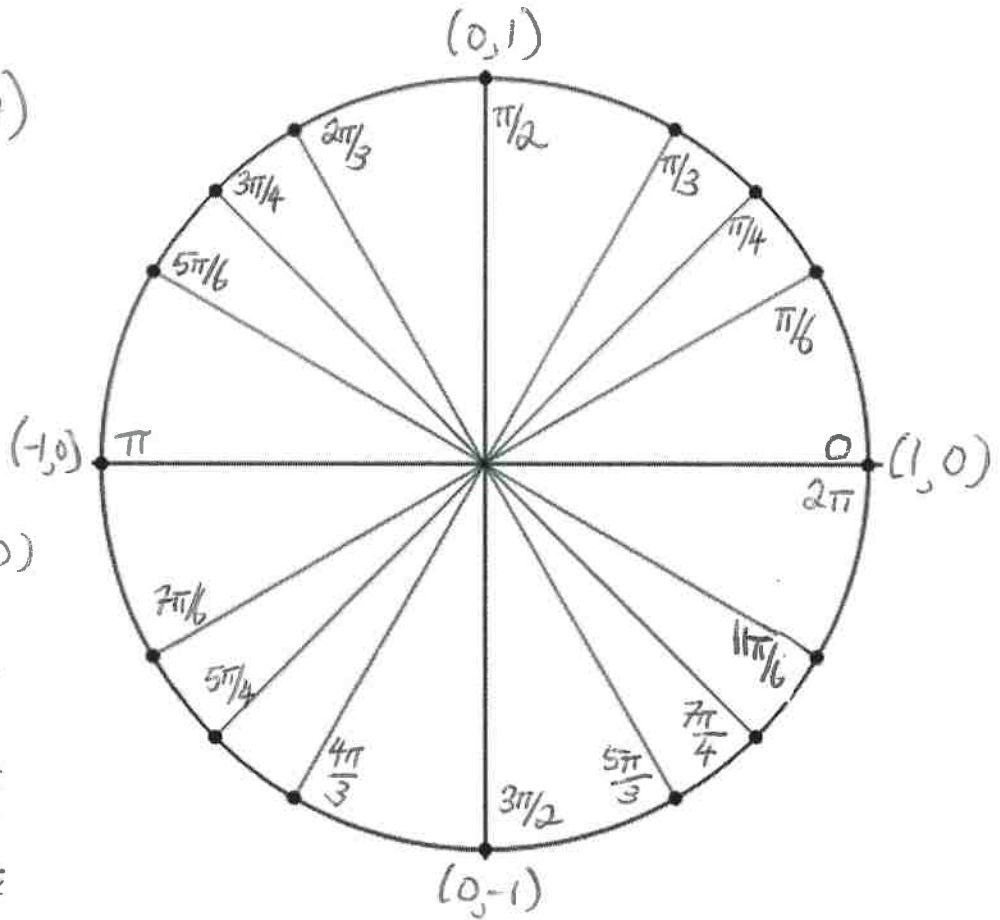
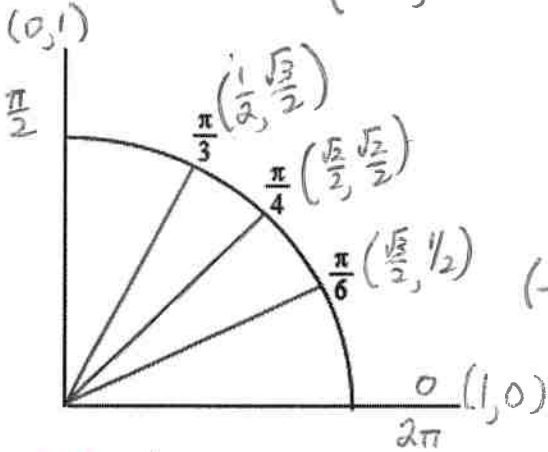


AB Calculus Ch. P Notes (Day 3) - Trig Review, Exponential Function Review, and Log Function Review

Trig Unit Circle Review

$(\cos \theta, \sin \theta)$

Quadrant I



(-,+)	S	A	(+,+)
(-, -)	T	C	(+,-)

$\tan \theta = \frac{y}{x}$   
 $\cot \theta = \frac{x}{y}$   
 $\sec \theta = \frac{1}{x}$   
 $\csc \theta = \frac{1}{y}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31.  $\sin \frac{\pi}{6} = \frac{1}{2}$

32.  $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

33.  $\sin 2\pi = 0$

34.  $\tan \pi = \frac{0}{-1} = 0$

35.  $\sec \frac{\pi}{2} = \frac{1}{0} \rightarrow \text{undefined}$

36.  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

37.  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

38.  $\sin \frac{3\pi}{2} = -1$

39.  $\tan \frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

40.  $\csc \frac{\pi}{2} \rightarrow \frac{1}{1} = 1$

41.  $\sin \pi = 0$

42.  $\cos \frac{\pi}{3} = \frac{1}{2}$

43. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$  and  $x = \frac{5\pi}{6}$

44. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

$\tan x = 0$

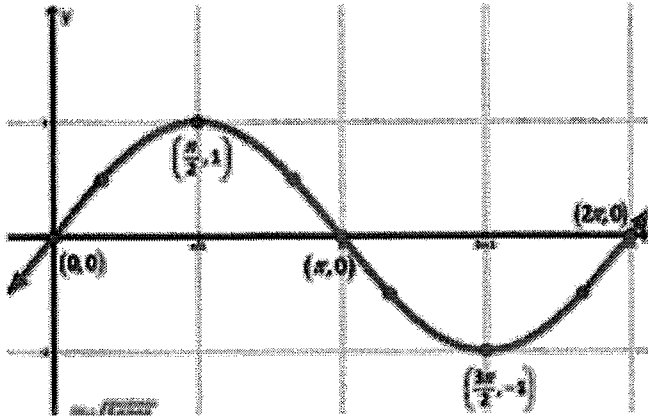
$x = 0, \pi, 2\pi$

45. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

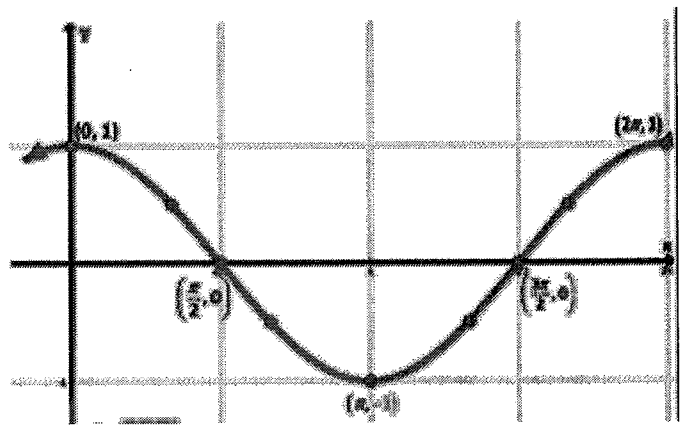
$\cos x = -1$

$x = \pi$

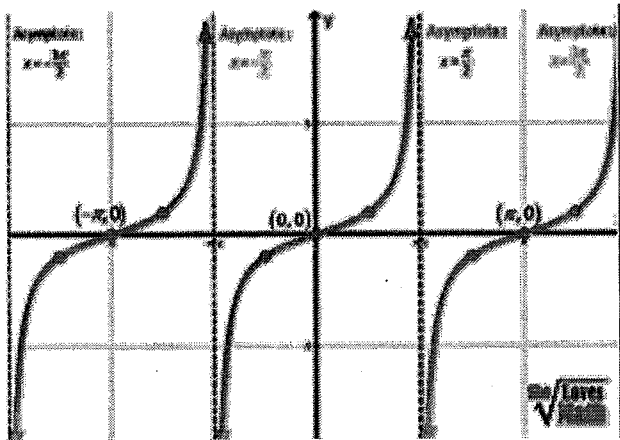
$$y = \sin(x)$$



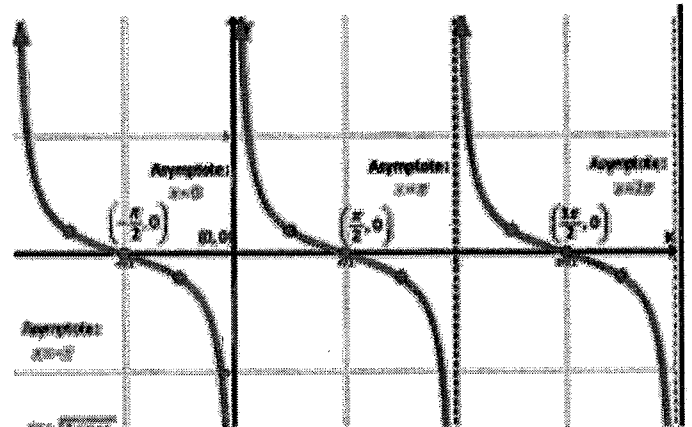
$$y = \cos(x)$$



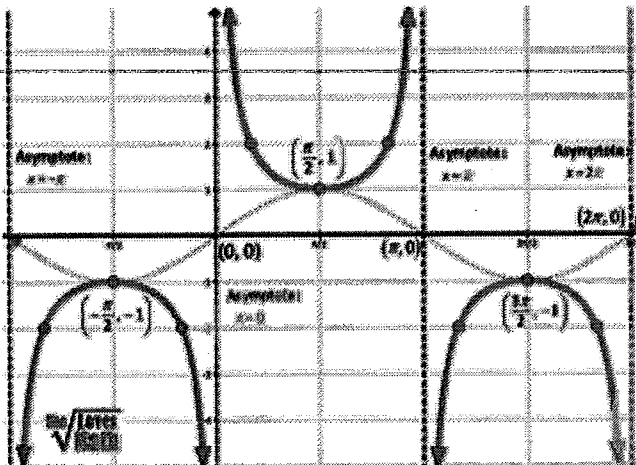
$$y = \tan(x)$$



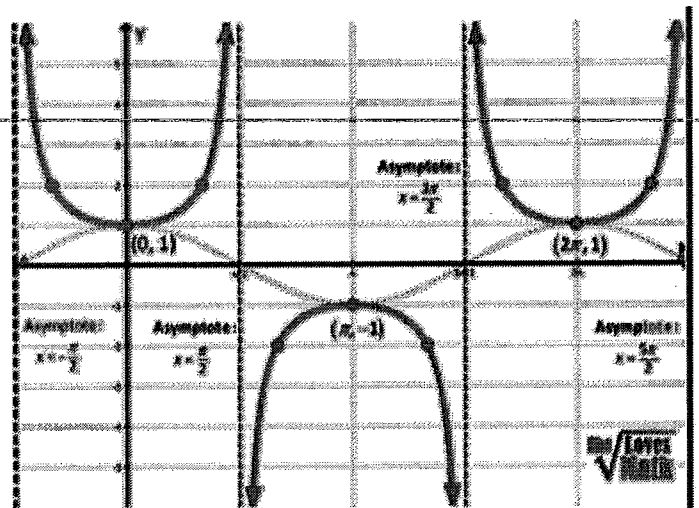
$$y = \cot(x)$$



$$y = \csc(x)$$



$$y = \sec(x)$$



Solve the following trig equations where  $0 \leq x \leq 2\pi$ .

55.  $\sin x = \frac{1}{2}$  Q1, Q2

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

56.  $\cos x = -1$

$$x = \pi$$

57.  $\cos x = \frac{\sqrt{3}}{2}$  Q1, Q4

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

58.  $2\sin x = -1$  Q3, Q4

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

59.  $\cos x = \frac{\sqrt{2}}{2}$  Q1, Q4

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

60.  $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$  Q1, Q4

$$\frac{x}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \frac{x}{2} = \frac{11\pi}{6}$$

$$\frac{x}{2} = \frac{\pi}{6} \quad x = \frac{\pi}{3} \quad \frac{x}{2} = \frac{11\pi}{3} \text{ outside interval}$$

61.  $\tan x = 0$

$$x = 0, \pi, 2\pi$$

62.  $\sin(2x) = 1$

$$2x = \sin^{-1}(1)$$

$$\frac{1}{2} [2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}]$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$$

63.  $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$  Q1, Q2

$$\frac{x}{4} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \frac{x}{4} = \frac{2\pi}{3}$$

$$\frac{x}{4} = \frac{\pi}{3} \quad 3x = 8\pi \quad x = \frac{8\pi}{3} \text{ outside interval}$$

$$3x = 4\pi \quad x = \frac{4\pi}{3}$$

Solve the following equations. Remember  $e^0 = 1$  and  $\ln 1 = 0$ .

46.  $e^x + 1 = 2$

$$e^x = 1 \quad x = 0$$

$$\ln e^x = \ln 1$$

$$x \ln e = 0$$

47.  $3e^x + 5 = 8$

$$3e^x = 3$$

$$e^x = 1$$

$$\ln e^x = \ln 1 \quad x = 0$$

$$x \ln e = 0$$

48.  $e^{2x} = 1$

$$\ln e^{2x} = \ln 1 \quad x = 0$$

$$2x \ln e = \ln 1$$

$$2x = 0$$

49.  $\ln x = 0$

$$\log_e x = 0$$

$$e^0 = x$$

$$1 = x \quad x = 1$$

50.  $3 - \ln x = 3$

$$0 = \ln x$$

$$\ln x = 0$$

$$\log_e x = 0 \quad x = 1$$

$$e^0 = x$$

51.  $\ln(3x) = 0$

$$\log_e(3x) = 0$$

$$e^0 = 3x$$

$$1 = 3x \quad x = \frac{1}{3}$$

**Review of Exponential and Log properties:**

$a^{m+n} = a^m * a^n$	$\ln(ab) = \ln a + \ln b$	$\ln x = \log_e x$
$a^{m-n} = \frac{a^m}{a^n}$	$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$	<ul style="list-style-type: none"> <li><math>\ln 1 = 0</math></li> <li><math>\ln e = 1</math></li> <li><math>\ln 0 = \text{does not exist}</math> (vertical asymptote)</li> </ul>
$(a^m)^n = a^{m*n}$	$\ln a^n = n * \ln a$	<p>If <math>y = b^x</math>, then <math>\log_b y = x</math></p> <p>If <math>\log_b y = x</math>, then <math>b^x = y</math></p>

Solve the following:

<p>52. <math>x^2 - 3x = 0</math></p> <p><math>x(x-3) = 0</math></p> <p><math>x=0, x=3</math></p>	<p>53. <math>e^x + xe^x = 0</math></p> <p><math>e^x(1+x) = 0</math></p> <p><math>e^x = 0</math>   <math>1+x = 0</math></p> <p><math>\text{None}</math>   <math>x = -1</math></p>	<p>54. <math>e^{2x} - e^x = 0</math></p> <p><math>e^x(e^x - 1) = 0</math></p> <p><math>e^x = 0</math>   <math>e^x - 1 = 0</math></p> <p><math>\ln e^x = \ln 0</math>   <math>e^x = 1</math>   <math>x = 0</math></p> <p><math>\text{None}</math>   <math>\ln e^x = \ln 1</math></p> <p><del><math>x \ln e = 0</math></del></p>
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55) Solve the following:

<p>a) <math>\log_{81} \sqrt{3} = x</math></p> <p><math>81^x = \sqrt{3}</math></p> <p><math>3^{4x} = 3^{1/2}</math></p> <p><math>4x = \frac{1}{2}</math>   <math>x = \frac{1}{8}</math></p> <p><math>x = \frac{1}{2} \cdot \frac{1}{4}</math></p>	<p>* If <math>b^x = b^y</math>, then <math>x = y</math></p> <p>b) <math>\log_x 64^{1/3} = \frac{1}{2}</math></p> <p><math>x^{1/2} = 64^{1/3}</math></p> <p><math>x^{1/2} = \sqrt[3]{64}</math></p> <p><math>x^{1/2} = 4</math>   <math>x = 16</math></p> <p><math>(x^{1/2})^2 = (4)^2</math></p>
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<p>c) <math>9 = 4 + \log_2(x+3)</math></p> <p><math>\log_2(x+3) = 5</math></p> <p><math>2^5 = x+3</math></p> <p><math>32 = x+3</math></p> <p><math>29 = x</math></p> <p><math>x = 29</math></p>	<p>d) <math>\frac{1}{3} \ln x = \ln 8</math></p> <p><math>\ln x^{1/3} = \ln 8</math></p> <p><math>x^{1/3} = 8</math></p> <p>* If <math>\ln a = \ln b</math>, then <math>a = b</math></p> <p><math>(x^{1/3})^3 = (8)^3</math></p> <p><math>x = 512</math></p>
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<p>e) <math>\log_b 8 = \log_b x + \log_b(x-2)</math></p> <p><math>\log_8 8 = \log_8 x(x-2)</math></p> <p><math>8 = x(x-2)</math></p> <p><math>8 = x^2 - 2x</math></p> <p><math>x^2 - 2x - 8 = 0</math></p> <p><math>(x-4)(x+2) = 0</math></p> <p><math>x = 4, x = -2</math> <i>Extraneous solution</i></p>	<p>f) <math>\frac{4 \ln(x+3)}{4} = \frac{12}{4}</math></p> <p><math>\ln(x+3) = 3</math></p> <p><math>\log_e(x+3) = 3</math></p> <p><math>e^3 = x+3</math></p> <p><math>e^3 - 3 = x</math></p> <p><math>x = e^3 - 3</math></p>
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<p>g) <math>e^{3x} = 6</math></p> <p><math>\ln e^{3x} = \ln 6</math></p> <p><del><math>3x \ln e = \ln 6</math></del></p> <p><math>3x = \ln 6</math></p> <p><math>x = \frac{\ln 6}{3}</math></p>	<p>h) <math>7^{(x-4)} = 100</math></p> <p><math>\ln 7^{(x-4)} = \ln 100</math></p> <p><math>(x-4) \ln 7 = \ln 100</math></p> <p><math>x-4 = \frac{\ln 100}{\ln 7}</math></p> <p><math>x = \frac{\ln 100}{\ln 7} + 4</math></p> <p><math>x \approx 6.367</math></p>
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