P.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Evaluating a Function In Exercises 1–10, evaluate the function at the given value(s) of the independent variable. Simplify the results.

1. $f(x) = 7x - 4$

(a) $f(0)$ (b) $f(-3)$

(c) $f(b)$ (d) $f(x - 1)$

2. $f(x) = \sqrt{x + 5}$

(a) $f(-4)$ (b) $f(11)$

(c) $f(4)$ (d) $f(x + \Delta x)$

3. $g(x) = 5 - x^2$

(a) $g(0)$ (b) $g(\sqrt{5})$

(c) $g(-2)$ (d) $g(t - 1)$

4. $g(x) = x^2(x - 4)$

(a) $g(4)$ (b) $g(\frac{3}{2})$

(c) $g(c)$ (d) $g(t + 4)$

5. $f(x) = \cos 2x$

(a) $f(0)$ (b) $f\left(-\frac{\pi}{4}\right)$

(c) $f\left(\frac{\pi}{3}\right)$ (d) $f(\pi)$

6. $f(x) = \sin x$

(a) $f(\pi)$ (b) $f\left(\frac{5\pi}{4}\right)$

(c) $f\left(\frac{2\pi}{3}\right)$ (d) $f\left(-\frac{\pi}{6}\right)$

7. $f(x) = x^3$

$\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

8. $f(x) = 3x - 1$

$\dfrac{f(x) - f(1)}{x - 1}$

9. $f(x) = \dfrac{1}{\sqrt{x - 1}}$

$\dfrac{f(x) - f(2)}{x - 2}$

10. $f(x) = x^3 - x$

$\dfrac{f(x) - f(1)}{x - 1}$

Finding the Domain and Range of a Function In Exercises 11–22, find the domain and range of the function.

11. $f(x) = 4x^2$

12. $g(x) = x^2 - 5$

13. $f(x) = x^3$

14. $h(x) = 4 - x^2$

15. $g(x) = \sqrt{6x}$

16. $h(x) = -\sqrt{x + 3}$

17. $f(x) = \sqrt{16 - x^2}$

18. $f(x) = |x - 3|$

19. $f(t) = \sec \dfrac{\pi t}{4}$

20. $h(t) = \cot t$

21. $f(x) = \dfrac{3}{x}$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Finding the Domain of a Function In Exercises 23–28, find the domain of the function.

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

25. $g(x) = \dfrac{2}{1 - \cos x}$

26. $h(x) = \dfrac{1}{\sin x - (1/2)}$

27. $f(x) = \dfrac{1}{|x + 3|}$

28. $g(x) = \dfrac{1}{|x^2 - 4|}$

Finding the Domain and Range of a Piecewise Function In Exercises 29–32, evaluate the function as indicated. Determine its domain and range.

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

30. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

31. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

32. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

Sketching a Graph of a Function In Exercises 33–40, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph.

33. $f(x) = 4 - x$

34. $g(x) = \dfrac{4}{x}$

35. $h(x) = \sqrt{x - 6}$

36. $f(x) = \frac{1}{4}x^3 + 3$

37. $f(x) = \sqrt{9 - x^2}$

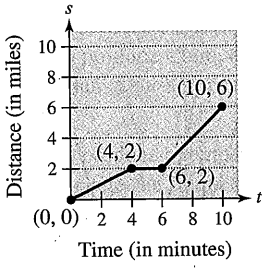
38. $f(x) = x + \sqrt{4 - x^2}$

39. $g(t) = 3 \sin \pi t$

40. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

WRITING ABOUT CONCEPTS

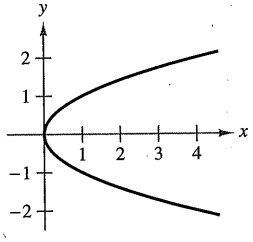
41. Describing a Graph The graph of the distance that a student drives in a 10-minute trip to school is shown in the figure. Give a verbal description of the characteristics of the student's drive to school.



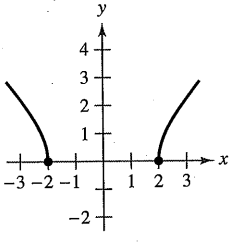
42. Sketching a Graph A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

Using the Vertical Line Test In Exercises 43–46, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to *MathGraphs.com*.

43. $x - y^2 = 0$

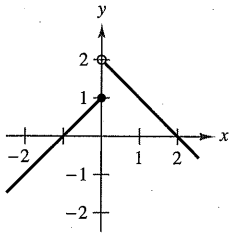


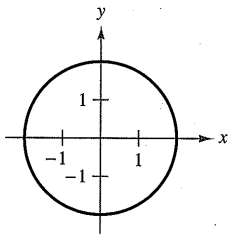
44. $\sqrt{x^2 - 4} - y = 0$



45. $y = \begin{cases} x + 1, & x \le 0 \\ -x + 2, & x > 0 \end{cases}$

46. $x^2 + y^2 = 4$





Deciding Whether an Equation Is a Function **In Exercises 47–50, determine whether y is a function of x.**

47. $x^2 + y^2 = 16$

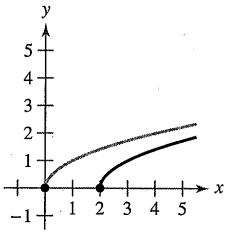
48. $x^2 + y = 16$

49. $y^2 = x^2 - 1$

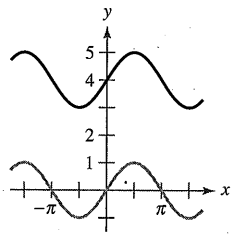
50. $x^2y - x^2 + 4y = 0$

Transformation of a Function **In Exercises 51–54, the graph shows one of the eight basic functions on page 22 and a transformation of the function. Describe the transformation. Then use your description to write an equation for the transformation.**

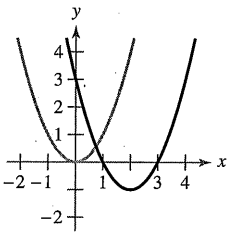
51.



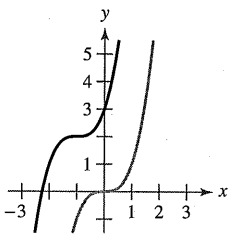
52.



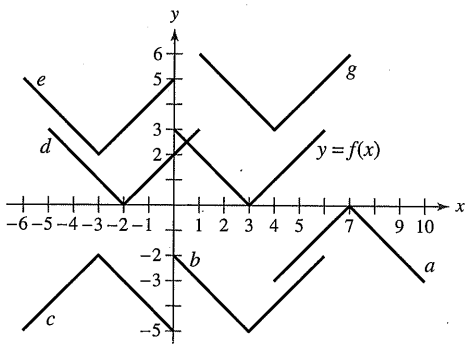
53.



54.



Matching **In Exercises 55–60, use the graph of $y = f(x)$ to match the function with its graph.**



55. $y = f(x + 5)$

56. $y = f(x) - 5$

57. $y = -f(-x) - 2$

58. $y = -f(x - 4)$

59. $y = f(x + 6) + 2$

60. $y = f(x - 1) + 3$

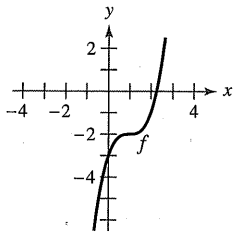
61. Sketching Transformations Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to *MathGraphs.com*.

(a) $f(x + 3)$ (b) $f(x - 1)$

(c) $f(x) + 2$ (d) $f(x) - 4$

(e) $3f(x)$ (f) $\frac{1}{4}f(x)$

(g) $-f(x)$ (h) $-f(-x)$



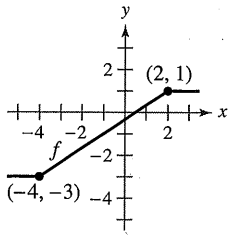
62. Sketching Transformations Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to *MathGraphs.com*.

(a) $f(x - 4)$ (b) $f(x + 2)$

(c) $f(x) + 4$ (d) $f(x) - 1$

(e) $2f(x)$ (f) $\frac{1}{2}f(x)$

(g) $f(-x)$ (h) $-f(x)$



Combinations of Functions **In Exercises 63 and 64, find (a) $f(x) + g(x)$, (b) $f(x) - g(x)$, (c) $f(x) \cdot g(x)$, and (d) $f(x)/g(x)$.**

63. $f(x) = 3x - 4$, $g(x) = 4$

64. $f(x) = x^2 + 5x + 4$, $g(x) = x + 1$

65. Evaluating Composite Functions Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$

(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

66. Evaluating Composite Functions Given $f(x) = \sin x$ and $g(x) = \pi x$, evaluate each expression.

(a) $f(g(2))$ (b) $f\left(g\left(\frac{1}{2}\right)\right)$ (c) $g(f(0))$

(d) $g\left(f\left(\frac{\pi}{4}\right)\right)$ (e) $f(g(x))$ (f) $g(f(x))$

Finding Composite Functions **In Exercises 67–70, find the composite functions $f \circ g$ and $g \circ f$. Find the domain of each composite function. Are the two composite functions equal?**

67. $f(x) = x^2$, $g(x) = \sqrt{x}$

68. $f(x) = x^2 - 1$, $g(x) = \cos x$

69. $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

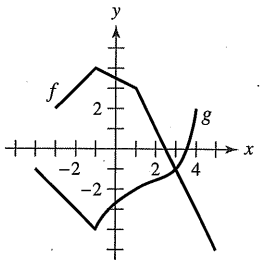
70. $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x + 2}$

71. Evaluating Composite Functions Use the graphs of f and g to evaluate each expression. If the result is undefined, explain why.

(a) $(f \circ g)(3)$ (b) $g(f(2))$

(c) $g(f(5))$ (d) $(f \circ g)(-3)$

(e) $(g \circ f)(-1)$ (f) $f(g(-1))$



72. Ripples A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in feet) of the outer ripple is given by $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.

Think About It **In Exercises 73 and 74, $F(x) = f \circ g \circ h$. Identify functions for f, g, and h. (There are many correct answers.)**

73. $F(x) = \sqrt{2x - 2}$ **74.** $F(x) = -4\sin(1 - x)$

Think About It **In Exercises 75 and 76, find the coordinates of a second point on the graph of a function f when the given point is on the graph and the function is (a) even and (b) odd.**

75. $\left(-\frac{3}{2}, 4\right)$ **76.** $(4, 9)$

77. Even and Odd Functions The graphs of f, g, and h are shown in the figure. Decide whether each function is even, odd, or neither.

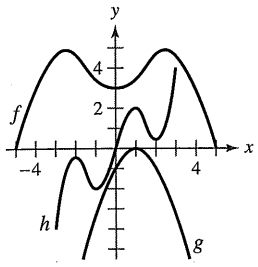


Figure for 77

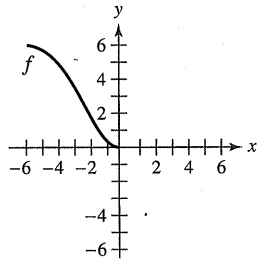


Figure for 78

78. Even and Odd Functions The domain of the function f shown in the figure is $-6 \le x \le 6$.

(a) Complete the graph of f given that f is even.

(b) Complete the graph of f given that f is odd.

Even and Odd Functions and Zeros of Functions **In Exercises 79–82, determine whether the function is even, odd, or neither. Then find the zeros of the function. Use a graphing utility to verify your result.**

79. $f(x) = x^2(4 - x^2)$ **80.** $f(x) = \sqrt[3]{x}$

81. $f(x) = x\cos x$ **82.** $f(x) = \sin^2 x$

Writing Functions **In Exercises 83–86, write an equation for a function that has the given graph.**

83. Line segment connecting $(-2, 4)$ and $(0, -6)$

84. Line segment connecting $(3, 1)$ and $(5, 8)$

85. The bottom half of the parabola $x + y^2 = 0$

86. The bottom half of the circle $x^2 + y^2 = 36$

Sketching a Graph **In Exercises 87–90, sketch a possible graph of the situation.**

87. The speed of an airplane as a function of time during a 5-hour flight

88. The height of a baseball as a function of horizontal distance during a home run

89. The amount of a certain brand of sneaker sold by a sporting goods store as a function of the price of the sneaker

90. The value of a new car as a function of time over a period of 8 years

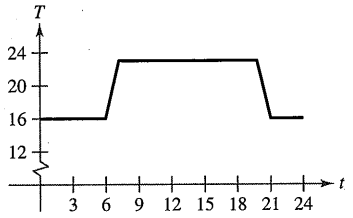
91. Domain Find the value of c such that the domain of $f(x) = \sqrt{c - x^2}$ is $[-5, 5]$.

92. Domain Find all values of c such that the domain of

$$f(x) = \frac{x + 3}{x^2 + 3cx + 6}$$

is the set of all real numbers.

93. Graphical Reasoning An electronically controlled thermostat is programmed to lower the temperature during the night automatically (see figure). The temperature T in degrees Celsius is given in terms of t, the time in hours on a 24-hour clock.

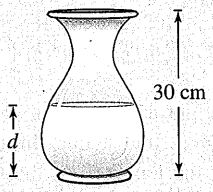


(a) Approximate $T(4)$ and $T(15)$.

(b) The thermostat is reprogrammed to produce a temperature $H(t) = T(t - 1)$. How does this change the temperature? Explain.

(c) The thermostat is reprogrammed to produce a temperature $H(t) = T(t) - 1$. How does this change the temperature? Explain.

94. HOW DO YOU SEE IT? Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown to answer the questions when d is the depth of the water in centimeters and t is the time in seconds (see figure).



(a) Explain why d is a function of t.

(b) Determine the domain and range of the function.

(c) Sketch a possible graph of the function.

(d) Use the graph in part (c) to approximate $d(4)$. What does this represent?

95. Modeling Data The table shows the average numbers of acres per farm in the United States for selected years. *(Source: U.S. Department of Agriculture)*

Year	1960	1970	1980	1990	2000	2010
Acreage	297	374	429	460	436	418

(a) Plot the data, where A is the acreage and t is the time in years, with $t = 0$ corresponding to 1960. Sketch a freehand curve that approximates the data.

(b) Use the curve in part (a) to approximate $A(25)$.

96. Automobile Aerodynamics

The horsepower H required to overcome wind drag on a certain automobile is approximated by

$$H(x) = 0.002x^2 + 0.005x - 0.029, \quad 10 \le x \le 100$$

where x is the speed of the car in miles per hour.

(a) Use a graphing utility to graph H.

(b) Rewrite the power function so that x represents the speed in kilometers per hour. [Find $H(x/1.6)$.]

97. Think About It Write the function $f(x) = |x| + |x - 2|$ without using absolute value signs. (For a review of absolute value, see Appendix C.)

98. Writing Use a graphing utility to graph the polynomial functions $p_1(x) = x^3 - x + 1$ and $p_2(x) = x^3 - x$. How many zeros does each function have? Is there a cubic polynomial that has no zeros? Explain.

99. Proof Prove that the function is odd.

$$f(x) = a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x$$

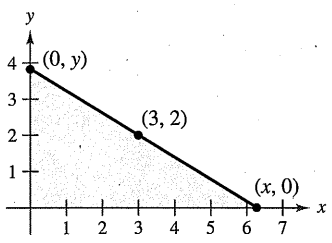
100. Proof Prove that the function is even.

$$f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$

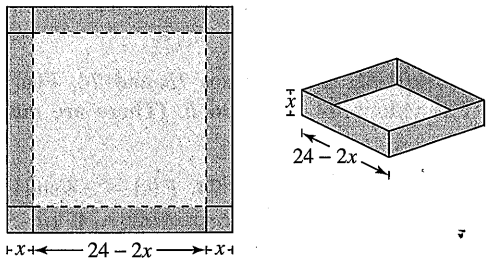
101. Proof Prove that the product of two even (or two odd) functions is even.

102. Proof Prove that the product of an odd function and an even function is odd.

103. Length A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(3, 2)$ (see figure). Write the length L of the hypotenuse as a function of x.



104. Volume An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure).



(a) Write the volume V as a function of x, the length of the corner squares. What is the domain of the function?

(b) Use a graphing utility to graph the volume function and approximate the dimensions of the box that yield a maximum volume.

(c) Use the *table* feature of a graphing utility to verify your answer in part (b). (The first two rows of the table are shown.)

Height, x	Length and Width	Volume, V
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$

True or False? In Exercises 105–110, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

105. If $f(a) = f(b)$, then $a = b$.

106. A vertical line can intersect the graph of a function at most once.

107. If $f(x) = f(-x)$ for all x in the domain of f, then the graph of f is symmetric with respect to the y-axis.

108. If f is a function, then

$$f(ax) = af(x).$$

109. The graph of a function of x cannot have symmetry with respect to the x-axis.

110. If the domain of a function consists of a single number, then its range must also consist of only one number.

PUTNAM EXAM CHALLENGE

111. Let R be the region consisting of the points (x, y) of the Cartesian plane satisfying both $|x| - |y| \le 1$ and $|y| \le 1$. Sketch the region R and find its area.

112. Consider a polynomial $f(x)$ with real coefficients having the property $f(g(x)) = g(f(x))$ for every polynomial $g(x)$ with real coefficients. Determine and prove the nature of $f(x)$.

These problems were composed by the Committee on the Putnam Prize Competition.