CHAPTER P
Preparation for Calculus

Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

   x-intercept: $(2, 0)$

   y-intercept: $(0, 3)$

   Matches graph (b).

2. $y = \sqrt{9 - x^2}$

   x-intercepts: $(-3, 0), (3, 0)$

   y-intercept: $(0, 3)$

   Matches graph (d).

3. $y = 3 - x^2$

   x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

   y-intercept: $(0, 3)$

   Matches graph (a).

4. $y = x^3 - x$

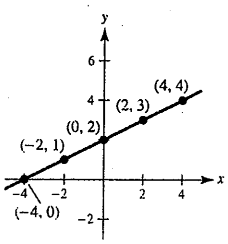
   x-intercepts: $(0, 0), (-1, 0), (1, 0)$

   y-intercept: $(0, 0)$

   Matches graph (c).

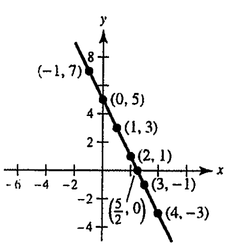
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



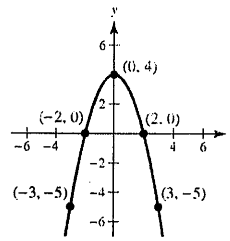
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



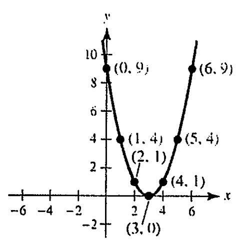
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



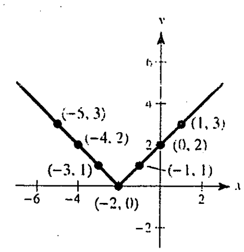
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



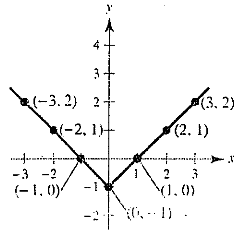
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



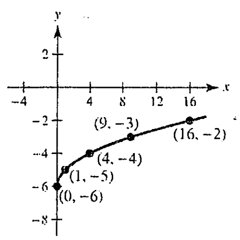
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



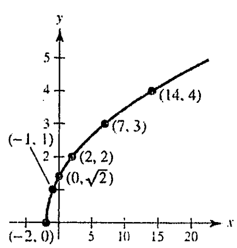
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



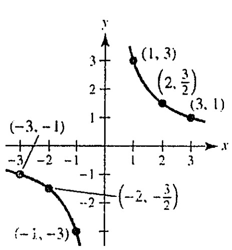
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



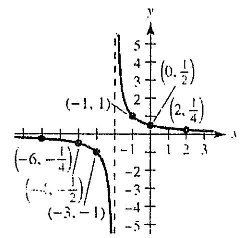
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

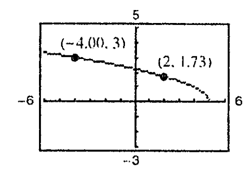


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



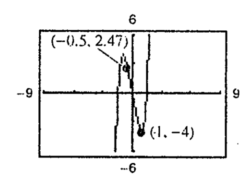
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept:
$$0 = 2x - 5$$
$$5 = 2x$$
$$x = \tfrac{5}{2}; \left(\tfrac{5}{2}, 0\right)$$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept:
$$0 = 4x^2 + 3$$
$$-3 = 4x^2$$

None. y cannot equal 0.

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2; (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1; (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0; (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2; (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0; (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4; (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1; (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2\ ;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4\ ;\ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3; (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0; (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0; (0; 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3}; \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x$.

30. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin because $(-x)(-y) = xy = 4$.

32. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10$.

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis because $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}$.

37. $y = |x^3 + x|$ is symmetric with respect to the y-axis because $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|$.

38. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

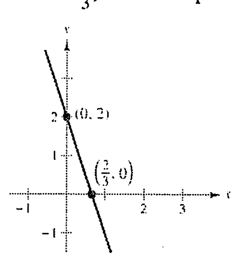
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2)$, $\left(\frac{2}{3}, 0\right)$

Symmetry: none



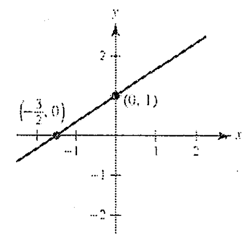
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1$, y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2}$, x-intercept

Intercepts: $(0, 1)$, $\left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

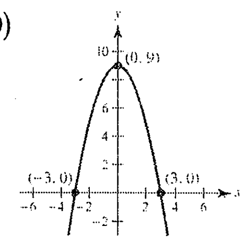
$y = 9 - (0)^2 = 9$, y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, x-intercepts

Intercepts: $(0, 9)$, $(3, 0)$, $(-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



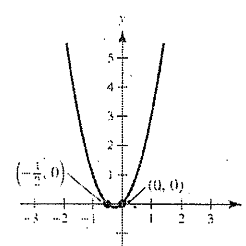
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0$, y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2}$, x-intercepts

Intercepts: $(0, 0)$, $\left(-\frac{1}{2}, 0\right)$

Symmetry: none



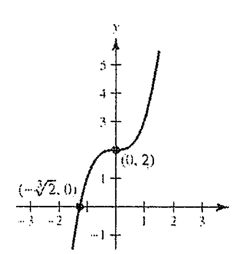
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right)$, $(0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

$y = 0^3 - 4(0) = 0$, y-intercept

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

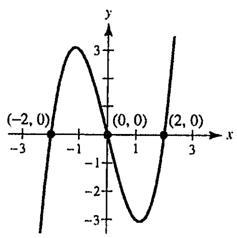
$x(x + 2)(x - 2) = 0$

$x = 0, \pm 2$, x-intercepts

Intercepts: $(0, 0)$, $(2, 0)$, $(-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



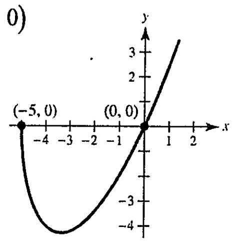
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0)$, $(-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$\sqrt{25 - x^2} = 0$

$25 - x^2 = 0$

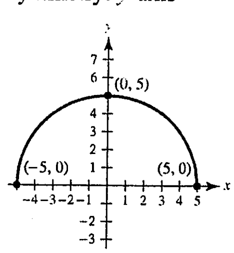
$(5 + x)(5 - x) = 0$

$x = \pm 5$, x-intercept

Intercepts: $(0, 5)$, $(5, 0)$, $(-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

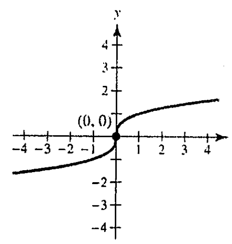
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



48. $x = y^2 - 4$

$y^2 - 4 = 0$

$(y + 2)(y - 2) = 0$

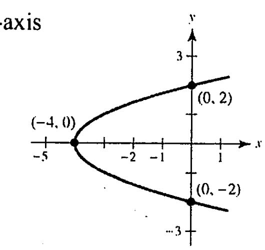
$y = \pm 2$, y-intercepts

$x = 0^2 - 4 = -4$, x-intercept

Intercepts: $(0, 2)$, $(0, -2)$, $(-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{8}{x}$

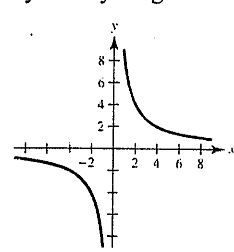
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



50. $y = \dfrac{10}{x^2 + 1}$

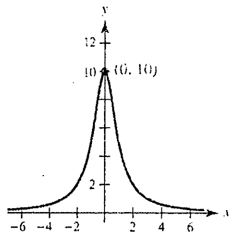
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: $(0, 10)$

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

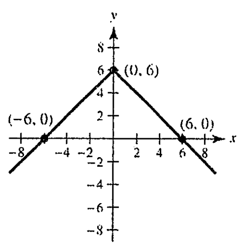
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

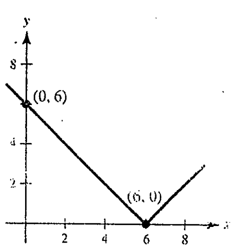
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: $(0, 6), (6, 0)$

Symmetry: none



53. $y^2 - x = 9$

$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

$y = \pm\sqrt{0 + 9} = \pm\sqrt{9} = \pm 3$, y-intercepts

$\pm\sqrt{x + 9} = 0$

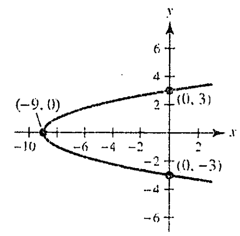
$x + 9 = 0$

$x = -9$, x-intercept

Intercepts: $(0, 3), (0, -3), (-9, 0)$

$(-y)^2 - x = 9 \Rightarrow y^2 - x = 9$

Symmetry: x-axis



54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

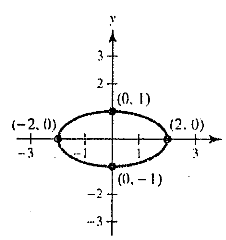
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



55. $x + 3y^2 = 6$

$3y^2 = 6 - x$

$y = \pm\sqrt{\dfrac{6-x}{3}}$

$y = \pm\sqrt{\dfrac{6-0}{3}} = \pm\sqrt{2}$, y-intercepts

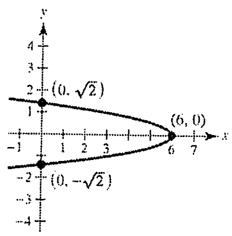
$x + 3(0)^2 = 6$

$x = 6$, x-intercept

Intercepts: $(6, 0)$, $(0, \sqrt{2})$, $(0, -\sqrt{2})$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\frac{3}{4}x - 2}$

$y = \pm\sqrt{\dfrac{3}{4}(0) - 2} = \pm\sqrt{-2}$

$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$3x - 4(0)^2 = 8$

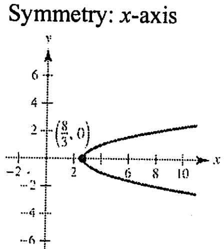
$3x = 8$

$x = \dfrac{8}{3}$, x-intercept

Intercept: $\left(\frac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$\dfrac{3x + 4}{2} = \dfrac{-4x - 10}{2}$

$3x + 4 = -4x - 10$

$7x = -14$

$x = -2$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$6 - x^2 = 4 - x$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2)$, $(-1, 5)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2)$, $(2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

$25 - x^2 = (3x + 15)^2$

$25 - x^2 = 9x^2 + 90x + 225$

$0 = 10x^2 + 90x + 200$

$0 = x^2 + 9x + 20$

$0 = (x + 5)(x + 4)$

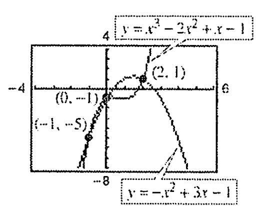
$x = -4$ or $x = -5$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection: $(-1, -5), (0, -1), (2, 1)$

Analytically, $x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

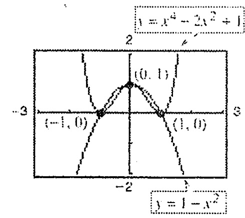
$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

$x = -1, 0, 2.$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



Points of intersection: $(-1, 0), (0, 1), (1, 0)$

Analytically, $1 - x^2 = x^4 - 2x^2 + 1$

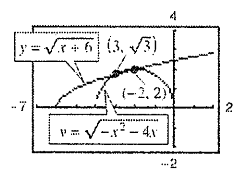
$0 = x^4 - x^2$

$0 = x^2(x + 1)(x - 1)$

$x = -1, 0, 1.$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically, $\sqrt{x + 6} = \sqrt{-x^2 - 4x}$

$x + 6 = -x^2 - 4x$

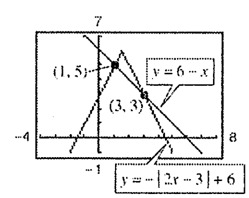
$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x = -3, -2.$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: $(3, 3), (1, 5)$

Analytically, $-|2x - 3| + 6 = 6 - x$

$|2x - 3| = x$

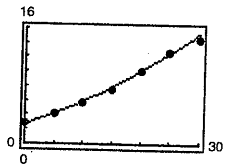
$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $\quad x = 1.$

67. (a) Using a graphing utility, you obtain

$y = 0.005t^2 + 0.27t + 2.7.$

(b)



(c) For 2020, $t = 40$.

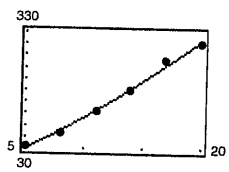
$$y = 0.005(40)^2 + 0.27(40) + 2.7$$
$$= 21.5$$

The GDP in 2020 will be \$21.5 trillion.

68. (a) Using a graphing utility, you obtain

$y = 0.24t^2 + 12.6t - 40.$

(b)



The model is a good fit for the data.

(c) For 2020, $t = 30$.

$$y = 0.24(30)^2 + 12.6(30) - 40$$
$$= 554$$

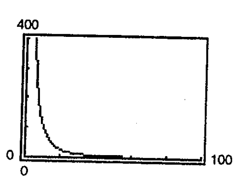
The number of cellular phone subscribers in 2020 will be 554 million.

69.

$$C = R$$
$$2.04x + 5600 = 3.29x$$
$$5600 = 3.29x - 2.04x$$
$$5600 = 1.25x$$
$$x = \frac{5600}{1.25} = 4480$$

To break even, 4480 units must be sold.

70. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

71. $y = kx^3$

(a) $(1, 4)$: $4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $-1 = k(-1)^3 = -k \Rightarrow k = 1$

72. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$
$1 = 4k$
$k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$
$16 = 8k$
$k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$
k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$
$9 = 12k$
$k = \frac{9}{12} = \frac{3}{4}$

73. Answers may vary. *Sample answer*:

$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

74. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

75. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.