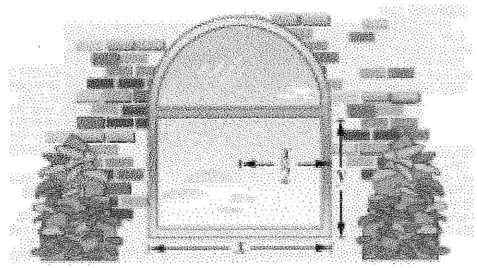
AP Calculus Worksheet 3-7 – Optimization

Ch. 3.7 Pg. 224

23. *Maximum Area* A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

$*$Find Max Area $A = xy + \frac{1}{2}(\pi r^2)$



$$P = x + 2y + \underbrace{\tfrac{1}{2}\left(2\pi\left(\tfrac{x}{2}\right)\right)}$$

½ circumference: $\frac{1}{2}(2\pi r)$

$$P = x + 2y + \frac{\pi x}{2}$$

$$2\left[16 = x + 2y + \frac{\pi x}{2}\right]$$

$$32 = 2x + 4y + \pi x$$

$$\frac{32 - 2x - \pi x}{4} = y$$

$$y = 8 - \frac{x}{2} - \frac{\pi x}{4}$$

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$A = xy + \frac{\pi x^2}{8}$$

$$A = x\left(8 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$A = 8x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$

$$A'(x) = 8 - \frac{1}{2}(2)x - \frac{\pi}{4}(2x) + \frac{\pi}{8}(2x)$$

$$A'(x) = 8 - x - \frac{\pi}{2}x + \frac{\pi}{4}x$$

$$0 = 8 + \frac{-4x - 2\pi x + \pi x}{4}$$

$$0 = 8 + \frac{-4x - \pi x}{4}$$

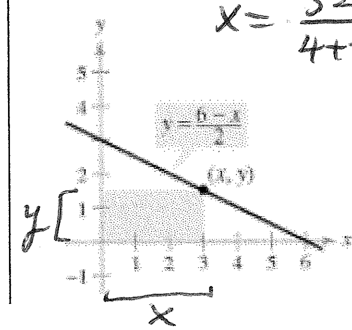
$$\frac{-4x - \pi x}{4} = -8 \qquad -4x - \pi x = -32$$

$$-x(4+\pi) = -32$$

$$x(4+\pi) = 32 \qquad \boxed{x = \frac{32}{4+\pi}}$$

Area is maximum when $x = \frac{32}{4+\pi}$ and $y = \frac{16}{4+\pi}$ ft.

24. *Maximum Area* A rectangle is bounded by the x- and y-axes and the graph of $y = (6 - x)/2$ (see figure). What length and width should the rectangle have so that its area is a maximum?



$$A = xy$$

$$A = x\left(\frac{6-x}{2}\right) = \frac{1}{2}(6x - x^2)$$

$$A = 3x - \frac{1}{2}x^2$$

$$A'(x) = 3 - x$$

$$0 = 3 - x$$

$$\boxed{3 = x}$$

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Max area occurs where $x = 3$

Area is at a maximum when $x = 3$ and $y = 3/2$