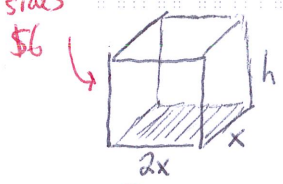
Key

Optimization Review Problem (Involving Cost)

1)

A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)



$S = 2x^2 + xh + xh + 2xh + 2xh$

$S = 2x^2 + 6xh$

$Cost = 10(2x^2) + 6(6xh)$

$*Cost = 20x^2 + 36xh$

$Volume = (2x)(x)(h)$

$10 = 2x^2h$

$\frac{10}{2x^2} = h$

$\frac{5}{x^2} = h$

$Cost = 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$C(x) = 20x^2 + \frac{180}{x}$

$C(x) = 20x^2 + 180x^{-1}$

$C'(x) = 40x - 180x^{-2}$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = \frac{40x}{1}$

$40x^3 = 180$

$x^3 = \frac{180}{40} = \frac{9}{2}$

$x^3 = 9/2$

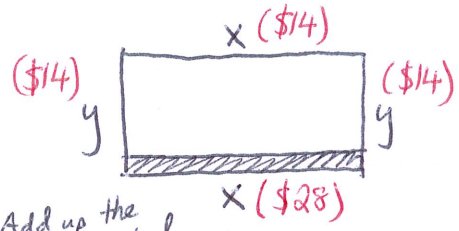
$x = \sqrt[3]{9/2} \approx 1.651$ meters

$C(1.651) = 20(1.651)^2 + \frac{180}{1.651}$

$= \boxed{\$163.54}$

2)

The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be? (perimeter)



Add up the costs associated with each side

$*Cost = 28x + 14x + 14y + 14y$

$C = 42x + 28y$

$C(x) = 42x + 28\left(\frac{600}{x}\right)$

$C(x) = 42x + 16800x^{-1}$

$Area = xy$

$600 = xy$

$\frac{600}{x} = y$

$C'(x) = 42 - 16800x^{-2}$

$0 = 42 - \frac{16800}{x^2}$

$\frac{16800}{x^2} = \frac{42}{1}$

$42x^2 = 16800$

$x^2 = \frac{16800}{42}$

$x^2 = 400$

$x = 20$ ft

$y = \frac{600}{x} \rightarrow \frac{600}{20} = 30$ ft

$Cost = 42(20) + 28(30)$

$\boxed{Cost = \$1680}$