Non-AP Calculus Spring Review WS #4 Show all appropriate work

1) Use left – handed rectangles to approximate $\int_{2}^{21} f(x)dx$ *using* 3 *subintervals*

x	2	4	7	9	16	17	21	23	27
f(x)	5	4	6	11	13	8	15	20	30

2) Use trapezoidal sum to approximate $\int_{5}^{25} f(x)dx$ *using* 3 *subintervals*

x	2	5	8	12	15	18	22	25	30
f(x)	0	11	6	9	30	6	12	8	15

3) $Find\ f'(x)\ for\ f(x) = \dfrac{2}{\sqrt[3]{(e^{5x} - \ln 15x^2)^8}}$

4) $Find\ \frac{dy}{dx}\ for\ y = \ln\left(3x^2\sqrt[5]{(4+5x^2)^3}\right)$

5) Use Implicit Differentiation to find $\dfrac{dy}{dx}$ for $\ln\left(\dfrac{4x}{\sqrt[7]{y^3}}\right) - 5e^{3y} = 3\pi x - 2x^5 - 3y^4$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$f(x) = 4e^{27-x^2}$ *at point* $(5, 4e^2)$

7)

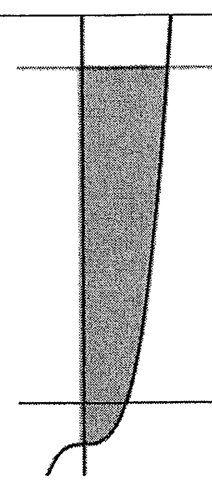
$$\int \frac{3\sqrt[4]{x^3} - 5x^4}{x^5} dx$$

8) $\int (\csc^2(4x) - \tan(2x))dx$

9) $$\int \frac{-3x^2}{\sqrt[5]{(4-2x^3)^6}} dx$$

10)

The diagram shows the shaded region bounded by y = 3x³ – 2, y = 22, and x = 0. Find the volume of the solid created by revolving the shaded region about the AOR of y = 25.



Non-AP Calculus Spring Review WS #4 Show all appropriate work

Key

1) Use left – handed rectangles to approximate $\int_{2}^{21} f(x)dx$ *using* 3 *subintervals*

5 9 5

x	2	4	7	9	16	17	21	23	27
f(x)	5	4	6	11	13	8	15	20	30

$$\int_2^{21} f(x)dx = 5(5) + 9(6) + 5(13) = \boxed{144}$$

2) Use trapezoidal sum to approximate $\int_{5}^{25} f(x)dx$ *using* 3 *subintervals*

*Area $= \frac{w}{2}[h_1+h_2]$

7 6 7

x	2	5	8	12	15	18	22	25	30
f(x)	0	11	6	9	30	6	12	8	15

$$\int_5^{25} f(x)dx = \frac{7}{2}[11+9] + \frac{6}{2}[9+6] + \frac{7}{2}[6+8] = \boxed{164}$$

3) *Find* $f'(x)$ *for* $f(x) = \frac{2}{\sqrt[3]{(e^{5x} - \ln 15x^2)^8}}$

*chain rule

$f(x) = 2(e^{5x} - \ln 15x^2)^{-8/3}$

outside: $2(\quad)^{-8/3}$

inside: $e^{5x} - \ln 15x^2$

$f'(x) = 2 \cdot \frac{-8}{3}(e^{5x} - \ln 15x^2)^{-11/3} \cdot \left(e^{5x} \cdot 5 - \frac{30x}{15x^2}\right)$

$$\boxed{f'(x) = \frac{-16\left(5e^{5x} - \frac{2}{x}\right)}{3(e^{5x} - \ln 15x^2)^{11/3}}}$$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln\left(3x^2 \sqrt[5]{(4+5x^2)^3}\right)$

*expand first: $\ln(ab) = \ln a + \ln b$

$y = \ln(3x^2) + \ln(4+5x^2)^{3/5}$

$y = \ln(3x^2) + \frac{3}{5}\ln(4+5x^2)$

$y' = \frac{6x}{3x^2} + \frac{3}{\cancel{5}} \cdot \frac{\cancel{10}^{2}x}{4+5x^2}$

$$\boxed{y' = \frac{2}{x} + \frac{6x}{4+5x^2}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{4x}{\sqrt[7]{y^3}}\right) - 5e^{3y} = 3\pi x - 2x^5 - 3y^4$

*expand first:

$\ln 4x - \ln y^{3/7} - 5e^{3y} = 3\pi x - 2x^5 - 3y^4$

$\ln 4x - \frac{3}{7}\ln y - 5e^{3y} = 3\pi x - 2x^5 - 3y^4$

$\frac{4}{4x} - \frac{3}{7} \cdot \frac{1}{y} \cdot \frac{dy}{dx} - 5e^{3y} \cdot 3\left(\frac{dy}{dx}\right) = 3\pi - 10x^4 - 12y^3\left(\frac{dy}{dx}\right)$

$12y^3\left(\frac{dy}{dx}\right) - \frac{3}{7y}\left(\frac{dy}{dx}\right) - 15e^{3y}\left(\frac{dy}{dx}\right) = 3\pi - 10x^4 - \frac{1}{x}$

$$\boxed{\frac{dy}{dx} = \frac{3\pi - 10x^4 - \frac{1}{x}}{12y^3 - \frac{3}{7y} - 15e^{3y}}}$$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$f(x) = 4e^{27-x^2}$ at point $(5, 4e^2)$

$f'(x) = 4e^{27-x^2} \cdot -2x = -8xe^{27-x^2}$

$f'(5) = -8(5)e^{27-25} = -40e^2$

point: $(5, 4e^2)$

slope: $m = -40e^2$

$\boxed{y - 4e^2 = -40e^2(x-5)}$

7) $\int \frac{3\sqrt[4]{x^3} - 5x^4}{x^5} dx$

$-\frac{20}{4}$

$\int (3x^{3/4} - 5x^4)x^{-5}\, dx$

$\int 3x^{-17/4} - 5x^{-1}\, dx$

$+4/4$

$3\int x^{-17/4} dx - 5\int \frac{1}{x} dx$

$3\cdot\left(\frac{x^{-13/4}}{-13/4}\right) - 5\ln|x| + C$

$3\cdot -\frac{4}{13}x^{-13/4} - 5\ln|x| + C$

$\boxed{\frac{-12}{13x^{13/4}} - 5\ln|x| + C}$

8) $\int (\csc^2(4x) - \tan(2x))dx$

$\int \csc^2(4x)\,dx - \int \tan(2x)\,dx$

$u = 4x$ $\quad dx = \frac{du}{4}$ $\quad \frac{du}{dx} = 4$

$u = 2x$ $\quad dx = \frac{du}{2}$ $\quad \frac{du}{dx} = 2$

$\int \csc^2(u)\cdot\frac{du}{4}$ $\qquad \int \tan u \cdot \frac{du}{2}$

$\frac{1}{4}\int \csc^2 u\, du$ $\qquad \frac{1}{2}\int \tan u\, du$

$\boxed{-\frac{1}{4}\cot(4x) - \left(-\frac{1}{2}\ln|\cos 2x|\right) + C}$

9) $\int \frac{-3x^2}{\sqrt[5]{(4-2x^3)^6}} dx$

$\int \frac{-3x^2}{(4-2x^3)^{6/5}} dx = \int -3x^2(4-2x^3)^{-6/5}\, dx$

$u = 4 - 2x^3$

$\frac{du}{dx} = -6x^2$

$dx = \frac{du}{-6x^2}$

$\int -3\cancel{x^2}\cdot u^{-6/5}\cdot\frac{du}{-6\cancel{x^2}}$

$\frac{1}{2}\int u^{-6/5}\, du$

$\frac{1}{2}\cdot\frac{u^{-1/5}}{-1/5} + C$

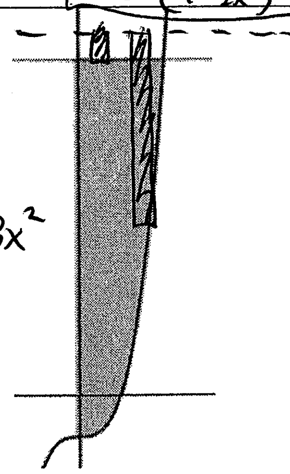
$-\frac{5}{2}u^{-1/5} + C$

$\boxed{\frac{-5}{2(4-2x^3)^{1/5}} + C}$

10)

The diagram shows the shaded region bounded by $y = 3x^3 - 2$, $y = 22$, and $x = 0$. Find the volume of the solid created by revolving the shaded region about the AOR of $y = 25$.

$\boxed{\text{AOR } y = 25}$



*washer method

*Find intersections:

$3x^3 - 2 = 22$

$3x^3 = 24$

$x^3 = 8$

$x = 2$

$R(x) = 25 - (3x^2 - 2) = 25 - 3x^2 + 2 = 27 - 3x^2$

$r(x) = 25 - 22 = 3$

$V = \pi\int_0^2 [27 - 3x^2]^2 - [3]^2\, dx$

$\boxed{V = 1065.6\pi \text{ units}^3}$