Non-AP Calculus Spring Review WS #5 Show all appropriate work

1) Use middle rectangles to approximate $\int_{2}^{21} f(x)dx$ *using 3 subintervals*

x	2	4	7	9	16	17	21	24	27
f(x)	6	1	2	4	6	7	12	7	10

2) Use trapezoidal sum to approximate $\int_{4}^{23} f(x)dx$ *using 2 subintervals*

x	2	4	7	11	18	19	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

3) *Find* $f'(x)$ *for* $f(x) = \dfrac{6}{\sqrt{(\sin(\pi x) - \ln 3x)^3}}$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln\left(\dfrac{x}{\sqrt{(5-3x^3)^7}}\right)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{\sqrt{y}}{x}\right) - e^{6y} = 3e^4 - 5y - 2x^2$

6. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$$f(x) = 2e^{1-x^2} \quad at\ point\ \left(2, \frac{2}{e^3}\right)$$

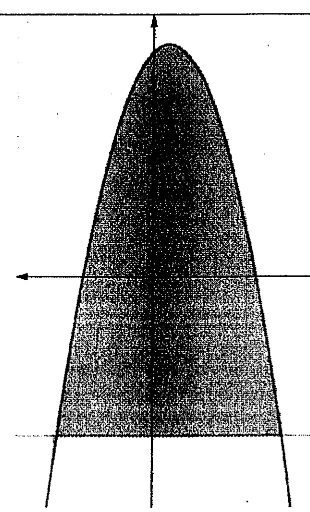
7)
$$\int \frac{2x^3 - 3x^2 + \sqrt{x}}{x^3} dx$$

8) $\int e^{5x} - \sec^2(2x)dx$

9) $\int \frac{7x^2}{\sqrt[5]{(3e - 2x^3)}} dx$

10)

The diagram shows the shaded region bounded by y = $-x^2 + x + 7$ and y = -5. Find the volume of the solid created by revolving the shaded region about the AOR of y = -5.



Non-AP Calculus Spring Review WS #5 Show all appropriate work

Key

1) Use middle rectangles to approximate $\int_2^{21} f(x)dx$ using 3 subintervals

5 9 5

x	2	4	7	9	16	17	21	24	27
f(x)	6	1	2	4	6	7	12	7	10

$$\int_2^{21} f(x)dx \approx 5(1)+9(4)+5(7) = \boxed{76}$$

2) Use trapezoidal sum to approximate $\int_4^{23} f(x)dx$ using 2 subintervals

$Area = \frac{w}{2}[h_1+h_2]$

14 5

x	2	4	7	11	18	19	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

$$\int_4^{23} f(x)dx \approx \frac{14}{2}[1+5]+\frac{5}{2}[5+7] = \boxed{72}$$

3) Find $f'(x)$ for $f(x) = \frac{6}{\sqrt{(\sin(\pi x) - \ln 3x)^3}}$

*chain rule

$f(x) = 6(\sin\pi x - \ln 3x)^{-3/2}$

outside: $6(\quad)^{-3/2}$

inside: $\sin\pi x - \ln 3x$

$f'(x) = 6\cdot -\frac{3}{2}(\quad)^{-5/2}\cdot\left(\cos\pi x\cdot\pi - \frac{3}{3x}\right)$

$$\boxed{f'(x) = \frac{-9\left(\pi\cos\pi x - \frac{1}{x}\right)}{(\sin\pi x - \ln 3x)^{5/2}}}$$

4) Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{x}{\sqrt{(5-3x^3)^7}}\right)$

*expand first: $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$y = \ln x - \ln(5-3x^3)^{7/2}$

$y = \ln x - \frac{7}{2}\ln(5-3x^3)$

$y' = \frac{1}{x} - \frac{7}{2}\cdot\frac{-9x^2}{5-3x^3}$

$$\boxed{y' = \frac{1}{x} + \frac{63x^2}{2(5-3x^3)}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{\sqrt{y}}{x}\right) - e^{6y} = 3e^4 - 5y - 2x^2$

*expand first:

$\ln y^{1/2} - \ln x - e^{6y} = 3e^4 - 5y - 2x^2$

$\frac{1}{2}\ln y - \ln x - e^{6y} = 3e^4 - 5y - 2x^2$

$\frac{1}{2}\cdot\frac{1}{y}\left(\frac{dy}{dx}\right) - \frac{1}{x} - e^{6y}\cdot 6\left(\frac{dy}{dx}\right) = 0 - 5\left(\frac{dy}{dx}\right) - 4x$

$\frac{1}{2y}\left(\frac{dy}{dx}\right) - 6e^{6y}\left(\frac{dy}{dx}\right) + 5\left(\frac{dy}{dx}\right) = \frac{1}{x} - 4x$

$\frac{dy}{dx}\left(\frac{1}{2y} - 6e^{6y} + 5\right) = \frac{1}{x} - 4x$

$$\boxed{\frac{dy}{dx} = \frac{\frac{1}{x} - 4x}{\frac{1}{2y} - 6e^{6y} + 5}}$$

6. Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$

$f(x) = 2e^{1-x^2}$ at point $\left(2, \frac{2}{e^3}\right)$

$f'(x) = 2e^{1-x^2}(-2x) = -4xe^{1-x^2}$

$f'(2) = -4(2)e^{1-2^2} = -8e^{-3} = -\frac{8}{e^3}$

point: $\left(2, \frac{2}{e^3}\right)$

slope: $m = \frac{-8}{e^3}$

$\boxed{y - \frac{2}{e^3} = \frac{-8}{e^3}(x-2)}$

7) $\int \frac{2x^3 - 3x^2 + \sqrt{x}}{x^3}\,dx$

$\int (2x^3 - 3x^2 + x^{1/2})x^{-3}\,dx$

$\int 2 - 3x^{-1} + x^{-5/2}\,dx$

$\int 2 - \frac{3}{x} + x^{-5/2 \; +2/2}\,dx$

$2x - 3\ln x + \left(\frac{x^{-3/2}}{-3/2}\right)$

$2x - 3\ln x - \frac{2}{3}x^{-3/2} + C$

$\boxed{2x - 3\ln x - \frac{2}{3x^{3/2}} + C}$

8) $\int e^{5x} - \sec^2(2x)\,dx$

$\int e^{5x}\,dx - \int \sec^2(2x)\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$\int e^u \cdot \frac{du}{5}$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int \sec^2 u \cdot \frac{du}{2}$

$\frac{1}{5}\int e^u\,du - \frac{1}{2}\int \sec^2 u\,du$

$\boxed{\frac{1}{2}e^{5x} - \frac{1}{2}\tan(2x) + C}$

9) $\int \frac{7x^2}{\sqrt[5]{(3e - 2x^3)}}\,dx$

$\int 7x^2(3e - 2x^3)^{-1/5}\,dx$

$u = 3e - 2x^3$

$\frac{du}{dx} = -6x^2$

$dx = \frac{du}{-6x^2}$

$\int 7\cancel{x^2} \cdot u^{-1/5} \cdot \frac{du}{-6\cancel{x^2}}$

$-\frac{7}{6}\int u^{-1/5 \; +5/5}\,du$

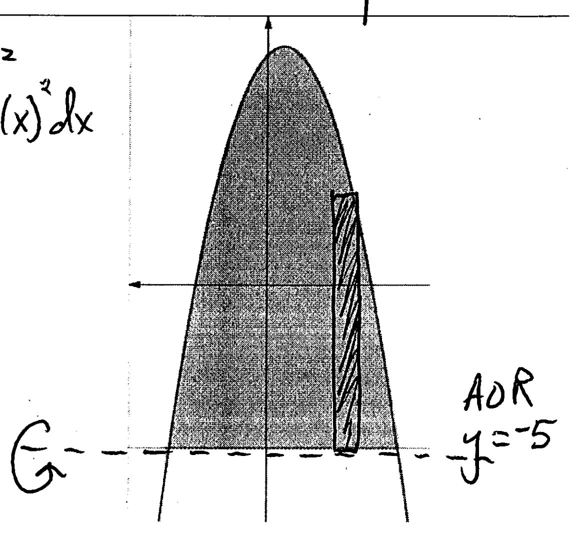
$-\frac{7}{6}\cdot\left(\frac{u^{4/5}}{4/5}\right) + C$

$-\frac{7}{6}\cdot\frac{5}{4}u^{4/5} + C$

$\boxed{-\frac{35}{24}(3e - 2x^3)^{4/5} + C}$

10) The diagram shows the shaded region bounded by y = –x² + x + 7 and y = – 5 . Find the volume of the solid created by revolving the shaded region about the AOR of y = –5.

Disc Method: $V = \pi\int_{x_1}^{x_2} R(x)^2\,dx$



*Disc Method

*find intersection

$-5 = -x^2 + x + 7$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, x = -3$

$R(x) = -x^2 + x + 7 - (-5)$

$= -x^2 + x + 7 + 5$

$R(x) = -x^2 + x + 12$

$V = \pi\int_{-3}^{4}\left[-x^2 + x + 12\right]^2 dx$

$\boxed{V = 560.233\pi \text{ or } \frac{16807}{30}\pi \text{ units}^3}$