2 + 5x + 1) = 8 + x^2 - 5x - 1 = x^2 - 5x + 7$

$V = \pi \int_{-1}^{5} [12 - x]^2 - [x^2 - 5x + 7]^2 dx$

$\boxed{V = 460.8\pi \text{ or } \frac{2304}{5}\pi \text{ units}^3}$