76. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0) (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

77. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

78. True. $f(4) = f(-4)$.

79. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

80. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

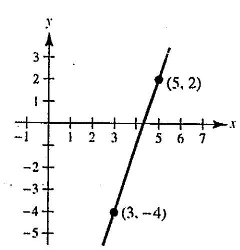
1. $m = 2$

2. $m = 0$

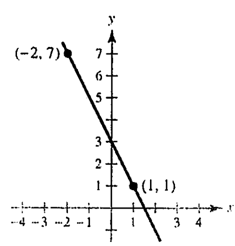
3. $m = -1$

4. $m = -12$

5. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

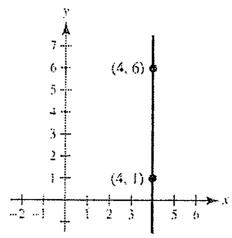


6. $m = \dfrac{7 - 1}{-2 - 1} = \dfrac{6}{-3} = -2$



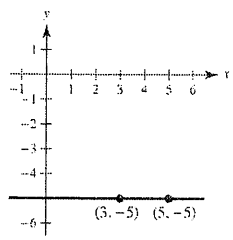
7. $m = \dfrac{1-6}{4-4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

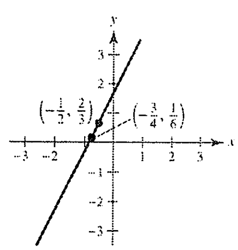


8. $m = \dfrac{-5-(-5)}{5-3} = \dfrac{0}{2} = 0$

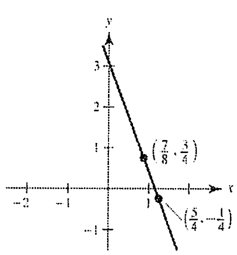
The line is horizontal.



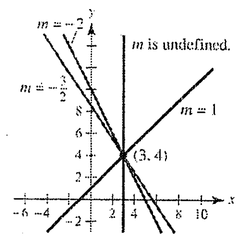
9. $m = \dfrac{\dfrac{2}{3} - \dfrac{1}{6}}{-\dfrac{1}{2} - \left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



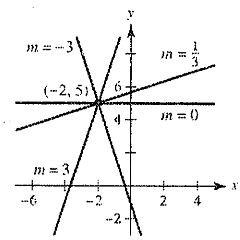
10. $m = \dfrac{\left(\dfrac{3}{4}\right) - \left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right) - \left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

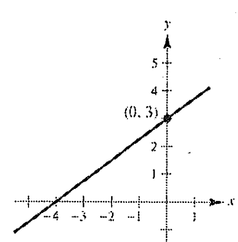
16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

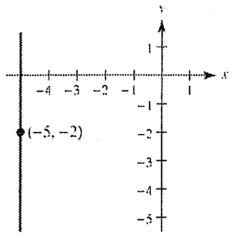
17. $y = \frac{3}{4}x + 3$

$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



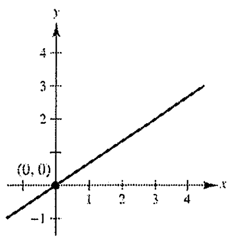
18. The slope is undefined so the line is vertical.

$$x = -5$$
$$x + 5 = 0$$



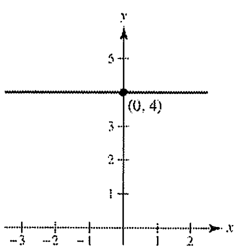
19. $y = \frac{2}{3}x$

$$3y = 2x$$
$$0 = 2x - 3y$$



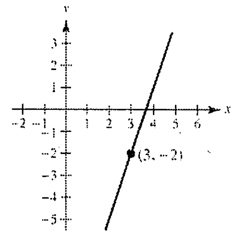
20. $y = 4$

$$y - 4 = 0$$



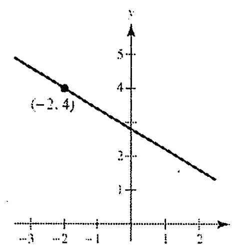
21. $y + 2 = 3(x - 3)$

$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$



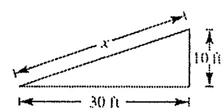
22. $y - 4 = -\frac{3}{5}(x + 2)$

$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

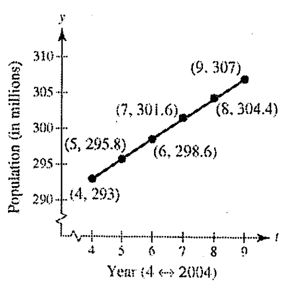
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

24. (a)



(b) The slopes are:

$$\frac{295.8 - 293.0}{5 - 4} = 2.8$$
$$\frac{298.6 - 295.8}{6 - 5} = 2.8$$
$$\frac{301.6 - 298.6}{7 - 6} = 3.0$$
$$\frac{304.4 - 301.6}{8 - 7} = 2.8$$
$$\frac{307.0 - 304.4}{9 - 8} = 2.6$$

The population increased least rapidly from 2008 to 2009.

(c) Average rate of change from 2004 to 2009:

$$\frac{307.0 - 293.0}{9 - 4} = \frac{14}{5}$$
$$= 2.8 \text{ million per yr}$$

(d) For 2020, $t = 20$ and $y \approx 16(2.8) + 293.0 = 337.8$ million.

[Equivalently, $y \approx 11(2.8) + 307.0 = 337.8$.]

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is $(0, 1)$.

27. $x + 5y = 20$

$y = -\frac{1}{5}x + 4$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

28. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

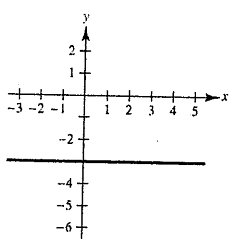
29. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

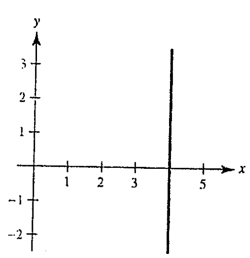
30. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

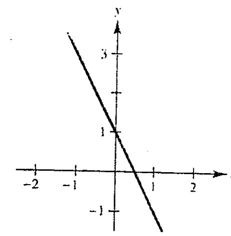
31. $y = -3$



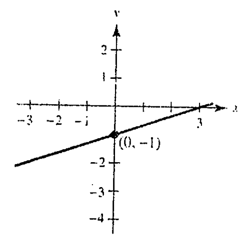
32. $x = 4$



33. $y = -2x + 1$

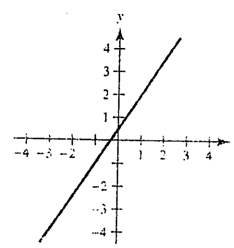


34. $y = \frac{1}{3}x - 1$



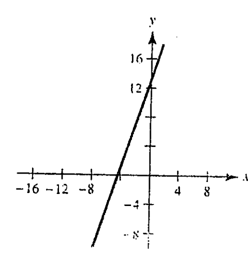
35. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



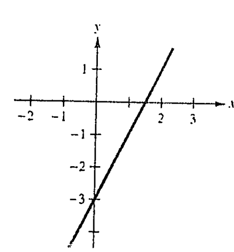
36. $y - 1 = 3(x + 4)$

$y = 3x + 13$



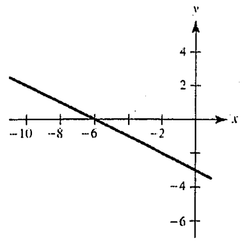
37. $2x - y - 3 = 0$

$y = 2x - 3$



38. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

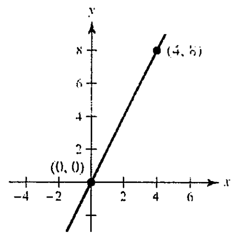


39. $m = \dfrac{8 - 0}{4 - 0} = 2$

$y - 0 = 2(x - 0)$

$y = 2x$

$0 = 2x - y$



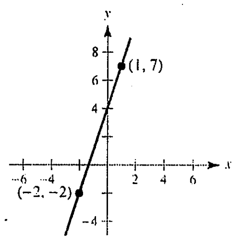
40. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y = 3x + 4$

$0 = 3x - y + 4$

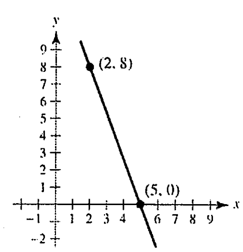


41. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$y - 0 = -\frac{8}{3}(x - 5)$

$y = -\frac{8}{3}x + \frac{40}{3}$

$8x + 3y - 40 = 0$

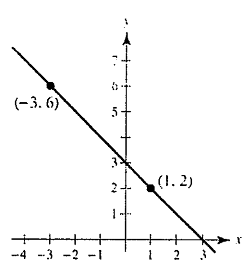


42. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

$x + y - 3 = 0$

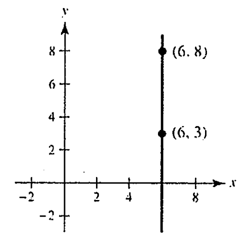


43. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

The line is horizontal.

$x = 6$

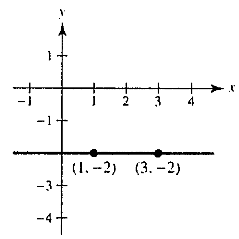
$x - 6 = 0$



44. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$y = -2$

$y + 2 = 0$

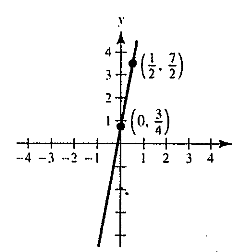


45. $m = \dfrac{\frac{7}{2} - \frac{3}{4}}{\frac{1}{2} - 0} = \dfrac{\frac{11}{4}}{\frac{1}{2}} = \dfrac{11}{2}$

$y - \frac{3}{4} = \frac{11}{2}(x - 0)$

$y = \frac{11}{2}x + \frac{3}{4}$

$0 = 22x - 4y + 3$

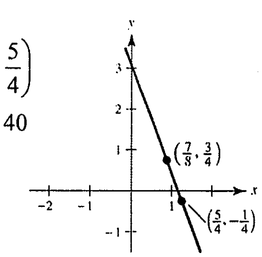


46. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$

$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$

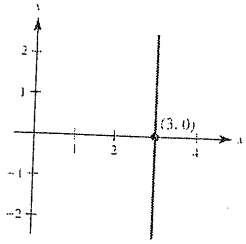
$12y + 3 = -32x + 40$

$32x + 12y - 37 = 0$



47. $x = 3$

$x - 3 = 0$

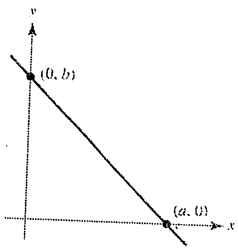


48. $m = -\dfrac{b}{a}$

$y = \dfrac{-b}{a}x + b$

$\dfrac{b}{a}x + y = b$

$\dfrac{x}{a} + \dfrac{y}{b} = 1$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

50. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$\dfrac{-3x}{2} - \dfrac{y}{2} = 1$

$3x + y = -2$

$3x + y + 2 = 0$

51. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{1}{a} + \dfrac{2}{a} = 1$

$\dfrac{3}{a} = 1$

$a = 3 \Rightarrow x + y = 3$

$x + y - 3 = 0$

52. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{-3}{a} + \dfrac{4}{a} = 1$

$\dfrac{1}{a} = 1$

$a = 1 \Rightarrow x + y = 1$

$x + y - 1 = 0$

53. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$\dfrac{9}{2a} + \dfrac{-2}{a} = 1$

$\dfrac{9 - 4}{2a} = 1$

$5 = 2a$

$a = \dfrac{5}{2}$

$\dfrac{x}{2\left(\frac{5}{2}\right)} + \dfrac{y}{\left(\frac{5}{2}\right)} = 1$

$\dfrac{x}{5} + \dfrac{2y}{5} = 1$

$x + 2y = 5$

$x + 2y - 5 = 0$

54. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$\dfrac{\left(-\frac{2}{3}\right)}{a} + \dfrac{(-2)}{-a} = 1$

$-\dfrac{2}{3} + 2 = a$

$a = \dfrac{4}{3}$

$\dfrac{x}{\left(\frac{4}{3}\right)} + \dfrac{y}{\left(-\frac{4}{3}\right)} = 1$

$x - y = \dfrac{4}{3}$

$3x - 3y - 4 = 0$

55. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

57. $x - y = -2$

$y = x + 2$

$m = 1$

(a) $y - 5 = 1(x - 2)$

$y - 5 = x - 2$

$x - y + 3 = 0$

(b) $y - 5 = -1(x - 2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

58. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$0 = x - y + 5$

59. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

$m = 2$

(a) $y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$0 = 2x - y - 3$

(b) $y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

60. $7x + 4y = 8$

$4y = -7x + 8$

$y = \frac{-7}{4}x + 2$

$m = -\frac{7}{4}$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$

$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$

$24y + 12 = -42x + 35$

$42x + 24y - 23 = 0$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$

$42y + 21 = 24x - 20$

$24x - 42y - 41 = 0$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$24x + 40y - 53 = 0$

62. $3x + 4y = 7$

$4y = -3x + 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

(a) $y - (-5) = -\frac{3}{4}(x - 4)$

$y + 5 = -\frac{3}{4}x + 3$

$4y + 20 = -3x + 12$

$3x + 4y + 8 = 0$

(b) $y - (-5) = \frac{4}{3}(x - 4)$

$y + 5 = \frac{4}{3}x - \frac{16}{3}$

$3y + 15 = 4x - 16$

$0 = 4x - 3y - 31$

63. The slope is 250.

$V = 1850$ when $t = 2$.

$V = 250(t - 2) + 1850$

$= 250t + 1350$

64. The slope is 4.50.

$V = 156$ when $t = 2$.

$V = 4.5(t - 2) + 156$

$= 4.5t + 147$

65. The slope is −1600.

$V = 17{,}200$ when $t = 2$.

$V = -1600(t - 2) + 17{,}200$

$= -1600t + 20{,}400$

66. The slope is −5600.

$V = 245{,}000$ when $t = 2$.

$V = -5600(t - 2) + 245{,}000$

$= -5600t + 256{,}200$

67. $m_1 = \dfrac{1 - 0}{-2 - (-1)} = -1$

$m_2 = \dfrac{-2 - 0}{2 - (-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

68. $m_1 = \dfrac{-6 - 4}{7 - 0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11 - 4}{-5 - 0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

69. Equations of perpendicular bisectors:

$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right)$$

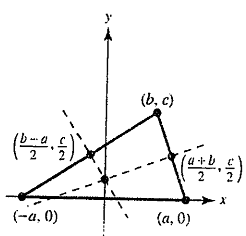
$$y - \frac{c}{2} = \frac{a + b}{-c}\left(x - \frac{b - a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.

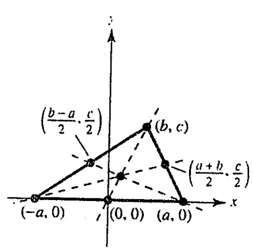


70. Equations of medians:

$$y = \frac{c}{b}x$$

$$y = \frac{c}{3a + b}(x + a)$$

$$y = \frac{c}{-3a + b}(x - a)$$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

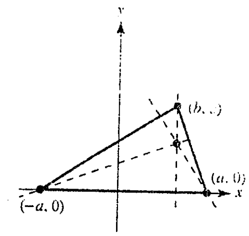
71. Equations of altitudes:

$$y = \frac{a - b}{c}(x + a)$$

$$x = b$$

$$y = -\frac{a + b}{c}(x - a)$$

Solving simultaneously, the point of intersection is $\left(b, \dfrac{a^2 - b^2}{c}\right)$.



72. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2 - b^2}{c}\right)$ is:

$$m_1 = \frac{\left[\left(a^2 - b^2\right)/c\right] - (c/3)}{b - (b/3)}$$

$$= \frac{\left(3a^2 - 3b^2 - c^2\right)/(3c)}{(2b)/3} = \frac{3a^2 - 3b^2 - c^2}{2bc}$$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$ is:

$$m_2 = \frac{\left[\left(-a^2 + b^2 + c^2\right)/(2c)\right] - (c/3)}{0 - (b/3)}$$

$$= \frac{\left(-3a^2 + 3b^2 + 3c^2 - 2c^2\right)/(6c)}{-b/3} = \frac{3a^2 - 3b^2 - c^2}{2bc}$$

$m_1 = m_2$

Therefore, the points are collinear.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$
$$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$
$$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$$

(e) $a = \frac{5}{2}$ and $b = 3$.

$$\frac{5}{2}x + 3y = 4$$
$$5x + 6y = 8$$

74. (a) Lines c, d, e and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.
Lines d and f appear parallel.

(d) Lines b and f appear perpendicular.
Lines b and d appear perpendicular.

75. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32$$

or

$$C = \frac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

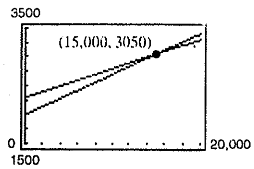
For $F = 72°$, $C \approx 22.2°$.

76. $C = 0.51x + 200$

For $x = 137$, $C = 0.51(137) + 200 = \$269.87$.

77. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b) 

Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050$.

When sales exceed \$15,000, the current job pays more.

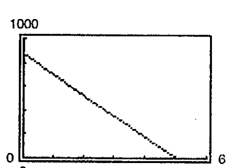
(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

78. (a) Depreciation per year:

$$\frac{875}{5} = \$175$$
$$y = 875 - 175x$$

where $0 \leq x \leq 5$.



(b) $y = 875 - 175(2) = \$525$

(c) $200 = 875 - 175x$

$$175x = 675$$
$$x \approx 3.86 \text{ years}$$

79. (a) Two points are (50, 780) and (47, 825).
The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

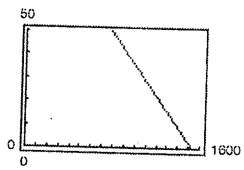
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



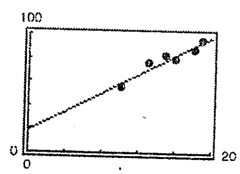
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

80. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score},\ y = \text{test score})$

(b)

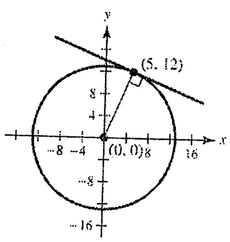


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

81. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

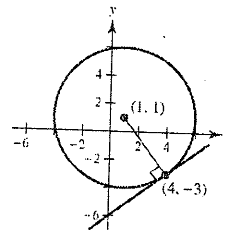
The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$5x + 12y - 169 = 0.$$

82. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$$\frac{1 + 3}{1 - 4} = \frac{-4}{3}.$$

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - 6$$

$$0 = 3x - 4y - 24$$

83. $x - y - 2 = 0 \Rightarrow d = \frac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}}$

$$= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

84. $4x + 3y - 10 = 0 \Rightarrow d = \frac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \frac{7}{5}$

85. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

86. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

87. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow \quad A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$
$$\left(A^2 + B^2\right)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow \quad ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$
$$\left(A^2 + B^2\right)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

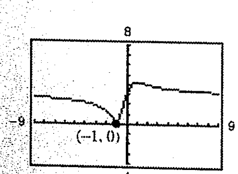
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$
$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$
$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{\left(A^2 + B^2\right)\left(C + Ax_1 + By_1\right)^2}{\left(A^2 + B^2\right)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

88. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).

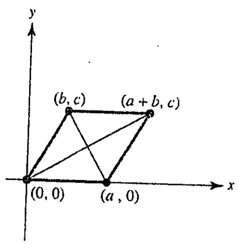


89. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a+b}$ and $m_2 = \dfrac{c}{b-a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a+b} \cdot \frac{c}{b-a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



90. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

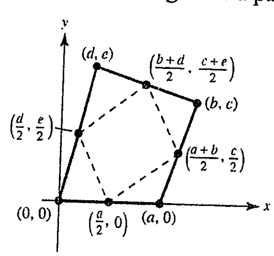
$$\left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{c}{2}\right), \left(\frac{b+d}{2}, \frac{c+e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a+b}{2} - \frac{a}{2}} = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

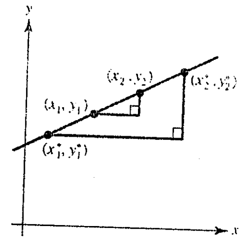
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c+e}{2}}{\frac{a+b}{2} - \frac{b+d}{2}} = -\frac{e}{a-d}$$

Therefore, the figure is a parallelogram.



91. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



92. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

93. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

94. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

95. True. The slope must be positive.

96. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section P.3 Functions and Their Graphs

1. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

2. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$