Non-AP Calculus 2.2-2.4 Derivatives Quiz Review WS #3

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5\sqrt[3]{x^4}(x^3 + 2) - \frac{2}{13\left(\sqrt[5]{x}\right)} - \sqrt{\pi^5}(1 + 0.44\pi^3) + \pi\sqrt{x^9}$

2. Find $\frac{dy}{dx}$ for $y = \frac{6}{\sqrt[4]{(5x^3-2x^2+3\pi^5)^3}}$

3. Find g'(x) for $g(x) = 2x^3(3 - 5x^3)^7$ (simplify fully in factored form)

4. Find $\frac{dy}{dx}$ for $y = \frac{2x}{\sqrt{2+3x^4}}$ (simplify fully in factored form)

5. Find $\frac{dy}{dx}$ for $y = \left(\frac{2x-1}{2-3x^2}\right)^5$ (simplify fully in factored form)

6. If $f(x) = \frac{3x^2-1}{x+4}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

7. Particle moves along the x-axis and its velocity at time t is given as $v(t) = (t+1)(t-1)^2(t-3)$ in in meters per minute for all Real numbers. Use this to answer the questions below. **Include units with your answers**

a) What is its velocity at $t = 4$ seconds?	b) What is its velocity at $t = 1$ seconds?
c) Find the average acceleration of particle in $[1, 4]$	d) When is the particle at rest?

e) When is the particle moving right? moving left? When does particle change directions? (Create Sign Line) Give justification.

Solution Key

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No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5\sqrt[3]{x^4}(x^3+2) - \frac{2}{13\left(\sqrt[5]{x}\right)} - \sqrt{\pi^5}(1+0.44\pi^3) + \pi\sqrt{x^9}$

$y = 5x^{4/3}(x^3+2) - \frac{2}{13}x^{-1/5} - \sqrt{\pi^5}(1+0.44\pi^3) + \pi x^{9/2}$

$y = 5x^{13/3} + 10x^{4/3} - \frac{2}{13}x^{-1/5} - \sqrt{\pi^5}(1+0.44\pi^3) + \pi x^{9/2}$

$y' = 5\cdot\frac{13}{3}x^{10/3} + 10\cdot\frac{4}{3}x^{1/3} - \frac{2}{13}\cdot\frac{-1}{5}x^{-6/5} - 0 + \pi\cdot\frac{9}{2}x^{7/2}$

$\boxed{\frac{dy}{dx} = \frac{65}{3}x^{10/3} + \frac{40}{3}x^{1/3} + \frac{2}{65x^{6/5}} + \frac{9\pi}{2}x^{7/2}}$

2. Find $\frac{dy}{dx}$ for $y = \frac{6}{\sqrt[4]{(5x^3-2x^2+3\pi^5)^3}}$

$y = 6\left(5x^3-2x^2+3\pi^5\right)^{3/4}$

* chain rule:
outside: $6(\ )^{3/4}$
inside: $5x^3-2x^2+3\pi^5$

$y' = 6\cdot\frac{3}{4}\left(5x^3-2x^2+3\pi^5\right)^{-1/4}\cdot(15x^2-4x+0)$

$\boxed{\frac{dy}{dx} = \frac{9(15x^2-4x)}{2\left(5x^3-2x^2+3\pi^5\right)^{1/4}}}$

3. Find g'(x) for $g(x) = 2x^3(3-5x^3)^7$ (simplify fully in factored form)

* ① product rule
* ② chain rule
outside: $(\ )^7$
inside: $3-5x^3$

$g'(x) = \underbrace{6x^2\cdot(3-5x^3)^7}_{6x^2(3-5x^3)^6} + \underbrace{2x^3\cdot 7(3-5x^3)^6(-15x^2)}_{6x^2(3-5x^3)^6}$ (-5)

$g'(x) = 6x^2(3-5x^3)^6\left[3-5x^3-35x^3\right]$

$\boxed{g'(x) = 6x^2(3-5x^3)^6(3-40x^3)}$

4. Find $\frac{dy}{dx}$ for $y = \frac{2x}{\sqrt{2+3x^4}} = \frac{2x}{(2+3x^4)^{1/2}}$ (simplify fully in factored form)

* ① quotient rule
* ② chain rule

$y' = \frac{2(2+3x^4)^{1/2} - 2x\cdot\frac{1}{2}(2+3x^4)^{-1/2}(12x^3)}{\left[(2+3x^4)^{1/2}\right]^2}$

$y' = \frac{2(2+3x^4)^{1/2} - \frac{12x^4}{(2+3x^4)^{1/2}}}{(2+3x^4)^1}$ $\cdot\ \frac{(2+3x^4)^{1/2}}{(2+3x^4)^{1/2}}$

$\frac{dy}{dx} = \frac{2(2+3x^4) - 12x^4}{(2+3x^4)^{3/2}}$ (2 + 6x^4)

$\boxed{\frac{dy}{dx} = \frac{4-6x^4}{(2+3x^4)^{3/2}}}$

5. Find $\frac{dy}{dx}$ for $y = \left(\frac{2x-1}{2-3x^2}\right)^5$ (simplify fully in factored form)

① * chain rule: outside: $(\ )^5$, inside: $\frac{2x-1}{2-3x^2}$
② * quotient rule

$y' = 5\left(\frac{2x-1}{2-3x^2}\right)^4 \cdot \left[\frac{2(2-3x^2)-(2x-1)(-6x)}{(2-3x^2)^2}\right]$ $\quad -(-6x)(2x-1)$

$y' = 5\left(\frac{2x-1}{2-3x^2}\right)^4 \left[\frac{4-6x^2+12x^2-6x}{(2-3x^2)^2}\right]$

$\frac{(2x-1)^4}{(2-3x^2)^4}$

$\boxed{\frac{dy}{dx} = \frac{5(2x-1)^4(6x^2-6x+4)}{(2-3x^2)^6}}$

6. If $f(x) = \frac{3x^2-1}{x+4}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$f'(x) = \frac{(6x)(x+4)-(3x^2-1)(1)}{(x+4)^2}$

$f'(x) = \frac{6x^2+24x-3x^2+1}{(x+4)^2}$

$f'(x) = \frac{3x^2+24x+1}{(x+4)^2}$

point: $f(1) = \frac{3(1)^2-1}{1+4} = \frac{2}{5}$

slope: $f'(1) = \frac{3(1)^2+24(1)+1}{(1+4)^2} = \frac{28}{25}$

point: $(1, \frac{2}{5})$

slope: $m = \frac{28}{25}$

$\boxed{y - \frac{2}{5} = \frac{28}{25}(x-1)}$

7. Particle moves along the x-axis and its velocity at time t is given as $v(t) = (t+1)(t-1)^2(t-3)$ in in meters per minute for all Real numbers. Use this to answer the questions below. **Include units with your answers**

a) What is its velocity at $t = 4$ seconds?

$v(4) = \boxed{45 \text{ meters/min}}$

b) What is its velocity at $t = 1$ seconds?

$v(1) = \boxed{0 \text{ meters/min}}$

c) Find the average acceleration of particle in [1, 4]

Avg. acceleration $= \frac{v(4)-v(1)}{4-1} = \frac{45-0}{4-1}$

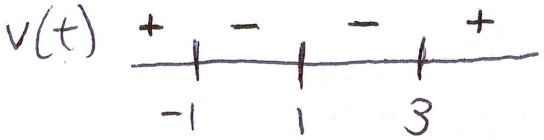
$= \boxed{15 \text{ meters/min}^2}$

d) When is the particle at rest? * set $v(t) = 0$

$0 = (t+1)(t-1)^2(t-3)$

$\boxed{t = -1, 1, 3 \text{ mins b/c } v(t) = 0}$

e) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification. $v(t) = (t+1)(t-1)^2(t-3)$



particle moves right $(-\infty, -1) \cup (3, \infty)$ b/c $v(t) > 0$

particle moves left $(-1, 1) \cup (1, 3)$ b/c $v(t) < 0$

particle change directions at $t = -1, 3$ mins b/c $v(t)$ change signs.