2017

P.1 Homework p. 8-9 # 19-25 odd 57-61 odd, 62

Find any intercepts:
* x-ints: set $y=0$, solve for x in numerator
* y-ints: set $x=0$, solve for y.

19) $y = x^2+x-2$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

$x = 1, -2$

$\boxed{\text{x-ints: } (1,0), (-2,0)}$

$y = 0^2+0-2$

$y = -2$

$\boxed{\text{y-int: } (0,-2)}$

21) $y = x\sqrt{16-x^2}$

$0 = x\sqrt{16-x^2}$

$0 = x\sqrt{(4-x)(4+x)}$

$x = 0, 4, -4$

$\boxed{\text{x-ints: } (0,0), (4,0)(-4,0)}$

y-int: $y = 0\sqrt{16-0^2} = 0$ $\boxed{\text{y-int: } (0,0)}$

23) $y = \dfrac{2-\sqrt{x}}{5x+1}$

x-int: $0 = 2-\sqrt{x}$

$\sqrt{x} = 2$

$x = 4$

$\boxed{\text{x-int: } (4,0)}$

$y = \dfrac{2-\sqrt{0}}{5(0)+1} = \dfrac{2}{1} = 2$

$\boxed{\text{y-int: } (0,2)}$

25) $x^2y - x^2 + 4y = 0$

x-ints: (set $y=0$)

$x^2(0) - x^2 + 4(0) = 0$

$-x^2 = 0$

$x = 0$

$\boxed{\text{x-int: } (0,0)}$

y-int: (set $x=0$)

$0^2(y) - (0)^2 + (4y) = 0$

$4y = 0 \quad y = 0$

$\boxed{\text{y-int: } (0,0)}$

Find pts of intersection: * solve system of equation using substitution method

57) $x+y=8$

$4x-y=7$

$x = \underbrace{8-y}$

$4x - y = 7$

$4(8-y) - y = 7$

$32 - 4y - y = 7$

$32 - 5y = 7$

$-5y = -25$

$y = 5$

$x = 8 - y$

$x = 8 - 5 = 3$

$\boxed{\text{Point of Intersection: } (3,5)}$

59) $x^2 + y = 6$

$x + y = 4$

$x = 4 - y$

$x^2 + y = 6$

$(4-y)^2 + y = 6$

$16 - 8y + y^2 + y = 6$

$y^2 - 7y + 10 = 0$

$(y-5)(y-2) = 0$

$y = 2, 5$

$x = 4 - y$

$x = 4 - 2 = 2$

$x = 4 - 5 = -1$

$\boxed{\text{Points of Intersection: } (2,2) \text{ and } (-1,5)}$

61) $x^2 + y^2 = 5$

$x - y = 1$

$x = \underbrace{1+y}$

$x^2 + y^2 = 5$

$((1+y))^2 + y^2 = 5$

$(1+y)(1+y) + y^2 = 5$

$1 + 2y + y^2 + y^2 = 5$

$2y^2 + 2y + 1 = 5$

$2y^2 + 2y - 4 = 0$

$2(y^2 + y - 2) = 0$

$2(y+2)(y-1) = 0$

$y = +1, -2$

$x = 1 + y$

$x = 1 + 1 = 2$

$x = 1 + -2 = -1$

$\boxed{\text{Points of Intersection: } (-1,-2) \text{ and } (2,1)}$

2017

P.1

62) $x^2+y^2=25$

$-3x+y=15$

$y=3x+15$

$x^2+y^2=25$

$x^2+(3x+15)^2=25$

$x^2+(3x+15)(3x+15)=25$

$x^2+9x^2+45x+45x+225=25$

$10x^2+90x+200=0$

$10(x^2+9x+20)=0$

$10(x+4)(x+5)=0$

$x=-4,-5$

$y=3x+15$

$y=3(-4)+15=3$

$y=3(-5)+15=0$

$\boxed{\text{Points of Intersection: } (-4,3),(-5,0)}$

P.2 HW p.16-17 # 27, 37, 39 (40), 41, 55, 59

Find slope, y-int:

27) $x+5y=20$

$5y=-1x+20$

$y=-\frac{1}{5}x+\frac{20}{5}$

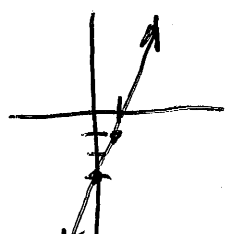
$y=-\frac{1}{5}x+4$

slope: $m=-1/5$ y-int: 4

37) $2x-y-3=0$

$-y=-2x+3$

$y=2x-3$



40) Find equation of line:

$(-2,-2),(1,7)$

$m=\frac{7-(-2)}{1-(-2)}=\frac{9}{3}=3$

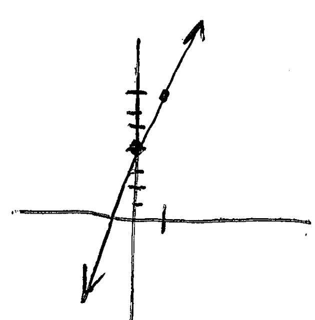
point $(1,7)$ slope: $m=3$

$y-y_1=m(x-x_1)$

$y-7=3(x-1)$

$y=3x-3+7$

$y=3x+4$

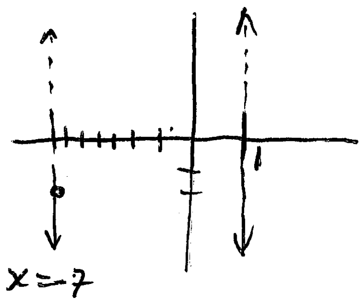


41) $(2,8),(5,0)$

$m = \frac{0-8}{5-2} = \frac{-8}{+3} = \frac{-8}{3}$

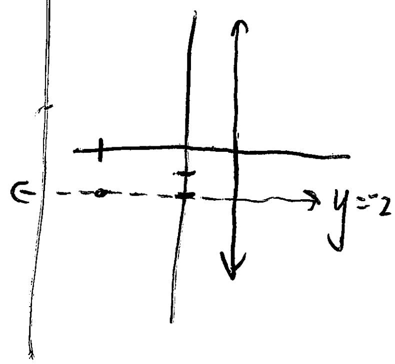
$y - 0 = \frac{-8}{3}(x-5)$

55) through $(-7,-2)$ parallel to $x=1$



$\boxed{x=-7}$

through $(-7,-2)$, $\perp$ to $x=1$



$\boxed{y=-2}$

59) $(2,1)$ parallel to $4x-2y=3$

$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{3}{-2}$

$y = 2x - \frac{3}{2}$

$m=2$ point $(2,1)$

$\boxed{y-1 = 2(x-2)}$

perpendicular to line

$m = \frac{-1}{2}$ point $(2,1)$

$\boxed{y-1 = \frac{-1}{2}(x-2)}$

2017

$\boxed{P.3}$ p.27–29 # 3, 7, 9, 15, 17, 23, 29, 61, 67, 69, 71, 79, 81, 82

3) $g(x) = 5 - x^2$

a) $g(0) = 5 - 0^2 = \boxed{5}$

b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = \boxed{0}$

c) $g(-2) = 5 - (-2)^2 = 5 - 4 = \boxed{1}$

d) $g(t-1) = 5 - (t-1)^2 = 5 - (t^2 - 2t + 1)$

$= 5 - t^2 + 2t - 1$

$= \boxed{4 - t^2 + 2t}$

7) $f(x) = x^3$ $\qquad f(x+\Delta x) = (x+\Delta x)^3$

pascal triangle

1
1 1
1 2 1
1 3 3 1

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{\cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - \cancel{x^3}}{\Delta x}$$

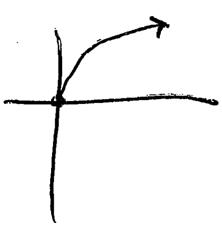
$$= \frac{\cancel{\Delta x}(3x^2 + 3x\Delta x + \Delta x^2)}{\cancel{\Delta x}} = \boxed{3x^2 + 3x\Delta x + \Delta x^2}$$

9) $f(x) = \dfrac{1}{\sqrt{x-1}}$ $\qquad f(2) = \dfrac{1}{\sqrt{2-1}} = \dfrac{1}{\sqrt{1}} = 1$

$$\frac{f(x) - f(2)}{x-2} = \frac{\frac{1}{\sqrt{x-1}} - 1}{x-2} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} = \frac{1 - \sqrt{x-1}}{(\sqrt{x-1})(x-2)} \cdot \frac{(1+\sqrt{x-1})}{(1+\sqrt{x-1})} =$$

$$= \frac{1-(x-1)}{(\sqrt{x-1})(x-2)(1+\sqrt{x-1})} = 1 - x + 1 = \frac{\overset{(-1)}{\cancel{2-x}}}{(\sqrt{x-1})\cancel{(x-2)}(1+\sqrt{x-1})} = \boxed{\frac{-1}{(\sqrt{x-1})(1+\sqrt{x-1})}}$$

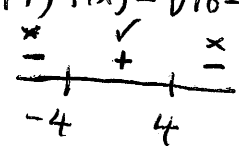
15) $g(x) = \sqrt{6x}$



Domain: $[0, \infty)$

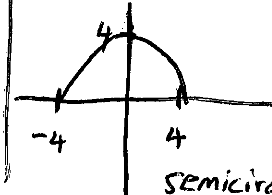
Range: $[0, \infty)$

17) $f(x) = \sqrt{16 - x^2} = \sqrt{(4-x)(4+x)}$



Domain: $[-4, 4]$

Range: $[0, 4]$



semicircle

23) $f(x) = \sqrt{x} + \sqrt{1-x}$ ← Domain: $(-\infty, 1]$

↖ Domain: $[0, \infty)$

* Domain is the overlap of these separate domains

$\boxed{\text{Domain: } [0, 1]}$

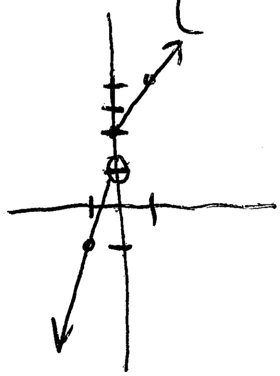
29) $f(x) = \begin{cases} 2x+1, & x < 0 \\ 2x+2, & x \geq 0 \end{cases}$

a) $f(-1) = \boxed{-1}$

b) $f(0) = \boxed{2}$

c) $f(2) = \boxed{6}$

d) $f(t^2+1) = 2(t^2+1)+2$
$= \boxed{2t^2+4}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

61) <u>Transformations:</u> $y = af(x+h) - K$

a) $f(x+3)$ → graph shifts 3 units left

c) $f(x)+2$ → graph shifts 2 units up

b) $f(x-1)$ → graph shifts 1 unit right

d) $f(x)-4$ → graph shifts 4 units down

e) $3f(x)$ → graph stretched vertically by factor of 3

f) $\frac{1}{4}f(x)$ → graph stretched (compressed) vertically by factor of 1/4

g) $-f(x)$ → graph is a reflection in x-axis

h) $-f(-x)$ → graph is reflection about origin.

Composite Functions: Find $f \circ g$ and $g \circ f$

$$f \circ g \downarrow f(g(x)) \qquad g \circ f \downarrow g(f(x))$$

67)

$f(x) = x^2 \quad g(x) = \sqrt{x}$

$f(g(x)) = (\sqrt{x})^2 = x$ Domain: $[0, \infty)$

$g(f(x)) = \sqrt{x^2} = |x|$ Domain: $(-\infty, \infty)$

the 2 composite functions are not equal, they have different domains.

69) $f(x) = \frac{3}{x} \quad g(x) = x^2 - 1$

$f(g(x)) = \frac{3}{x^2-1} = \frac{3}{(x-1)(x+1)} \quad x \neq 1, -1$ $\boxed{\text{Domain: } (-\infty, -1) \cup (-1, 1) \cup (1, \infty)}$

$g(f(x)) = g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2} \quad x \neq 0$

composite functions are not equal

$\boxed{\text{Domain: } (-\infty, 0) \cup (0, \infty)}$

71) a) $(f \circ g)(3) = f[g(3)] = f[-1] = \boxed{4}$

b) $g(f(2)) = g[1] = \boxed{-2}$

c) $g(f(5)) = g[-5] =$ undefined

d) $(f \circ g)(-3) = f[g(-3)] = f[-2] = 3$

e) $(g \circ f)(-1) = g[f(-1)] = g[4] = 2$

f) $f[g(-1)] = f[-4] =$ undefined.

Determine if even, odd, neither

79) $f(x) = x^2[4-x^2]$

* even if $f(-x) = f(x)$
* odd if $f(-x) = -f(x)$

$f(-x) = (-x)^2[4-(-x)^2]$

$= x^2[4-x^2]$ Since $f(-x) = f(x)$, $f(x)$ is even.

$f(x) = x^2[4-x^2]$

$0 = x^2[2-x][2+x]$ $\boxed{\text{Zeros } x = 0, -2, 2}$

81) $f(x) = x\cos x$

* $\cos x$ is even function $\cos(-x) = \cos x$
* $\sin x$ is odd function $\sin(-x) = -\sin x$

$f(-x) = (-x)\cdot\cos(-x)$

$= -x\cdot\cos x$

$f(-x) = -x\cos x$

Since $f(-x) = -f(x)$, $\boxed{f(x) \text{ is an odd function.}}$

zeros: $0 = x\cos x$ $x = 0$, $\cos x = 0 \rightarrow x = \pi/2 \xrightarrow{+\pi} 3\pi/2 \xrightarrow{+\pi} \ldots$

$\boxed{\text{Zeros: } x = 0, \frac{\pi}{2} + n\pi}$

82) $f(x) = \sin^2 x = [\sin x]^2$

$f(-x) = [\sin(-x)]^2 = [-\sin x]^2 = [-\sin x]\cdot[-\sin x] = (\sin x)^2$

since $f(-x) = f(x)$, $\boxed{f \text{ is even}}$

zeros: $0 = [\sin x]^2$ $\sin x = 0$ $x = 0, \pi, 2\pi, \ldots$ $\boxed{x = n\pi}$