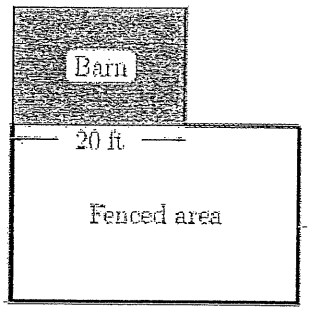
Optimization Test Review Questions WS

10) A farmer wishes to erect a fence enclosing a rectangular area adjacent to a barn which is 20 feet long. The diagram illustrates his plan for the fenced area. Find the largest area that can be enclosed if 96 feet of fencing material is available. Justify your answer.



11) We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.

12) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

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Key

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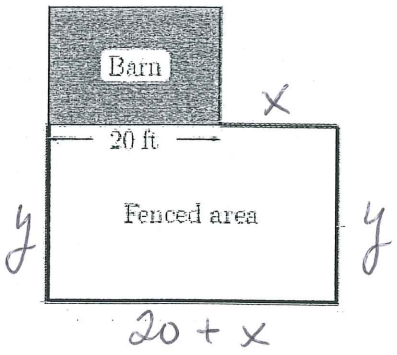

$*A = y(20+x)$

$P = x + 2y + 20 + x$

$P = 2x + 2y + 20$

$96 = 2x + 2y + 20$

$76 - 2x = 2y$

$\frac{76-2x}{2} = y$

$38 - x = y$

$A = (38-x)(20+x)$

$A = 760 - 20x + 38x - x^2$

$A = -x^2 + 18x + 760$

$A'(x) = -2x + 18$

$0 = -2x + 18 \qquad 2x = 18 \qquad x = 9$ ft.

$y = 38 - x$

$y = 38 - 9 = 29$ ft

$Area_{(max)} = y(20+x) = 29(20+9)$

$Area_{(max)} = 841\ ft^2$

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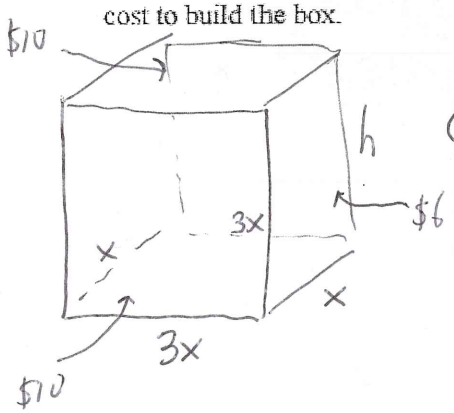

top and bottom | smaller vertical sides | larger vertical sides

$$C(x) = 10(3x^2) + 10(3x^2) + 6xh + 6xh + 6(3xh) + 6(3xh)$$

$$C(x) = 60x^2 + 12xh + 36xh$$

$$C(x) = 60x^2 + 48xh$$

$\text{Volume} = 3x \cdot x \cdot h$

$V = 3x^2h$

$50 = 3x^2h$

$\frac{50}{3x^2} = h$

$$C(x) = 60x^2 + 48x\left(\frac{50}{3x^2}\right) = 60x^2 + 800x^{-1}$$

$$C'(x) = 120x - 800x^{-2}$$

$$0 = 120x - \frac{800}{x^2}$$

$$\frac{800}{x^2} = \frac{120x}{1} \qquad 120x^3 = 800 \qquad x^3 = \frac{20}{3}$$

$$x \approx 1.882$$

$$h \approx \frac{50}{3(1.882)^2} = 4.706$$

Dimensions: 5.646 ft × 1.882 ft × 4.706 ft.

Minimum cost: $637.64

12) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

$P = (\#\text{ of trees})(\text{apple output per tree})$

$P = (50+x)(800-10x)$

$P = 40{,}000 + 300x - 10x^2$

$P'(x) = 300 - 2x$

$0 = 300 - 2x$

$0 = 20(15 - x)$

$x = 15$ trees

15 trees should be added to total 65 trees