Non-AP Calculus Chapter 1 Limits Test Review Worksheet #1

1) For the function $g(x)=\begin{cases}\dfrac{x^2-6x-7}{x+1}, & x\neq -1\\ 9 & , x=-1\end{cases}$

Use continuity condition to show that $g(x)$ is discontinuous at $x = -1$ and state why it is discontinuous there. Determine type of discontinuity if function is not continuous at x = -1

2) If $f(x) = \begin{cases} x^2 - 1, x < 2 \\ 5, \quad x = 2 \\ 4x - 5, x > 2 \end{cases}$, then find the following

a) $\lim\limits_{x\to 2} f(x) =$

b) f(2) =

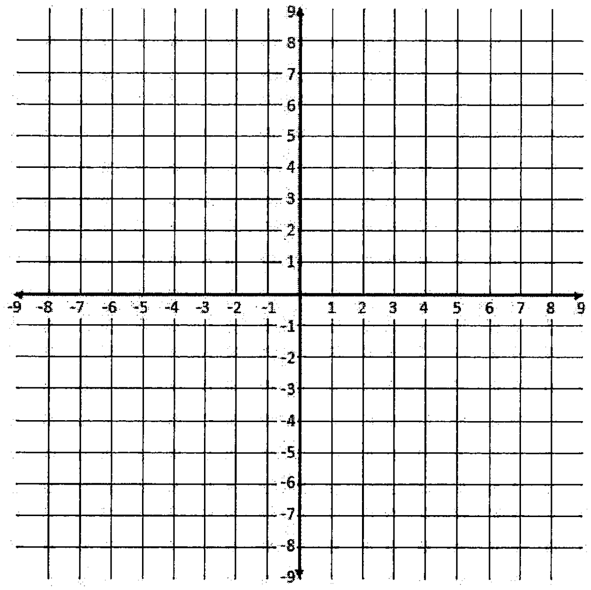
c) Use continuity condition to determine if function is continuous or not continuous at x = 2 (If not continuous, determine type of discontinuity)

3) Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem

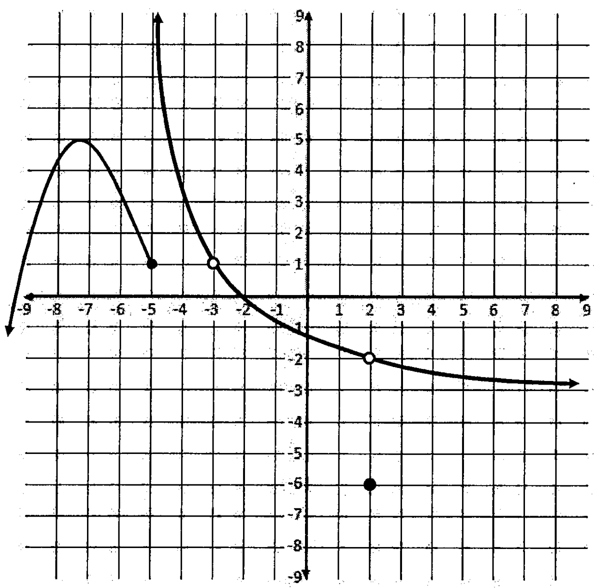
$f(x) = x^2 - 5x - 3$ in the interval [0, 7] f(c) = 3

4) . Sketch a possible graph for <u>function</u> $f(x)$ with the following properties:

- $f(-3) = -4$
- $\lim_{x \to -3} f(x) = 2$
- $\lim_{x \to -4} f(x) = +\infty$
- $f(1) = 5$
- $\lim_{x \to 1^+} f(x) = 4$
- $\lim_{x \to -\infty} f(x) = 6$
- $\lim_{x \to \infty} f(x) = -\infty$



5) Find the following:



a) $\lim_{x \to -5^-} f(x) =$ ____________ b) $\lim_{x \to -5^+} f(x) =$ ____________ c) $f(-5) =$ ____________

d) $\lim_{x \to -5} f(x) =$ ____________ e) $\lim_{x \to -3} f(x) =$ ____________ f) $f(-3) =$ ____________

g) $\lim_{x \to 2} f(x) =$ ____________ h) $\lim_{x \to \infty} f(x) =$ ____________ i) $\lim_{x \to -\infty} f(x) =$ ____________

Find the following:

6) $$\lim_{x\to 0^-} \frac{\frac{1}{x+6}-\frac{1}{6}}{x}$$

7) $$\lim_{x\to -\infty} \frac{5x^2-3x-4}{(2x-3)^2} =$$

8) $$\lim_{x\to 5^+} \frac{4-\sqrt{11+x}}{x-5}$$

9) $$\lim_{x\to \infty} \frac{3x^2-1}{x^2-9} =$$

10) $$\lim_{x\to 3^+} \frac{3x^2-1}{x^2-9}$$

11) $$\lim_{x\to -\infty} \frac{3x^2-1}{4x^3-9}$$

12) $$\lim_{x\to -\infty} \frac{x^3-16}{4-x^2} =$$

13) $$\lim_{x\to \infty} \frac{x^2-3}{4-x} =$$

14)
$$\lim_{x \to 0^+} \frac{\frac{1}{x+7} - \frac{1}{7}}{x}$$

15) $\lim_{x \to \infty} \frac{6-5x^2}{2x-3} =$

16)
$$\lim_{x \to 7^-} \frac{5 - \sqrt{18+x}}{x-7}$$

17) $\lim_{x \to -\infty} \frac{17x^2-1}{36x^2-9} =$

18) $\lim_{x \to 2^+} \frac{3x^2-1}{x^2-4}$

19) $\lim_{x \to -\infty} \frac{129x^8-5000x^7-120x^6+120000}{2500x^6-400x^9-9x^5}$

20) $\lim_{x \to 2} \frac{x^3-16}{4-x^2} =$

21) $\lim_{x \to 2^+} \frac{6x^2-14x+4}{x-2} =$

Key

Non-AP Calculus Chapter 1 Limits Test Review Worksheet #1

1) For the function $g(x)=\begin{cases}\dfrac{x^2-6x-7}{x+1}, & x\neq -1\\ 9, & x=-1\end{cases}$

Use continuity condition to show that $g(x)$ is discontinuous at $x = -1$ and state why it is discontinuous there. Determine type of discontinuity if function is not continuous at x = -1

i) $g(-1)=9$

ii) $\lim\limits_{x\to -1^-}\dfrac{x^2-6x-7}{x+1} = \dfrac{0}{0}\ \dfrac{(x-7)\cancel{(x+1)}}{\cancel{(x+1)}} = -8$ $\quad \lim\limits_{x\to -1^+}\dfrac{x^2-6x-7}{x+1} = -8$ $\quad \lim\limits_{x\to -1} g(x) = -8$

iii) $g(-1) \neq \lim\limits_{x\to -1} g(x)$ Removable Discontinuity at $x=-1$

2) If $f(x)=\begin{cases}x^2-1, & x<2\\ 5, & x=2\\ 4x-5, & x>2\end{cases}$, then find the following

a) $\lim\limits_{x\to 2} f(x) =$

$\lim\limits_{x\to 2^-} x^2-1 = 4-1=3$

$\lim\limits_{x\to 2^+} 4x-5 = 8-5=3$

$\lim\limits_{x\to 2} f(x) = 3$

b) f(2) = 5

c) Use continuity condition to determine if function is continuous or not continuous at x = 2 (If not continuous, determine type of discontinuity)

i) $f(2)=5$

ii) $\lim\limits_{x\to 2} f(x) = 3$

iii) $f(2) \neq \lim\limits_{x\to 2} f(x)$ Removable Discontinuity at $x=2$

3) Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem

$f(x) = x^2 - 5x - 3$ in the interval [0, 8] f(c) = 3

f(x) continuous on $[0,8]$

$f(0) = -3$

$f(8) = 21$

IVT applies; By IVT, $f(c)=3$ in $[0,8]$

$3 = x^2-5x-3$

$0 = x^2-5x-6$

$0 = (x-6)(x+1)$

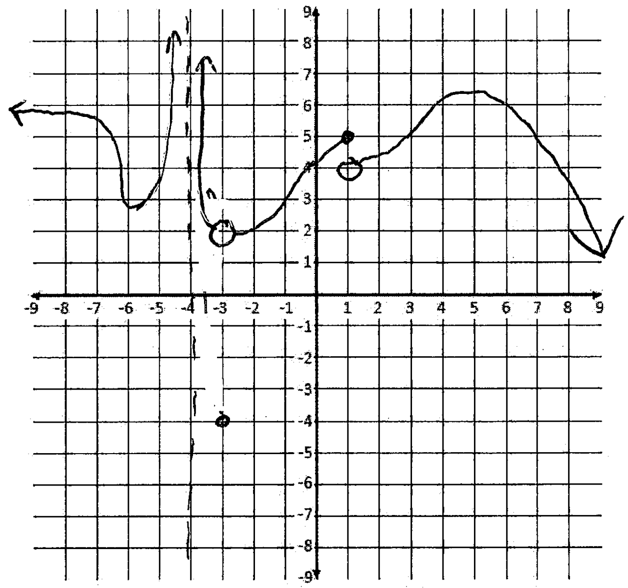
$x=6$, $\cancel{x=-1}$

$\boxed{c=6}$ in $[0,8]$

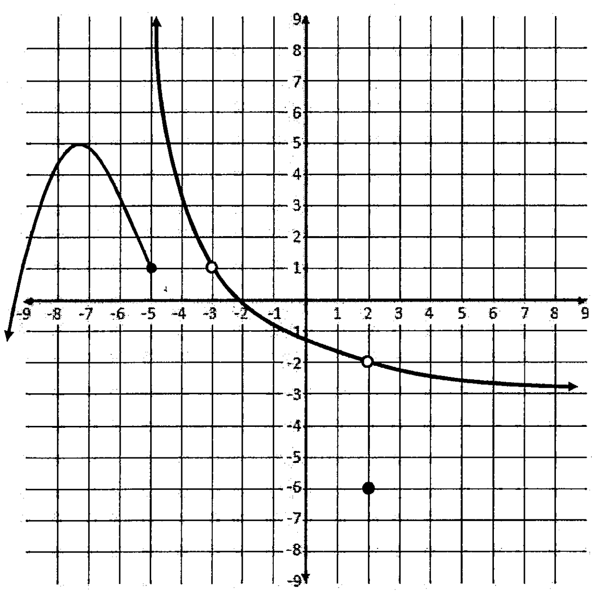
-6, 1, 1, -6, -5

4) . Sketch a possible graph for <u>function</u> $f(x)$ with the following properties:

- $f(-3) = -4$ ✓
- $\lim_{x\to-3} f(x) = 2$ ✓
- $\lim_{x\to-4} f(x) = +\infty$ ✓
- $f(1) = 5$ ✓
- $\lim_{x\to1^+} f(x) = 4$ ✓
- $\lim_{x\to-\infty} f(x) = 6$ ✓
- $\lim_{x\to\infty} f(x) = -\infty$ ✓



5) Find the following:



a) $\lim_{x\to-5^-} f(x) =$ 1

b) $\lim_{x\to-5^+} f(x) =$ $+\infty$

c) $f(-5) =$ 1

d) $\lim_{x\to-5} f(x) =$ DNE

e) $\lim_{x\to-3} f(x) =$ 1

f) $f(-3) =$ undefined

g) $\lim_{x\to2} f(x) =$ -2

h) $\lim_{x\to\infty} f(x) =$ -3

i) $\lim_{x\to-\infty} f(x) =$ $-\infty$

Find the following:

6) $\lim\limits_{x\to 0^-} \dfrac{\frac{1}{x+6}-\frac{1}{6}}{x}$ $\quad \frac{0}{0}$ $\quad 6(x+6)$

$\lim\limits_{x\to 0^-} \dfrac{6-(x+6)}{6x(x+6)}$

$\lim\limits_{x\to 0^-} \dfrac{\cancel{6}-\cancel{x}-\cancel{6}}{6\cancel{x}(x+6)}$

$\lim\limits_{x\to 0^-} \dfrac{-1}{6(x+6)} = \dfrac{-1}{6(6)} = \boxed{\dfrac{-1}{36}}$

7) $\lim\limits_{x\to -\infty} \dfrac{5x^2-3x-4}{(2x-3)^2} = \lim\limits_{x\to -\infty} \dfrac{5x^2-3x-4}{4x^2-12x+9}$

$\boxed{\dfrac{5}{4}}$

8) $\lim\limits_{x\to 5^+} \dfrac{4-\sqrt{11+x}}{x-5}$ $\quad \frac{0}{0}$ $\quad \cdot \dfrac{4+\sqrt{11+x}}{4+\sqrt{11+x}}$

$\lim\limits_{x\to 5^+} \dfrac{16-(11+x)}{(x-5)(4+\sqrt{11+x})} = \lim\limits_{x\to 5^+} \dfrac{16-11-x}{(x-5)(4+\sqrt{11+x})}$

$\lim\limits_{x\to 5^+} \dfrac{\cancel{(5-x)}\,-1}{\cancel{(x-5)}(4+\sqrt{11+x})} = \dfrac{-1}{4+\sqrt{16}} = \dfrac{-1}{8}$

9) $\lim\limits_{x\to \infty} \dfrac{3x^2-1}{x^2-9} = \quad = \boxed{3}$

10) $\lim\limits_{x\to 3^+} \dfrac{3x^2-1}{x^2-9} = \dfrac{26}{0}$ $\nearrow +\infty$ $\searrow -\infty$

3.1 $\quad \dfrac{3(3.1)^2-1}{(3.1)^2-9} = \dfrac{+}{+} = \boxed{+\infty}$

11) $\lim\limits_{x\to -\infty} \dfrac{3x^2-1}{4x^3-9} = \boxed{0}$

12) $\lim\limits_{x\to -\infty} \dfrac{x^3-16}{4-x^2} =$ $\nearrow +\infty$ $\searrow -\infty$

$\dfrac{(-\infty)^3-16}{4-(-\infty)^2} = \dfrac{-}{-} = \boxed{+\infty}$

13) $\lim\limits_{x\to \infty} \dfrac{x^2-3}{4-x} =$ $\nearrow +\infty$ $\searrow -\infty$

$\dfrac{(\infty)^2-3}{4-(\infty)} = \dfrac{+}{-} = \boxed{-\infty}$

14) $\lim_{x\to 0^+} \dfrac{\frac{1}{x+7} - \frac{1}{7}}{x}$ $\quad \frac{0}{0} \quad 7(x+7)$

$\lim_{x\to 0^+} \dfrac{7-(x+7)}{x(7)(x+7)} = \lim_{x\to 0^+} \dfrac{\cancel{7}-\cancel{x}-\cancel{7}}{\cancel{x}(7)(x+7)} = \dfrac{-1}{7(x+7)}$

$= \boxed{\dfrac{-1}{49}}$

15) $\lim_{x\to\infty} \dfrac{6-5x^2}{2x-3} = \begin{cases} \nearrow +\infty \\ \searrow -\infty \end{cases}$

$\dfrac{6-5(\infty)^2}{2(\infty)-3} = \dfrac{-}{+} = \boxed{-\infty}$

16) $\lim_{x\to 7^-} \dfrac{5-\sqrt{18+x}}{x-7}$ $\quad \frac{0}{0} \quad \cdot \dfrac{5-\sqrt{18+x}}{5+\sqrt{18+x}}$

$\lim_{x\to 7^-} \dfrac{25-(18+x)}{(x-7)(5+\sqrt{18+x})}$

$\lim_{x\to 7^-} \dfrac{25-18-\cancel{x}}{(x-7)(5+\sqrt{18+x})}$

$\lim_{x\to 7^-} \dfrac{\cancel{(7-x)}-1}{\cancel{(x-7)}(5+\sqrt{18+x})}$

$= \dfrac{-1}{5+\sqrt{25}} = \boxed{\dfrac{-1}{10}}$

17) $\lim_{x\to-\infty} \dfrac{17x^2-1}{36x^2-9} = \boxed{\dfrac{17}{36}}$

18) $\lim_{x\to 2^+} \dfrac{3x^2-1}{x^2-4}$ $\quad \dfrac{11}{0} \begin{cases} \nearrow +\infty \\ \searrow -\infty \end{cases}$

2.1

$\dfrac{3(2.1)^2-1}{(2.1)^2-4} = \dfrac{+}{+} = \boxed{+\infty}$

19) $\lim_{x\to-\infty} \dfrac{129x^8-5000x^7-120x^6+120000}{2500x^6-400x^9-9x^5} = \boxed{0}$

20) $\lim_{x\to 2} \dfrac{x^3-16}{4-x^2} = \dfrac{8-16}{0} = \dfrac{-8}{0}$

$\boxed{\text{DNE}}$

Does Not Exist

21) $\lim_{x\to 2^+} \dfrac{6x^2-14x+4}{x-2} = \quad \frac{0}{0} \quad \dfrac{2(3x^2-7x+2)}{x-2}$

$\lim_{x\to 2^+} \dfrac{2\cancel{(x-2)}(3x-1)}{\cancel{(x-2)}}$

-6 6 -1
3 3 -7

$= 2(5) = \boxed{10}$