Non-AP Calculus 2.2-2.4 Derivatives Quiz Review WS #2

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 3x^4\left(\sqrt{x} - 1\right) - \frac{3}{17x^2} + \frac{6}{\sqrt{\pi^5}} + \frac{12}{\sqrt[5]{x}} + 5\sqrt{x^3}$

2. Find $\frac{dy}{dx}$ for $y = \frac{2}{\sqrt[3]{6x^3 - 11x + 7}}$

3. Find g'(x) for $g(x) = 5x^2(3 - 2x^3)^5$ (simplify fully in factored form)

4. Find $\frac{dy}{dx}$ for $y = \frac{x}{\sqrt{3 - 4x^3}}$ (simplify fully in factored form)

5. If $f(x) = \frac{x^2+4}{x+2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = -1$ in point-slope form.

6. Particle moves along the x-axis so that its position at time t is given $x(t) = \frac{2}{3}t^3 - 9t^2 + 28t - 3$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

b) What is its velocity at $t = 1$ seconds?

c) What is its acceleration at t = 3 seconds?

d) Find the average velocity of particle in [1, 5]

e) When is the particle at rest?

f) When is the particle moving left? When does particle change directions? (Create Sign Line) Give justification.

g) What is displacement of particle from t = 3 to t = 8? Show work.

h) What is the total distance of particle from t = 3 to t = 8? Show work.

i) Is the speed increasing or decreasing at t = 5? Justify.

j) Is velocity increasing or decreasing at t = 2? Justify.

Solution Key

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No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 3x^4(\sqrt{x} - 1) - \frac{3}{17x^2} + \frac{6}{\sqrt{\pi^5}} + \frac{12}{\sqrt[5]{x}} + 5\sqrt{x^3}$

$$y = 3x^4(x^{1/2}) - 3x^4 - \frac{3}{17}x^{-2} + \frac{6}{\sqrt{\pi^5}} + 12x^{-1/5} + 5x^{3/2}$$

$$y = 3x^{9/2} - 3x^4 - \frac{3}{17}x^{-2} + \frac{6}{\sqrt{\pi^5}} + 12x^{-1/5} + 5x^{3/2}$$

$$\frac{dy}{dx} = 3\cdot\frac{9}{2}x^{7/2} - 3\cdot 4x^3 - \frac{3}{17}\cdot -2x^{-3} + 0 + 12\cdot\frac{-1}{5}x^{-6/5} + 5\cdot\frac{3}{2}x^{1/2}$$

$$\boxed{\frac{dy}{dx} = \frac{27}{2}x^{7/2} - 12x^3 + \frac{6}{17x^3} - \frac{12}{5x^{6/5}} + \frac{15}{2}x^{1/2}}$$

2. Find $\frac{dy}{dx}$ for $y = \frac{2}{\sqrt[3]{6x^3 - 11x + 7}} = \frac{2}{(6x^3-11x+7)^{1/3}}$

* chain rule
outside: $2(\ )^{-1/3}$
inside: $6x^3 - 11x + 7$

$$y = 2(6x^3 - 11x + 7)^{-1/3}$$

$$\frac{dy}{dx} = 2\cdot\frac{-1}{3}(6x^3-11x+7)^{-4/3}\cdot(18x^2-11)$$

$$\boxed{\frac{dy}{dx} = \frac{-2(18x^2-11)}{3(6x^3-11x+7)^{4/3}}}$$

3. Find g'(x) for $g(x) = 5x^2(3 - 2x^3)^5$ (simplify fully in factored form)

* ① product rule
* ② chain rule

$$g'(x) = \underbrace{10x(3-2x^3)^5}_{10x(3-2x^3)^4} + \underbrace{5x^2\cdot\cancel{5}(3-2x^3)^4(\cancel{-6x^2})}_{10x(3-2x^3)^4}$$

$-3x$

$$g'(x) = 10x(3-2x^3)^4[3 - 2x^3 - 15x^3]$$

$$\boxed{g'(x) = 10x(3-2x^3)^4(3 - 17x^3)}$$

4. Find $\frac{dy}{dx}$ for $y = \frac{x}{\sqrt{3-4x^3}} = \frac{x}{(3-4x^3)^{1/2}}$ (simplify fully in factored form)

* ① quotient rule
* ② chain rule

$$y' = \frac{(1)(3-4x^3)^{1/2} - (x)\cdot\frac{1}{2}(3-4x^3)^{-1/2}(-12x^2)}{[(3-4x^3)^{1/2}]^2}$$

$$y' = \frac{(3-4x^3)^{1/2} + \frac{6x}{(3-4x^3)^{1/2}}}{(3-4x^3)^1}$$

$(3-4x^3)^{1/2}$

$(3-4x^3)^{1/2}$

$$\boxed{\frac{dy}{dx} = \frac{3-4x^3+6x}{(3-4x^3)^{3/2}}}$$

5. If $f(x) = \frac{x^2+4}{x+2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = -1$ in point-slope form.

$$f'(x) = \frac{(2x)(x+2) - (x^2+4)(1)}{(x+2)^2}$$

$$f'(x) = \frac{2x^2+4x-x^2-4}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-4}{(x+2)^2}$$

point: $f(-1) = \frac{(-1)^2+4}{(-1)+2} = \frac{5}{1} = 5$

slope: $f'(-1) = \frac{(-1)^2+4(-1)-4}{(-1+2)^2} = \frac{-7}{1} = -7$

point: $(-1, 5)$

slope: $m = -7$

$$\boxed{y-5 = -7(x+1)}$$

6. Particle moves along the x-axis so that its position at time t is given $x(t) = \frac{2}{3}t^3 - 9t^2 + 28t - 3$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

$v(t) = \frac{2}{3} \cdot 3t^2 - 18t + 28$

$v(t) = 2t^2 - 18t + 28 = 2(t^2 - 9t + 14)$

$a(t) = 4t - 18 = 2(2t - 9)$

b) What is its velocity at $t = 1$ seconds?

$v(1) = 2(1)^2 - 18(1) + 28 = \boxed{12 \text{ ft/s}}$

c) What is its acceleration at t = 3 seconds?

$a(3) = 4(3) - 18 = \boxed{-6 \text{ ft/s}^2}$

d) Find the average velocity of particle in [1, 5]

Avg. velocity $= \frac{\text{change in position}}{\text{change in time}} = \frac{x(5) - x(1)}{5-1}$

Avg. velocity $= \frac{-14/3 - 50/3}{4} = \boxed{\frac{-16}{3} \text{ ft/s}}$

e) When is the particle at rest? * set $v(t) = 0$

$v(t) = 2(t^2 - 9t + 14)$

$0 = 2(t-7)(t-2)$

$\boxed{t = 2, 7 \text{ seconds b/c } v(t) = 0}$

f) When is the particle moving left? When does particle change directions? (Create Sign Line) Give justification.

$v(t) = 2(t-2)(t-7)$

$v(t)$: + (0 to 2, test 1), − (2 to 7, test 3), + (after 7, test 8)

particle moving left $(2, 7)$ b/c $v(t) < 0$

particle change directions $t = 2, 7$ seconds b/c $v(t)$ change signs

g) What is displacement of particle from t = 3 to t = 8? Show work.

$x(3) = 18$

$x(8) = -\frac{41}{3}$

Displacement $= x(8) - x(3)$

$= -\frac{41}{3} - 18 = \boxed{-\frac{95}{3}}$

h) What is the total distance of particle from t = 3 to t = 8? Show work. * changes direction at $t = 7$

$x(3) = 18$

$x(7) = -58/3$ $\Big] = \frac{112}{3}$

$x(8) = -41/3$ $\Big] = +\frac{17}{3}$

$\frac{112}{3} + \frac{17}{3} = \boxed{\frac{129}{3}}$ ft.

i) Is the speed increasing or decreasing at t = 5? Justify.

$v(5) = -12$ ft/s

$a(5) = 2$ ft/s^2

speed decreasing at $t = 5$ b/c $v(t)$ and $a(t)$ have opposite signs

j) Is velocity increasing or decreasing at t = 2? Justify.

Since $a(2) = -10$ ft/s^2

velocity is decreasing at $t = 2$ since $a(t) < 0$