Non-AP Calculus Spring Review WS #6 Show all appropriate work

1) Use middle rectangles to approximate $\int_{2}^{31} f(x)dx$ *using* 2 *subintervals*

x	2	4	5	9	12	17	21	24	31
f(x)	6	1	2	4	6	7	12	7	10

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2) Use trapezoidal sum to approximate $\int_{7}^{21} f(x)dx$ *using* 2 *subintervals*

x	2	4	7	11	18	19	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

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3) *Find* $f'(x)$ *for* $f(x) = \dfrac{6}{\sqrt{\left(e^{(\pi x)} - \ln 2x\right)^3}}$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln(x(2 - 3x^3)^3)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{\sqrt{x}}{y^2}\right) - e^{6y} = 3x^4 - \cos y - 2y$

6. Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$

$f(x) = 3e^{1-2x^2}$ at point $\left(1, \frac{3}{e}\right)$

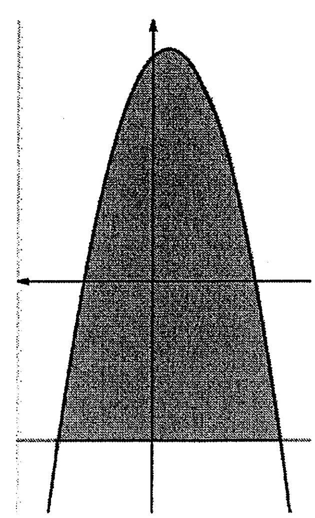
7) $\displaystyle\int \frac{2x^6 - 3x^2 + \sqrt{x}}{x^3}\,dx$

8) $\displaystyle\int 2e^{3x} - \csc^2(5x)\,dx$

9) $\displaystyle\int \frac{2x^2}{\sqrt[3]{(3e - 5x^3)}}\,dx$

10)

The diagram shows the shaded region bounded by $y = -x^2 + x + 7$ and $y = -5$. Find the volume of the solid created by revolving the shaded region about the AOR of $y = -8$.



Non-AP Calculus Spring Review WS #6 Show all appropriate work

Key

1) Use middle rectangles to approximate $\int_{2}^{31} f(x)dx$ *using 2 subintervals*

10 19

x	2	4	5	9	12	17	21	24	31
f(x)	6	1	2	4	6	7	12	7	10

$$\int_2^{31} f(x)dx \approx 10(2) + 19(12) = \boxed{248}$$

2) Use trapezoidal sum to approximate $\int_{7}^{21} f(x)dx$ *using 2 subintervals*

*Area $= \frac{W}{2}[h_1 + h_2]$

11 3

x	2	4	7	11	18	19	21	23	30
f(x)	6	1	2	4	5	7	12	7	10

$$\int_7^{21} f(x)dx = \frac{11}{2}[2+5] + \frac{3}{2}[5+12] = \boxed{64}$$

3) *Find* $f'(x)$ *for* $f(x) = \dfrac{6}{\sqrt{\left(e^{(\pi x)} - \ln 2x\right)^3}}$

$f(x) = 6(e^{\pi x} - \ln 2x)^{-3/2}$

outside: $6(\quad)^{-3/2}$

inside: $e^{\pi x} - \ln 2x$

$f'(x) = 6 \cdot \frac{-3}{2}(e^{\pi x} - \ln 2x)^{-5/2}\left(\pi e^{\pi x} - \frac{2}{2x}\right)$

$$\boxed{f'(x) = \frac{-9\left(\pi e^{\pi x} - \frac{1}{x}\right)}{(e^{\pi x} - \ln 2x)^{5/2}}}$$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln(x(2-3x^3)^3)$

*expand first: $\ln(ab) = \ln a + \ln b$

$y = \ln x + \ln(2-3x^3)^3$

$y = \ln x + 3\ln(2-3x^3)$

$y' = \frac{1}{x} + 3 \cdot \frac{-9x^2}{2-3x^3}$

$$\boxed{y' = \frac{1}{x} - \frac{27x^2}{2-3x^3}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln\left(\frac{\sqrt{x}}{y^2}\right) - e^{6y} = 3x^4 - \cos y - 2y$

$\ln x^{1/2} - \ln y^2 - e^{6y} = 3x^4 - \cos(y) - 2y$

$\frac{1}{2}\ln x - 2\ln y - e^{6y} = 3x^4 - \cos(y) - 2y$

$\frac{1}{2}\left(\frac{1}{x}\right) - 2\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) - e^{6y}(6)\left(\frac{dy}{dx}\right) = 12x^3 - (-\sin y)\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(\frac{-2}{y} - 6e^{6y} - \sin(y) + 2\right) = 12x^3 - \frac{1}{2x}$

$$\boxed{\frac{dy}{dx} = \frac{12x^3 - \frac{1}{2x}}{\frac{-2}{y} - 6e^{6y} - \sin(y) + 2}}$$

6. Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$

$f(x) = 3e^{1-2x^2}$ at point $\left(1, \frac{3}{e}\right)$

$f'(x) = 3e^{1-2x^2}\cdot(-4x) = -12xe^{1-2x^2}$

$f'(1) = -12(1)e^{1-2} = -12e^{-1} = -\frac{12}{e}$

point: $\left(1, \frac{3}{e}\right)$

Slope: $m = \frac{-12}{e}$

$\boxed{y - \frac{3}{e} = -\frac{12}{e}(x-1)}$

7) $\int \frac{2x^6 - 3x^2 + \sqrt{x}}{x^3}dx$

$\int (2x^6 - 3x^2 + x^{1/2})x^{-3}dx$

$\int 2x^3 + 3x^{-1} + x^{-5/2}dx$

$\int 2x^3dx + 3\int \frac{1}{x}dx + \int x^{-5/2}dx$

$\frac{2x^4}{4} + 3\ln|x| + \frac{x^{-3/2}}{-3/2} + c$

$\boxed{\frac{x^4}{2} + 3\ln|x| - \frac{2}{3x^{3/2}} + C}$

8) $\int 2e^{3x} - \csc^2(5x)dx$

$\int 2e^{3x}dx - \int \csc^2(5x)dx$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$u = 5x \quad dx = \frac{du}{5}$

$\frac{du}{dx} = 5$

$\int 2e^{u}\cdot\frac{du}{3}$

$\frac{2}{3}\int e^{u}du$

$\int \csc^2(u)\frac{du}{5}$

$-\frac{1}{5}\int \csc^2 u\,du$

$= \boxed{\frac{2}{3}e^{3x} + \frac{1}{5}\cot(5x) + c}$

9) $\int \frac{2x^2}{\sqrt[3]{(3e - 5x^3)}}dx$ *u-sub

$\int 2x^2(3e-5x^3)^{-1/3}dx$

$u = 3e - 5x^3$

$\frac{du}{dx} = -15x^2$

$dx = \frac{du}{-15x^2}$

$\int 2\cancel{x^2}(u)^{-1/3}\cdot\frac{du}{-15\cancel{x^2}}$

$\frac{-2}{15}\int u^{-1/3}du$ $+3/3$

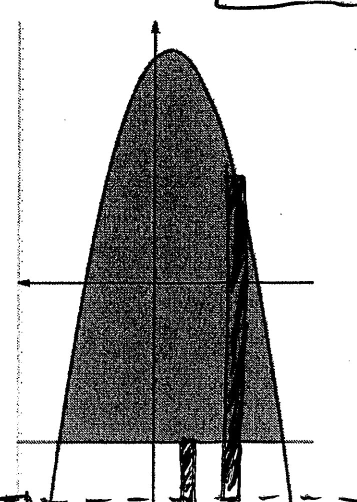
$\frac{-2}{15}\cdot\left(\frac{u^{2/3}}{2/3}\right) + C$

$\frac{-2}{15}\cdot\frac{3}{2}u^{3/2} + c$

$\frac{-1}{5}u^{3/2} + c \rightarrow \boxed{\frac{-1}{5}(3e-5x^3)^{3/2} + C}$

10)

The diagram shows the shaded region bounded by y = –x² + x + 7 and y = – 5 . Find the volume of the solid created by revolving the shaded region about the AOR of y = –8.



*intersection:

$-5 = -x^2 + x + 7$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, -3$

*washer method:

$R(x) = -x^2 + x + 7 - (-8) = -x^2 + x + 15$

$R(x) = -x^2 + x + 15$

$r(x) = -5 - (-8) = 3$

$V = \pi\int_{-3}^{4}[-x^2+x+15]^2 - [3]^2dx$

$\boxed{\text{AOR } y = -8}$

$\boxed{V = 903.233 = \frac{27097}{30}\pi \text{ units}^3}$