**Non-AP Calculus Spring Review WS #7** Show all appropriate work

1) Use left rectangles to approximate $\int_{2}^{27} f(x)dx$ *using* 2 *subintervals*

| x | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 27 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 6 | 1 | 2 | 4 | 6 | 7 | 12 | 7 | 10 |

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2) Use trapezoidal sum to approximate $\int_{4}^{17} f(x)dx$ *using* 2 *subintervals*

| x | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 30 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 6 | 1 | 2 | 4 | 5 | 7 | 12 | 7 | 10 |

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3) *Find* $f'(x)$ *for* $f(x) = \dfrac{6}{\sqrt{(\sin(3x) - \ln 2x)^5}}$

4) *Find* $\frac{dy}{dx}$ *for* $y = \ln\left(\frac{x}{\sqrt[5]{(2-3x)}}\right)$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln(x\sqrt{y}) - 2e^{6y} = 3e^2 - 5y - 3x^2$

**6**. Find the equation of a tangent line:

$y - y_1 = m(x - x_1)$

$$f(x) = 2e^{3-5x} \quad at\ point\ \left(1, \frac{2}{e^2}\right)$$

7)

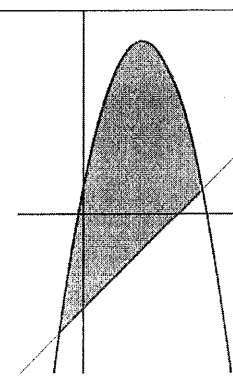
$$\int \frac{9x^3 - 3x + \sqrt{x}}{2x^2} dx$$

8) $\int e^{5x} - \sec(2x)dx$

9) $\int \frac{7x^2}{\sqrt[5]{(e^4 - 2x^3)^3}} dx$

10)

The diagram shows the shaded region bounded by y = $-x^2 + 5x + 1$ and y = x – 4. Find the volume of the solid created by revolving the shaded region about the AOR of y = – 11.



**Non-AP Calculus Spring Review WS #7** Show all appropriate work

Key

1) Use left rectangles to approximate $\int_2^{27} f(x)dx$ using 2 subintervals

$\overbrace{\quad}^{14}$ $\overbrace{\quad}^{11}$

| x | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 27 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 6 | 1 | 2 | 4 | 6 | 7 | 12 | 7 | 10 |

$$\int_2^{27} f(x)dx \approx 14(6)+11(6) = \boxed{150}$$

2) Use trapezoidal sum to approximate $\int_4^{17} f(x)dx$ using 2 subintervals

$A = \frac{w}{2}[h_1+h_2]$

$\overbrace{\quad}^{5}$ $\overbrace{\quad}^{8}$

| x | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 30 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 6 | 1 | 2 | 4 | 5 | 7 | 12 | 7 | 10 |

$$\int_4^{17} f(x)dx \approx \frac{5}{2}[1+4]+\frac{8}{2}[4+7] = \boxed{56.5 \text{ or } \frac{113}{2}}$$

3) Find $f'(x)$ for $f(x) = \frac{6}{\sqrt{(\sin(3x)-\ln 2x)^5}}$

*chain rule:

outside: $6(\quad)^{-5/2}$

inside: $\sin 3x - \ln 2x$

$f(x) = 6(\sin 3x - \ln 2x)^{-5/2}$

$$f'(x) = 6\cdot -\frac{5}{2}(\sin 3x - \ln 2x)^{-7/2}\left(\cos 3x \cdot 3 - \frac{2}{2x}\right)$$

$$\boxed{f'(x) = \frac{-15\left(3\cos 3x - \frac{1}{x}\right)}{(\sin 3x - \ln 2x)^{7/2}}}$$

4) Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{x}{\sqrt[5]{(2-3x)}}\right) \rightarrow \ln\left[\frac{x}{(2-3x)^{1/5}}\right]$

* expand: $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$y = \ln x - \ln(2-3x)^{1/5}$

$y = \ln x - \frac{1}{5}\ln(2-3x)$

$y' = \frac{1}{x} - \frac{1}{5}\cdot\frac{-3}{2-3x}$

$$\boxed{y' = \frac{1}{x} + \frac{3}{5(2-3x)}}$$

5) Use Implicit Differentiation to find $\frac{dy}{dx}$ for $\ln(x\sqrt{y}) - 2e^{6y} = 3e^2 - 5y - 3x^2$

$\ln x + \ln y^{1/2} - 2e^{6y} = 3e^2 - 5y - 3x^2$

$\ln x + \frac{1}{2}\ln y - 2e^{6y} = 3e^2 - 5y - 3x^2$

$\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{y}\left(\frac{dy}{dx}\right) - 2e^{6y}\cdot 6\left(\frac{dy}{dx}\right) = 0 - 5\left(\frac{dy}{dx}\right) - 6x$

$\frac{1}{2y}\left(\frac{dy}{dx}\right) - 12e^{6y}\left(\frac{dy}{dx}\right) + 5\left(\frac{dy}{dx}\right) = -\frac{1}{x} - 6x$

$\frac{dy}{dx}\left(\frac{1}{2y} - 12e^{6y} + 5\right) = -\frac{1}{x} - 6x$

$$\boxed{\frac{dy}{dx} = \frac{-\frac{1}{x} - 6x}{\frac{1}{2y} - 12e^{6y} + 5}}$$

**6.** Find the equation of a tangent line:
$y - y_1 = m(x - x_1)$

$f(x) = 2e^{3-5x}$ at point $\left(1, \frac{2}{e^2}\right)$

$f'(x) = 2e^{3-5x} \cdot (-5) = -10e^{3-5x}$

$f'(1) = -10e^{3-5} = -10e^{-2} = \frac{-10}{e^2}$

point: $\left(1, \frac{2}{e^2}\right)$

slope: $m = \frac{-10}{e^2}$

$$\boxed{y - \frac{2}{e^2} = \frac{-10}{e^2}(x-1)}$$

7)
$$\int \frac{9x^3 - 3x + \sqrt{x}}{2x^2} dx$$

$\frac{1}{2}\int \left(9x^3 - 3x + x^{1/2}\right) x^{-2} dx$

$\frac{1}{2}\int 9x - 3x^{-1} + x^{-3/2} dx$

$\int \frac{9}{2}x - \int \frac{3}{2} \cdot \frac{1}{x} + \int \frac{1}{2} x^{-3/2} dx$

$\frac{9}{2}\left(\frac{x^2}{2}\right) - \frac{3}{2}\ln|x| + \frac{1}{2}\cdot\left(\frac{x^{-1/2}}{-1/2}\right) + C$

$\frac{9}{4}x^2 - \frac{3}{2}\ln|x| - 1x^{-1/2} + C$

$$\boxed{\frac{9}{4}x^2 - \frac{3}{2}\ln|x| - \frac{1}{x^{1/2}} + C}$$

8) $\int e^{5x} - \sec(2x)dx$

$\int e^{5x} dx - \int \sec(2x) dx$

$u = 5x \quad dx = \frac{du}{5}$ ; $u = 2x \quad dx = \frac{du}{2}$

$\frac{du}{dx} = 5$ ; $\frac{du}{dx} = 2$

$\int e^u \cdot \frac{du}{5}$ ; $\int \sec u \cdot \frac{du}{2}$

$\frac{1}{5}\int e^u du - \frac{1}{2}\int \sec u du$

$$\boxed{\frac{1}{5}e^{5x} - \frac{1}{2}\ln|\sec 2x + \tan 2x| + C}$$

9) $\int \frac{7x^2}{\sqrt[5]{(e^4 - 2x^3)^3}} dx$

$\int 7x^2 (e^4 - 2x^3)^{-3/5} dx$

$u = e^4 - 2x^3$

$\frac{du}{dx} = -6x^2$

$dx = \frac{du}{-6x^2}$

$\int \cancel{7x^2}(u)^{-3/5} \cdot \frac{du}{-6\cancel{x^2}}$

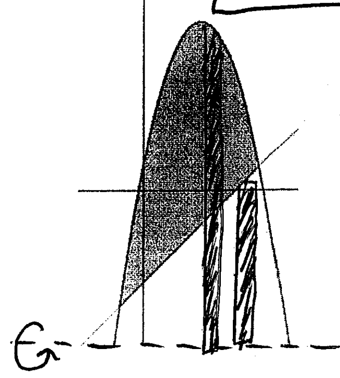
$-\frac{7}{6}\int u^{-3/5} du$ $\quad +5/5$

$-\frac{7}{6}\cdot\left(\frac{u^{+2/5}}{+2/5}\right) + C$

$-\frac{7}{6}\cdot\frac{+5}{2}u^{+2/5} + C$

$$\boxed{\frac{-35}{12}(e^4 - 2x^3)^{2/5} + C}$$

10) The diagram shows the shaded region bounded by y = -x² + 5x + 1 and y = x – 4. Find the volume of the solid created by revolving the shaded region about the AOR of y = – 11.



$\boxed{\text{AOR } y = -11}$

*intersection:

$x - 4 = -x^2 + 5x + 1$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, x = -1$

*washer method:

$R(x) = -x^2 + 5x + 1 - (-11)$

$= -x^2 + 5x + 12$

$r(x) = x - 4 - (-11)$

$= x - 4 + 11$

$r(x) = x + 7$

$$V = \pi \int_{-1}^{5} \left[-x^2 + 5x + 12\right]^2 - \left[x + 7\right]^2 dx$$

$$\boxed{V = 907.2\pi \text{ or } \frac{4536}{5}\pi \text{ units}^3}$$