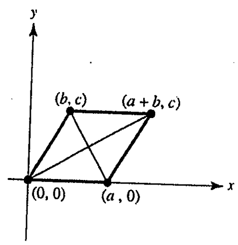
89. For simplicity, let the vertices of the rhombus be $(0, 0)$, $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \frac{c}{a+b}$ and $m_2 = \frac{c}{b-a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a+b} \cdot \frac{c}{b-a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



90. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

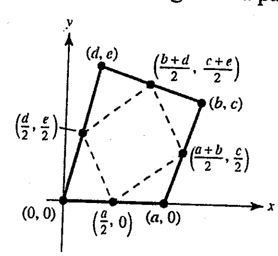
$$\left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{c}{2}\right), \left(\frac{b+d}{2}, \frac{c+e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a+b}{2} - \frac{a}{2}} = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

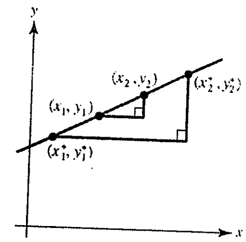
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c+e}{2}}{\frac{a+b}{2} - \frac{b+d}{2}} = -\frac{e}{a-d}$$

Therefore, the figure is a parallelogram.



91. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



92. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

93. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

94. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

95. True. The slope must be positive.

96. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section P.3 Functions and Their Graphs

1. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

2. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

3. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

4. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g(\frac{3}{2}) = (\frac{3}{2})^2(\frac{3}{2} - 4) = \frac{9}{4}(-\frac{5}{2}) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

5. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

(d) $f(\pi) = \cos(2(\pi)) = 1$

6. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(d) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

7. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

8. $$\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3,\ x \neq 1$$

9. $$\frac{f(x) - f(2)}{x - 2} = \frac{(1/\sqrt{x - 1} - 1)}{x - 2}$$
$$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}(1 + \sqrt{x - 1})} = \frac{-1}{\sqrt{x - 1}(1 + \sqrt{x - 1})},\ x \neq 2$$

10. $$\frac{f(x) - f(1)}{x - 1} = \frac{x^3 - x - 0}{x - 1} = \frac{x(x + 1)(x - 1)}{x - 1} = x(x + 1),\ x \neq 1$$

11. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

12. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

13. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

17. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

18. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

19. $f(t) = \sec\dfrac{\pi t}{4}$

$\dfrac{\pi t}{4} \neq \dfrac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n an integer

Range: $(-\infty, -1] \cup [1, \infty)$

20. $h(t) = \cot t$

Domain: all $t = n\pi$, n an integer

Range: $(-\infty, \infty)$

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Domain: all $x \neq -4$

Range: all $y \neq 1$

[**Note:** You can see that the range is all $y \neq 1$ by graphing f.]

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x - 2)(x - 1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

26. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \neq 0$

$\sin x \neq \dfrac{1}{2}$

Domain: all $x \neq \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi$, n integer

27. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

28. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x - 2)(x + 2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

30. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

31. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \ge 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

32. $f(x) = \begin{cases} \sqrt{x + 4}, & x \le 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

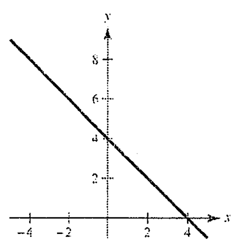
Domain: $[-4, \infty)$

Range: $[0, \infty)$

33. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

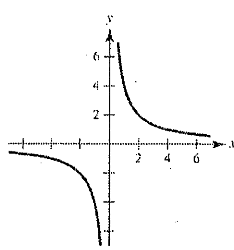
Range: $(-\infty, \infty)$



34. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



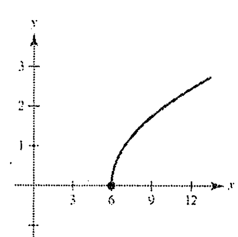
35. $h(x) = \sqrt{x - 6}$

Domain:

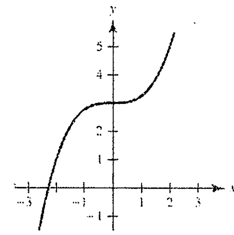
$x - 6 \ge 0$

$x \ge 6 \Rightarrow [6, \infty)$

Range: $[0, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$



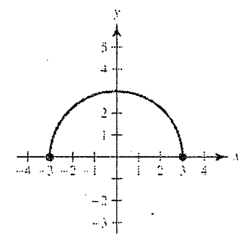
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



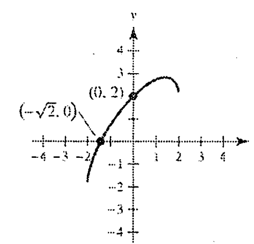
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

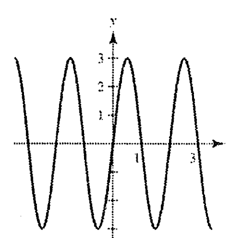
Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $\left(-\sqrt{2}, 0\right)$



39. $g(t) = 3 \sin \pi t$



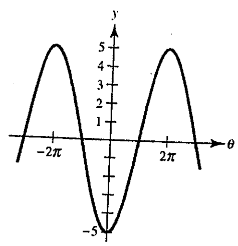
Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

40. $h(\theta) = -5\cos\frac{\theta}{2}$

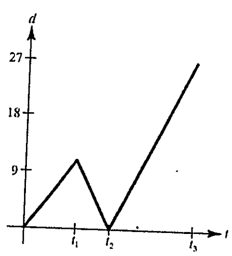
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



41. The student travels $\frac{2-0}{4-0} = \frac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the next 2 minutes. Finally, the student travels $\frac{6-2}{10-6} = 1$ mi/min during the final 4 minutes.

42.



43. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

44. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

45. y is a function of x. Vertical lines intersect the graph at most once.

46. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

47. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

48. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

49. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

50. $x^2y - x^2 + 4y = 0 \Rightarrow y = \frac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

51. The transformation is a horizontal shift two units to the right.

Shifted function: $y = \sqrt{x - 2}$

52. The transformation is a vertical shift 4 units upward.

Shifted function: $y = \sin x + 4$

53. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward.

Shifted function: $y = (x - 2)^2 - 1$

54. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward.

Shifted function: $y = (x + 1)^3 + 2$

55. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

56. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

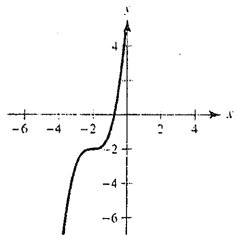
57. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

58. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

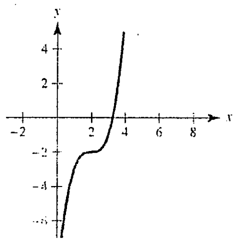
59. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

60. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

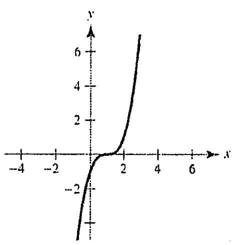
61. (a) The graph is shifted 3 units to the left.



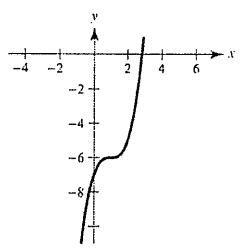
(b) The graph is shifted 1 unit to the right.



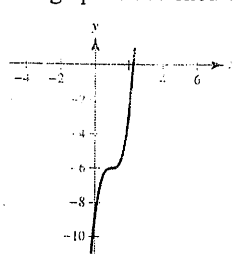
(c) The graph is shifted 2 units upward.



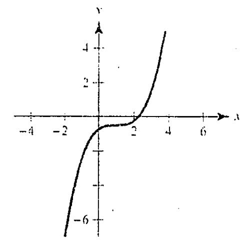
(d) The graph is shifted 4 units downward.



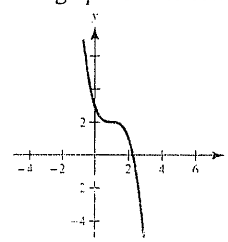
(e) The graph is stretched vertically by a factor of 3.



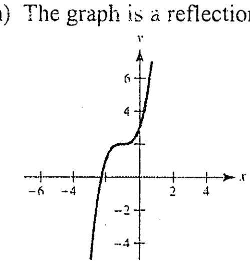
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the x-axis.



(h) The graph is a reflection about the origin.

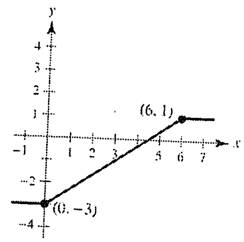


62. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

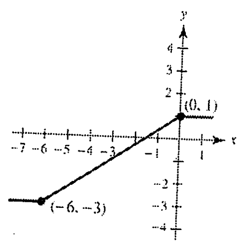


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

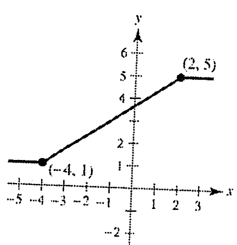


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

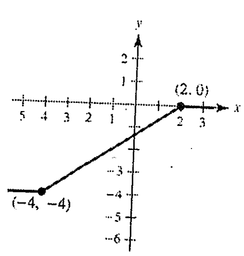


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

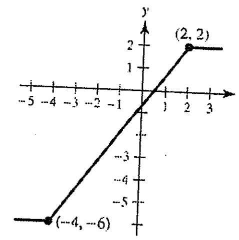


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

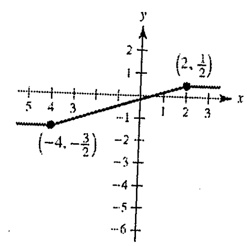


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

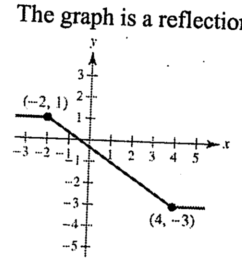


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the y-axis.

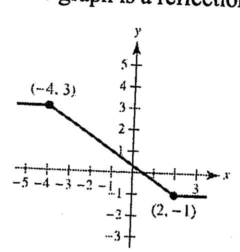


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the x-axis.



63. $f(x) = 3x - 4, \quad g(x) = 4$

(a) $f(x) + g(x) = (3x - 4) + 4 = 3x$

(b) $f(x) - g(x) = (3x - 4) - 4 = 3x - 8$

(c) $f(x) \cdot g(x) = (3x - 4)(4) = 12x - 16$

(d) $f(x)/g(x) = \dfrac{3x - 4}{4} = \dfrac{3}{4}x - 1$

64. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1)$

$= x^3 + 5x^2 + 4x + x^2 + 5x + 4$

$= x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

65. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

66. $f(x) = \sin x, \ g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\dfrac{1}{2}\right)\right) = f\left(\dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right) = g\left(\sin\left(\dfrac{\pi}{4}\right)\right)$

$= g\left(\dfrac{\sqrt{2}}{2}\right) = \pi\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

67. $f(x) = x^2, \ g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x, \ x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

68. $f(x) = x^2 - 1, \ g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

69. $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \frac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

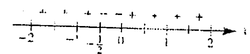
No, $f \circ g \neq g \circ f$.

70. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \frac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x} + 2} = \sqrt{\frac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

71. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

72. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

73. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

74. $F(x) = -4\sin(1 - x)$

Let $f(x) = -4x$, $g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4\sin(1 - x) = F(x)$.

[Other answers possible]

75. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

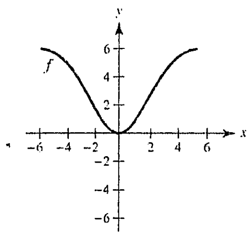
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

76. (a) If f is even, then $(-4, 9)$ is on the graph.

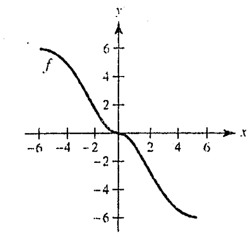
(b) If f is odd, then $(-4, -9)$ is on the graph.

77. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

78. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



79. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

80. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

81. $f(x) = x\cos x$

$f(-x) = (-x)\cos(-x) = -x\cos x = -f(x)$

f is odd.

$f(x) = x\cos x = 0$

Zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

82. $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

f is even.

$\sin^2 x = 0 \Rightarrow \sin x = 0$

Zeros: $x = n\pi$, where n is an integer

83. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

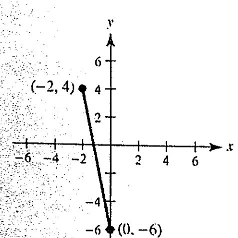
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, \ -2 \le x \le 0$



84. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

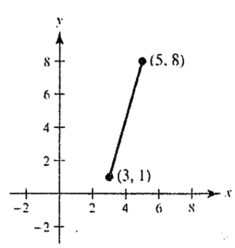
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, \ 3 \le x \le 5$

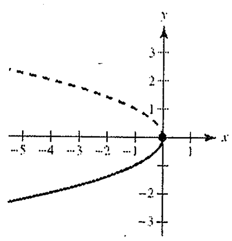


85. $x + y^2 = 0$

$y^2 = -x$

$y = -\sqrt{-x}$

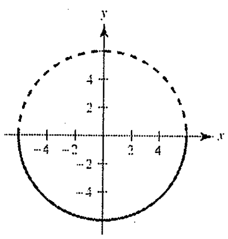
$f(x) = -\sqrt{-x},\ x \le 0$



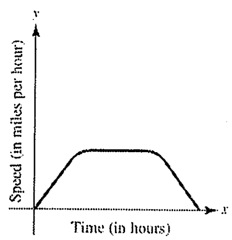
86. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

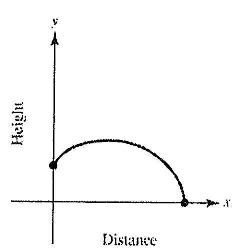
$y = -\sqrt{36 - x^2},\ -6 \le x \le 6$



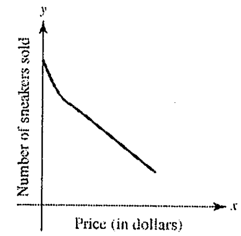
87. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:



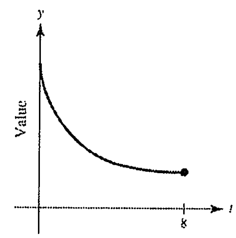
88. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



89. Answers will vary. *Sample answer*: In general, as the price decreases, the store will sell more.



90. Answers will vary. *Sample answer*: As time goes on, the value of the car will decrease



91. $y = \sqrt{c - x^2}$

$y^2 = c - x^2$

$x^2 + y^2 = c$, a circle.

For the domain to be $[-5, 5]$, $c = 25$.

92. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$

$$9c^2 < 24$$

$$c^2 < \tfrac{8}{3}$$

$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$

$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

93. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

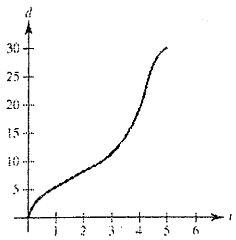
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

94. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

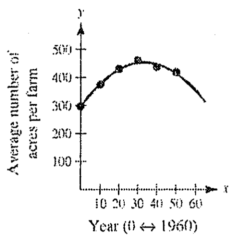
Range: $0 \le d \le 30$

(c)



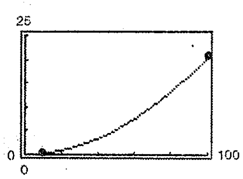
(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

95. (a)



(b) $A(25) \approx 445$ (Answers will vary.)

96. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.002\left(\dfrac{x}{1.6}\right)^2 + 0.005\left(\dfrac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

97. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x \le 0 \\ 2, & 0 < x < 2. \\ 2x - 2, & x \ge 2 \end{cases}$$

98. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

99. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right]$

$= -f(x)$

Odd

100. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

101. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$. So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then $F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$. So, $F(x)$ is even.

102. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then $F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$. So, $F(x)$ is odd.

103. By equating slopes, $\frac{y-2}{0-3} = \frac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

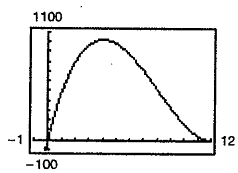
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

104. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

105. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

106. True

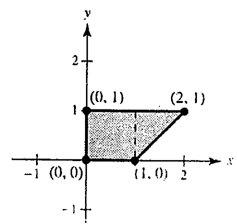
107. True. The function is even.

108. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

109. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

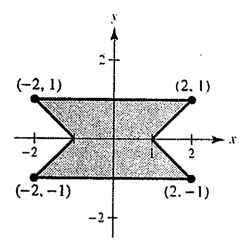
110. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

111. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

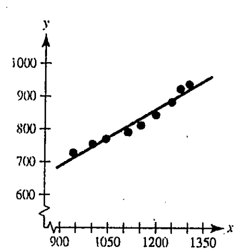
112. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Fitting Models to Data

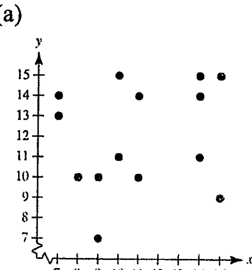
1. (a) and (b)



Yes, the data appear to be approximately linear. The data can be modeled by equation $y = 0.6x + 150$. (Answers will vary).

(c) When $x = 1075$, $y = 0.6(1075) + 150 = 795$.

2. (a)



The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.