Key

AB Calculus – Ch. P Miscellaneous Practice problems (Day 4)

1)

For the piecewise function $p(x) = \begin{cases} \sqrt{x+4}, x \le 5 \\ (x-5)^2, x > 5 \end{cases}$ find each of the following:

a) $p(-3)$

$p(-3) = \sqrt{-3+4} = \sqrt{1}$

$= \boxed{1}$

b) $p(0)$

$p(0) = \sqrt{0+4}$

$= \sqrt{4}$

$= \boxed{2}$

c) $p(5)$

$p(5) = \sqrt{5+4}$

$= \sqrt{9}$

$= \boxed{3}$

d) $p(10)$

$p(10) = (10-5)^2$

$= \boxed{25}$

2) Simplify the following given that all angles are between 0 and π:

a) $\sin\left(2 \underbrace{\arccos\frac{\sqrt{2}}{2}}_{\theta}\right)$

$\theta = \arccos\left(\frac{\sqrt{2}}{2}\right)$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$\sin\left(2 \cdot \frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{2}\right) = \boxed{1}$

b) $\cos\left(\arccos 0 + \arcsin\frac{1}{2}\right)$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6$

$\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \to \cos\left(\frac{3\pi}{6} + \frac{\pi}{6}\right) \to \cos\left(\frac{4\pi}{6}\right)$

$\to \cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

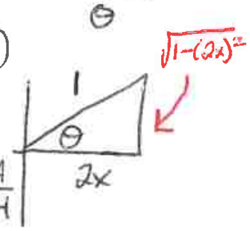
3) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) $\sin(\underbrace{\arccos 2x}_{\theta})$

$\theta = \arccos(2x)$

$\cos\theta = 2x$

$\cos\theta = \frac{2x}{1} \to \frac{A}{H}$



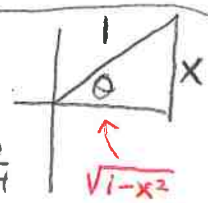
$\sin(\arccos(2x)) \to \frac{O}{H} \to \frac{\sqrt{1-4x^2}}{1} \to \boxed{\sqrt{1-4x^2}}$

b) $\cot(\underbrace{\arcsin x}_{\theta})$

$\theta = \arcsin x$

$\sin\theta = x$

$\sin\theta = \frac{x}{1} \to \frac{O}{H}$

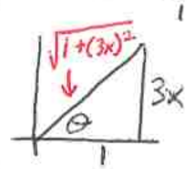


$\cot(\arcsin x) \to \frac{A}{O} \to \boxed{\frac{\sqrt{1-x^2}}{x}}$

c) $\sin(\underbrace{\arctan 3x}_{\theta})$

$\theta = \arctan(3x)$

$\tan\theta = \frac{3x}{1} \to \frac{O}{A}$



$\sin(\arctan 3x) \to \frac{O}{H}$

$\to \boxed{\frac{3x}{\sqrt{1+9x^2}}}$

4) Solve the following (use identities where necessary) given that $0 \le x < 2\pi$

a) $\cos^2 x - \cos x + 1 = \sin^2 x$

$\sin^2 x + \cos^2 x = 1$

$\downarrow$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x - \cos x + 1 = 1 - \cos^2 x$

$2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$ | $2\cos x - 1 = 0$

$x = \frac{\pi}{2}$ | $\cos x = 1/2$

$x = 3\pi/2$ | $x = \pi/3, 5\pi/3$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}}$

b) $\sin x \tan x = \sin x$

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$ | $\tan x - 1 = 0$

$x = 0,$ | $\tan x = 1$

$x = \pi,$ | $x = \pi/4, 5\pi/4$

$\cancel{x = 2\pi}$

$\boxed{x = 0, \pi, \pi/4, 5\pi/4}$

c) $\sin x = \cos 2x - 1$

*double angle identity

$\cos 2x = 1 - 2\sin^2 x$

$\sin x = 1 - 2\sin^2 x - 1$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ | $2\sin x + 1 = 0$

$x = 0$ | $\sin x = -1/2$

$x = \pi$ | $x = 7\pi/6, 11\pi/6$

$\boxed{x = 0, \pi, 7\pi/6, \frac{11\pi}{6}}$

d) $\sin 4x = \frac{1}{2}$ Solve for $0 \le x < \pi$

$4x = \sin^{-1}\left(\frac{1}{2}\right)$

$\frac{1}{4}\left[4x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}\right]$

$x = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \cancel{\frac{25\pi}{24}}, \cancel{\frac{29\pi}{24}}$

e) $\cot^2 x - \csc x = 1$

$* \ 1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$(\csc^2 x - 1) - \csc x = 1$

$\csc^2 x - 1 - \csc x - 1 = 0$

$\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$ | $\csc x = -1$

$\sin x = 1/2$ | $\sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

f) $\sec^2 x + 2\sec x = 0$

$\sec x(\sec x + 2) = 0$

$\sec x = 0$ (undefined) | $\sec x + 2 = 0$

$\sec x = -2$

$\cos x = -1/2$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

g) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$ ← Q1, Q3

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

h) $\sin 2x = \cos x$

$* \ \sin 2x = 2\sin x\cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ | $2\sin x - 1 = 0$

$x = \pi/2, \frac{3\pi}{2}$ | $\sin x = 1/2$

$x = \pi/6, 5\pi/6$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

i) $\cot x\cos^2 x = 2\cot x$

$\cot x\cos^2 x - 2\cot x = 0$

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$ | $\cos^2 x - 2 = 0$

$\frac{\cos x}{\sin x} = 0$ | $\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$ (undefined)

$\cos x = 0$

$x = \pi/2, 3\pi/2$

$\boxed{x = \pi/2, 3\pi/2}$

5) Write the equation of each line in point-slope form:

a. given a point and the slope: $(1, -2)$ $m = 3$

$y - y_1 = m(x - x_1)$

$y - (-2) = 3(x - 1)$

$\boxed{y + 2 = 3(x - 1)}$

b. given two points: $(1, -3)$ $(-7, 1)$

Slope: $m = \frac{1 - (-3)}{-7 - 1} \rightarrow \frac{4}{-8} = \frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$\boxed{y + 3 = \frac{-1}{2}(x - 1) \quad \text{or} \quad y - 1 = \frac{-1}{2}(x + 7)}$

c. given the point $(-1, -2)$ and is perpendicular to the line $y - 2 = 3(x + 1)$.

slope $m_1 = 3$

$m_2 = \frac{-1}{3}$ (perpendicular)

$y - y_1 = m(x - x_1)$

$\boxed{y + 2 = \frac{-1}{3}(x + 1)}$

d. given the point $(-1, -2)$ and is parallel to the line $3x + 2y = 1$.

$2y = -3x + 1$

$y = \frac{-3}{2}x + \frac{1}{2}$

slope: $m = \frac{-3}{2}$

$y - y_1 = m(x - x_1)$

$\boxed{y + 2 = \frac{-3}{2}(x + 1)}$

6)

(point-slope form)

Find the equation of the line in ~~slope-intercept~~ form, passing through the following points.

a. $(-3,6)$ and $(-1,2)$

$m = \frac{2-6}{-1-(-3)} \rightarrow \frac{-4}{2} = -2$

$\boxed{y-2=-2(x+1) \quad \text{or} \quad y-6=-2(x+3)}$

b. $(-7,1)$ and $(3,-4)$

$m = \frac{-4-1}{3-(-7)} \rightarrow \frac{-5}{10} \rightarrow \frac{-1}{2}$

$\boxed{y-1=\frac{-1}{2}(x+7) \quad \text{or} \quad y+4=\frac{-1}{2}(x-3)}$

c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

$m = \frac{1-\frac{2}{3}}{\frac{1}{2}-(-2)} \rightarrow \frac{\frac{1}{3}}{\frac{5}{2}} \rightarrow \frac{1}{3}\cdot\frac{2}{5} \rightarrow \frac{2}{15}$

$\boxed{y-\frac{2}{3}=\frac{2}{15}(x+2) \quad \text{or} \quad y-1=\frac{2}{15}\left(x-\frac{1}{2}\right)}$

7)

Write equations of the line through the given point a) parallel and b) normal to the given line.

a. $(5,-3)$, $x+y=4$

$y=-1x+4$

a) $m_1=-1$

$\boxed{y+3=-1(x-5)}$

b) $m_2=1$

$\boxed{y+3=1(x-5)}$

b. $(-6,2)$, $5x+2y=7$

$2y=-5x+7$

$y=\frac{-5}{2}x+\frac{7}{2}$

a) $m_1=\frac{-5}{2}$

$\boxed{y-2=\frac{-5}{2}(x+6)}$

b) $m_2=\frac{2}{5}$

$\boxed{y-2=\frac{2}{5}(x+6)}$

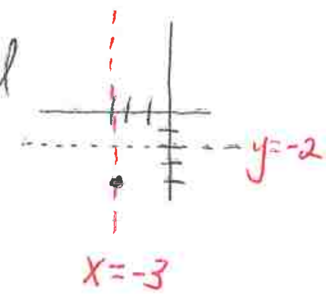
c. $(-3,-4)$, $y=-2$

slope: $m_1=0$

a) $y+4=0(x+3) \rightarrow \boxed{y=-4}$

b) slope $m_2 \rightarrow$ undefined

$\boxed{x=-3}$



8)

Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

x_1 y_1

slope: $m = \frac{3-4}{2-(-1)} \rightarrow \frac{-1}{3}$

$\boxed{y+2=\frac{-1}{3}(x-4)}$

9)

Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

$\frac{-5y}{-5}=\frac{-3x+9}{-5}$

$y=\boxed{\frac{3}{5}}x-\frac{9}{5}$

$ky=-2x+11$

$y=\boxed{\frac{-2}{k}}x+\frac{11}{k}$

a) If parallel, set slopes equal:

$\frac{3}{5}=\frac{-2}{k}$

$3k=-10$

$\boxed{k=\frac{-10}{3}}$

b) If perpendicular, set one as opp reciprocal.

$\frac{3}{5}=\frac{+k}{2}$

$6=5k$

$5k=6$

$\boxed{k=\frac{6}{5}}$

10)

Condense and write as a single logarithm.

a. $\log_4 3+5\log_4 x$

$\log_4 3+\log_4 x^5$

$\boxed{\log_4(3x^5)}$

b. $\log 3-5\log x$

$\log 3-\log x^5$

$\boxed{\log\left(\frac{3}{x^5}\right)}$

c. $\ln 2+4\ln x-3\ln y-\ln 8$

$\ln 2+\ln x^4-\ln y^3-\ln 8$

$\ln\left(\frac{2x^4}{8y^3}\right) \rightarrow \boxed{\ln\left(\frac{x^4}{4y^3}\right)}$

d. $2\ln 4-\frac{1}{2}\ln x+\ln y-3\ln 2$

$\ln 4^2-\ln x^{1/2}+\ln y-\ln 2^3$

$\ln\left(\frac{16y}{8\sqrt{x}}\right) \rightarrow \boxed{\ln\left(\frac{2y}{\sqrt{x}}\right)}$

11)

Expand each logarithmic expression.

a. $\log_7\frac{5x}{y^4}$

$\log_7 5+\log x-\log_7 y^4$

$\boxed{\log_7 5+\log x-4\log_7 y}$

b. $\log\frac{x^3}{9y^2}$

$\log x^3-\log 9-\log y^2$

$\boxed{3\log x-\log 9-2\log y}$

c. $\ln 27\sqrt[4]{a}$

$\ln 27+\ln a^{1/4}$

$\ln 3^3+\ln a^{1/4}$

$\boxed{3\ln 3+\frac{1}{4}\ln a}$

d. $\ln\frac{(3x^2+1)}{(2x^3-3x^2)}$

$\ln(3x^2+1)-\ln(2x^3-3x^2)$

$\ln(3x^2+1)-\ln[x^2(2x-3)]$

$\boxed{\ln(3x^2+1)-2\ln x-\ln(2x-3)}$