Key

AB Calculus – Chapter P (Day 1) – Functions, Function Properties, and their Graphs

Evaluating a Function:

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$f(-2) = (-2)^2 - 2(-2) + 5$

$f(-2) = 4 + 4 + 5$

$\boxed{f(-2) = 13}$

2. $f(x+2) =$

$f(\ \ ) = (\ \ )^2 - 2(\ \ ) + 5$

$f(x+2) = (x+2)^2 - 2(x+2) + 5$

$f(x+2) = (x+2)(x+2) - 2x - 4 + 5$

$f(x+2) = x^2 + 4x + 4 - 2x - 4 + 5$

$\boxed{f(x+2) = x^2 + 2x + 5}$

3. $f(x+h) =$

$f(\ \ ) = (\ \ )^2 - 2(\ \ ) + 5$

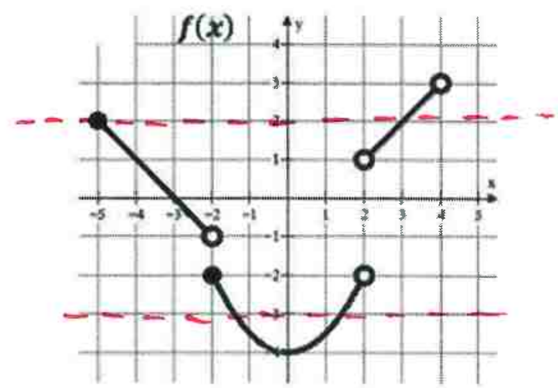
$f(x+h) = (x+h)^2 - 2(x+h) + 5$

$f(x+h) = (x+h)(x+h) - 2x - 2h + 5$

$f(x+h) = x^2 + 2xh + h^2 - 2x - 2h + 5$

$\boxed{f(x+h) = x^2 + 2xh + h^2 - 2x - 2h + 5}$

Use the graph $f(x)$ to answer the following.



4. $f(0) = \boxed{-4}$ $f(4) = \boxed{\text{undefined}}$

$f(-1) = \boxed{-3.5}$ $f(-2) = \boxed{-2}$

$f(2) = \boxed{\text{undefined}}$ $f(3) = \boxed{2}$

$f(x) = 2$ when $x = ?$

$\boxed{x = -5 \text{ and } x = 3}$

$f(x) = -3$ when $x = ?$

$\boxed{x = -1.5 \text{ and } x = 1.5}$

Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

5. slope $= 3$ and $(4, -2)$

point: $(4, -2)$ (x_1, y_1)

slope: $m = 3$

$y - (-2) = 3(x - 4)$

$\boxed{y + 2 = 3(x - 4)}$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

point: $(-5, 7)$ (x_1, y_1)

slope: $m = -3/2$

$y - 7 = \frac{-3}{2}(x - -5)$

$\boxed{y - 7 = \frac{-3}{2}(x + 5)}$

7. $f(4) = -8$ and $f(-3) = 12$

slope: $\frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{12 - (-8)}{-3 - 4} \rightarrow \frac{20}{-7}$

slope: $m = \frac{-20}{7}$

point: $(4, -8)$ (x_1, y_1)

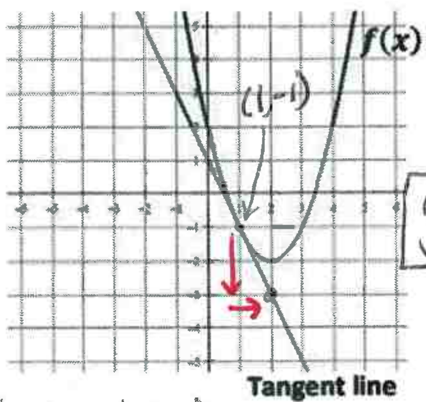
*choose either point. Either works

$y - -8 = \frac{-20}{7}(x - 4)$

$\boxed{y + 8 = \frac{-20}{7}(x - 4) \text{ or } y - 12 = \frac{-20}{7}(x + 3)}$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



$y-(-1) = -2(x-1)$

$\boxed{y+1 = -2(x-1)}$

slope (of tangent line): $m = \frac{-2}{1} = -2$

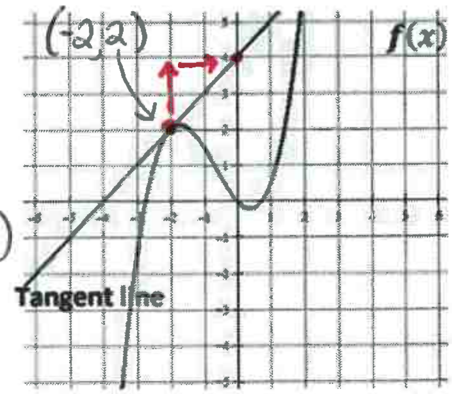
point: $(1,-1)$

9. The line tangent to $f(x)$ at $x = -2$

slope: $\frac{2}{2} = 1$

$m = 1$

point: $(-2,2)$



$\boxed{y-2 = 1(x+2)}$

MULTIPLE CHOICE! Remember slope $= \frac{y_2-y_1}{x_2-x_1}$

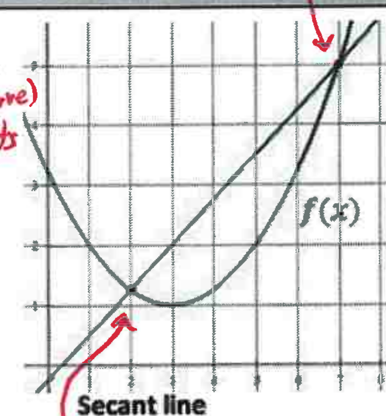
10. Which choice represents the slope of the secant line shown?

→ Intersects the graph (curve) at 2 points

A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\boxed{\frac{f(7)-f(2)}{7-2}}$

slope: $m = \frac{y_2-y_1}{x_2-x_1} \rightarrow \boxed{\frac{f(7)-f(2)}{7-2}}$

$(7, f(7))$



$(2, f(2))$

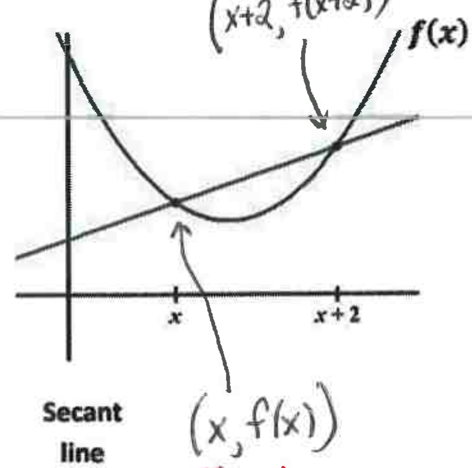
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\boxed{\frac{f(x+2)-f(x)}{x+2-x}}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

$$\boxed{\text{slope: } m = \frac{\overset{y_2\ -\ y_1}{f(x+2)-f(x)}}{\underset{x_2\ -\ x_1}{x+2-x}} \rightarrow \frac{f(x+2)-f(x)}{2}}$$

$\underset{x_2}{(x+2}, \underset{y_2}{f(x+2))}$



$(\underset{x_1}{x}, \underset{y_1}{f(x)})$

Find all Vertical Asymptotes, Horizontal Asymptotes, Holes, and x-intercepts (for Rational Functions)

I. **To Find Vertical Asymptotes:** Set Denominator Factors equal to zero and solve for x. (Make sure factors do not cancel with numerator)

II. **To Find Holes in graph:** Identify factors that cancels out between numerator and denominator. Set factor equal to zero and solve for x. To find the point (ordered pair), find y-value using the original function graph.

III. **To Find Horizontal Asymptote:** Compare Degrees between Numerator (N) and Denominator (D)

a) If **N = D**, then horizontal asymptote is **y = (ratio of leading coefficients)**

b) If **N < D**, the horizontal asymptote is **y = 0**

c) If **N > D**, there is **no horizontal asymptote.**

9. $f(x) = \frac{x+2}{3-x}$

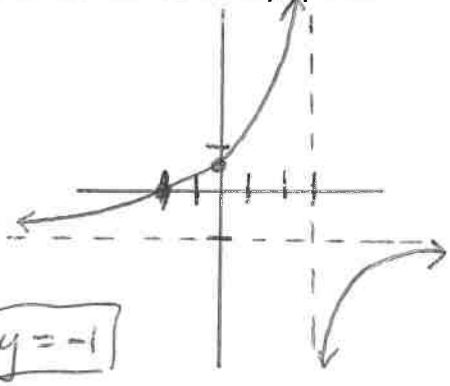
V.A: $3-x=0$

$\boxed{x=3}$

H.A. $y=\frac{1x}{-1x} \rightarrow \boxed{y=-1}$

x-int: $x+2=0 \rightarrow \boxed{x=-2} \rightarrow \boxed{(-2,0)}$

y-int: $y=\frac{0+2}{3-0} \rightarrow \frac{+2}{3} \rightarrow (0, {+2/3})$



Holes: none

Vertical Asymptotes: $x=3$

Horizontal Asymptote: $y=-1$

x-intercept: $(-2,0)$

10. $f(x) = \frac{4x-4}{x^2-9} \rightarrow \frac{4(x-1)}{(x+3)(x-3)}$

x-int: $x-1=0$

$x=1$

$(1,0)$

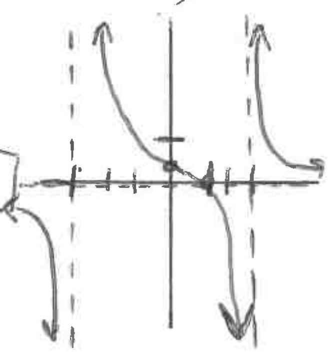
VA: $x+3=0$, $x-3=0$

$\boxed{x=-3, x=3}$

HA: $\frac{4x}{1x^2} \rightarrow \boxed{y=0}$

y-int: $y=\frac{0-4}{0-9}=\frac{4}{9}$

$(0, 4/9)$



Holes: none

Vertical Asymptotes: $x=3$, $x=-3$

Horizontal Asymptote: $y=0$

x-intercept: $(1,0)$

11. $f(x) = \frac{x^2-2x}{x^3-5x^2+6x}$

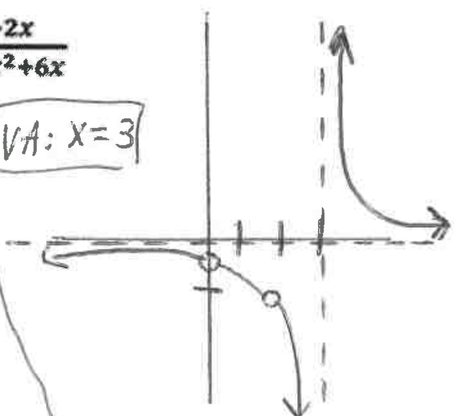
$y=\frac{x(x-2)}{x(x^2-5x+6)}$

$y=\frac{\cancel{x}\cancel{(x-2)}\,1}{\cancel{x}\cancel{(x-2)}(x-3)}$

hole at $x=0$, $x=2$

$\boxed{VA: x=3}$

H.A: $y=\frac{1x^2}{1x^3} \rightarrow y=0$



Holes: $(0, -1/3)$, $(2, -1)$

Vertical Asymptotes: $x=3$

Horizontal Asymptote: $y=0$

x-intercept: none

12. $f(x) = \frac{5x^2+2}{3x^2-12} \rightarrow \frac{5x^2+2}{3(x^2-4)} \rightarrow \frac{5x^2+2}{3(x+2)(x-2)}$

VA: $x+2=0$, $x-2=0$

$\boxed{x=-2, x=2}$

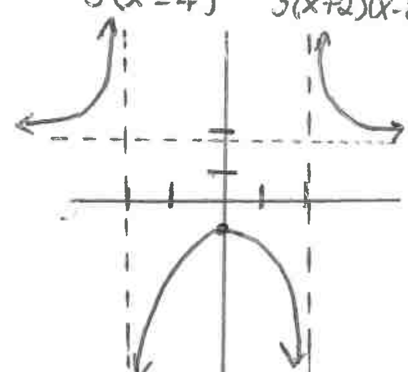
H.A: $y=\frac{5x^2}{3x^2} \rightarrow \frac{5}{3}$

$\boxed{y=5/3}$

x-int: $5x^2+2 \neq 0$

none

y-int: $(0, -1/6)$



Holes: none

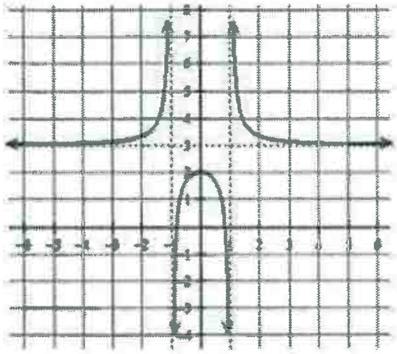
Vertical Asymptotes: $x=2$, $x=-2$

Horizontal Asymptote: $y=5/3$

x-intercept: none

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



VA: $x=-1$, $x=1$

Domain: $(-\infty,-1),(-1,1),(1,\infty)$

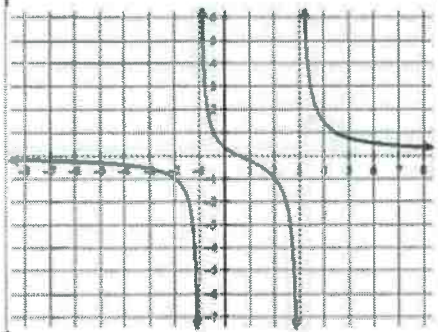
HA: $y=3$

Range: $(-\infty,2],(3,\infty)$

Horizontal Asymptote(s): $\boxed{y=3}$

Vertical Asymptotes(s): $\boxed{x=1, x=-1}$

15.



Domain: $(-\infty,-1),(-1,3),(3,\infty)$

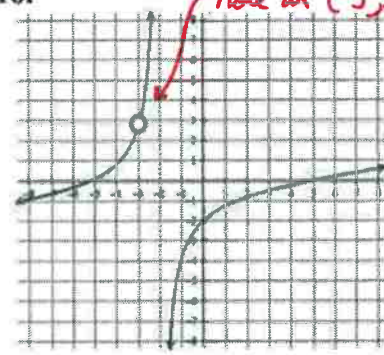
Range: $(-\infty,\infty)$

Horizontal Asymptote(s): $\boxed{y=0}$

Vertical Asymptotes(s): $\boxed{x=-1, x=3}$

16.

hole at $(-3,3)$



Domain: $(-\infty,-3),(-3,-2),(-2,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): $\boxed{\text{none}}$

Vertical Asymptotes(s): $\boxed{x=-2}$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$ → $\frac{x+5}{(x+2)(x-2)}$

(B) $\frac{x^2-16}{x-4}$ $\frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$

(C) $\frac{4x}{x+1}$

$\boxed{\text{(D)}}$ $\frac{x+6}{x^2-7x+12}$ → $\boxed{\frac{x+6}{(x-4)(x-3)}}$ $\boxed{\text{VA at } x=4}$

(E) None of the above

18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

✗ I. $f(x)$ has a vertical asymptote of $x = 2$
✓ II. $f(x)$ has a vertical asymptote of $x = -2$
✓ III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

$\boxed{\text{(D)}}$ II and III only

(E) I, II and III

$y=\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+2)}$

hole at $x=2$
$(2,\frac{-1}{4})$

VA: $x+2=0$
$\boxed{x=-2}$

HA: $y=\frac{x^2}{x^2}$
$\boxed{y=1}$

x-int: $x-3=0$
$\boxed{(3,0)}$

y-int: $(0,\frac{6}{-4})$
$\boxed{(0,-\frac{3}{2})}$