The Magical World of Number Theory!

Review:

Let a and b be two positive integers such that

$$a = p_1^{e_1} p_2^{e_2} \dots p_n^{e_n}$$

$$b = p_1^{f_1} p_2^{f_2} \dots p_n^{f_n}$$

Where $e_1, e_2, \dots, e_n$ and $f_1, f_2, \dots, f_n$ are exponents in the range $[1, \infty)$. The greatest common denominator may be written as

$$\gcd(a, b) = p_1^{\min\{e_1, f_1\}} p_2^{\min\{e_2, f_2\}} \dots p_n^{\min\{e_n, f_n\}}$$

And the least common multiple may be written as

$$\operatorname{lcm}(a, b) = p_1^{\max\{e_1, f_1\}} p_2^{\max\{e_2, f_2\}} \dots p_n^{\max\{e_n, f_n\}}$$

The Euclidean algorithm can speed up calculation of the gcd:

$$\gcd(m, n) = \gcd(m - n, n)$$

How many positive integers less than 101 are multiples of either 5 or 7, but not both at once?

20 multiples of 5 < 101 14 multiples of $7 < 101$

also multiples of 7: $7 \cdot 5$, $7 \cdot 10$, ~~7.15~~

$14 + 20 - 4 =$ ~~18 32 multiples~~ Magical :.

$= \boxed{30 \text{ multiples}}$

Suppose that Agni bakes 252 cookies, Kumar bakes 105 cookies, and Someone bakes 168 cookies. Each person's cookies are packaged separately in boxes with equal numbers of cookies. What is the greatest number of cookies that could be in each package?

$\gcd(252, 105)$
$= \gcd(147, 105)$
$= \gcd(42, 105)$
$= \gcd(42, 63)$
$= \gcd(42, 21)$
$= \boxed{21}$

$\gcd(105, 168)$
$= \gcd(105, 63)$
$= \gcd(42, 63)$
$= \gcd(42, 21)$
$= \boxed{21}$

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What is the smallest positive integer N such that the value $7 + 30N$ is not a prime number?

Let $7+30N=k$.

$30=2^1 3^1 5^1$

$\Rightarrow$ k must be multiple of 7

$30\cdot 7+7=7^1 31^1$

$30\cdot 6+7=187=11^1 17^1$

$30\cdot 5+7=157=157^1$ (prime!)

$30\cdot 4+7=127=127^1$ (prime again!)

$\boxed{N=6}$

11
22
33
⋮
121
132

How many even four digit palindromes are there? (Hint: Palindromes are of the form ~~mnnm~~.) ← (In this case. Can also be mm, mnm, mnpnm, etc.)

To be even, $m=2,4,6,8$ ∴ 4 options for m

n can be any ~~value~~ digit ∴ 10 options for n

$4\cdot 10=$ $\boxed{40 \text{ even palindromes}}$

This is what I get for being sleep deprived.

If $C = 19!$, then express $21! - 20!$ in terms of C.

$21!-20! = 19!(21\cdot 20-20)=20C(21-\cancel{20}1)$

$=\boxed{400C}$

The Magical World of Number Theory!

How to convert a number from base N to base 10:

1. Expand each number as the sum of digit bundles. Multiply each digit by its corresponding power of N.

   $$\text{Ex: } 537_9 = 5 \cdot 9^2 + 3 \cdot 9^1 + 7 \cdot 9^0$$

2. Find the sum.

   $$5 \cdot 9^2 + 3 \cdot 9^1 + 7 \cdot 9^0 = 405 + 27 + 7 = 439_{10}$$

Determine if 117_9 is a multiple of 5.

$117_9 = 1 \cdot 9^2 + 1 \cdot 9^1 + 7 \cdot 9^0 = 81 + 9 + 7 = \boxed{97}$

$\boxed{\text{not multiple of 5}}$

Find the base b in which the equation 4 · 12 = 103 is valid.

$4_b \cdot 12_b = 103_b$

$4_b = 4 \cdot b^0 = 4$

$12_b = 1 \cdot b^1 + 2 \cdot b^0 = b + 2$

$103_b = 1 \cdot b^2 + 0 \cdot b^1 + 3 \cdot b^0 = b^2 + 3$

$4(b+2) = b^2 + 3$

$4b + 8 = b^2 + 3$

$0 = b^2 - 4b - 5$

$\boxed{b = 5}$

How many positive integers less than 100 have a remainder of 1 when divided by 2 and also when divided by 3?

R1 when divided by 2 aka an even #, which is aka odd

50 odd #s less than 100

R1 when divided by 3:

3	4
6	7
9	10
12	13
15	16
18	19
21	22

odd only after even multiples of 3

33 multiples of 3 less than 100

of which $\boxed{\text{17 multiples}}$ are odd

#noscope360

$\frac{10}{20}$

$10 = 2^1 \cdot 5^1$

$20 = 2^2 \cdot 5^1$

$\gcd(10, 20) = 2^1 5^1$

$\gcd(15, 45)$

$= \gcd(15, 30)$

$= \gcd(15, 15) = \boxed{15}$