Particle Motion / PVA Practice WS

A particle moves along a horizontal line so that its position at any time is given by $s(t) = t^3 - 12t^2 + 36t,\ t \geq 0,$ where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at time t and at $t = 3$ seconds.

(b) Find the average velocity in interval [1, 2]

(c) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.

(d) When is the particle changing direction?

(e) Find the displacement of the particle after the first 8 seconds.

(f) Find the total distance traveled by the particle during the first 8 seconds.

(g) Find the acceleration of the particle at time t and at $t = 3$ seconds.

(h) Is the particle speeding up or slowing down at t = 1? Justify answer.

(i) Is the particle's velocity increasing or decreasing at t = 2? Justify answer.

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$v(t) = 3t^2 - 24t + 36$

$a(t) = 6t - 24$

Key

1 A particle moves along a horizontal line so that its position at any time is given by $s(t) = t^3 - 12t^2 + 36t$, $t \geq 0$, where s is measured in meters and t in seconds.

$v(t) = 3t^2 - 24t + 36$

(a) Find the instantaneous velocity at time t and at $t = 3$ seconds.

$v(3) = 3(3)^2 - 24(3) + 36 = \boxed{-9 \text{ m/s}}$

(b) Find the average velocity in interval [1, 2] $\rightarrow \frac{\text{change in position}}{\text{change in time}} = \frac{s(2) - s(1)}{2 - 1} = \frac{32 - 25}{2 - 1}$

$s(1) = 1 - 12 + 36 = 25$

$s(2) = 8 - 48 + 72 = 32$

$= \boxed{7 \text{ m/s}}$

(c) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.

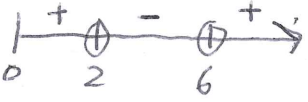
*set $v(t) = 0$

$0 = 3t^2 - 24t + 36$

$0 = 3(t^2 - 8t + 12)$

$0 = 3(t - 6)(t - 2)$

$t = 2, 6$



moving right: $[0, 2) \cup (6, \infty)$ because $v(t) > 0$

moves left: $(2, 6)$ because $v(t) < 0$

(d) When is the particle changing direction?

at $t = 2, 6$ seconds.

(e) Find the displacement of the particle after the first 8 seconds.

$s(0) = 0$

$s(8) = 32$

$\boxed{32 \text{ m}}$

$s(8) - s(0) = 32 - 0 = 32$

$s(0) = 0$

$s(8) = 32$

$= \boxed{32 \text{ m}}$

(f) Find the total distance traveled by the particle during the first 8 seconds.

$s(0) = 0 \rightarrow 32$ m

$s(2) = 32 \rightarrow 32$ m

$s(6) = 0$

$s(8) = 32 \rightarrow 32$ m

$32 + 32 + 32 = \boxed{96 \text{ m}}$

(g) Find the acceleration of the particle at time t and at $t = 3$ seconds.

$a(t) = 6t - 24$ $\quad a(3) = 6(3) - 24 = \boxed{-6 \text{ m/s}^2}$

(h) Is the particle speeding up or slowing down at t = 1? Justify answer.

$v(1) = 15$ m/s particle is slowing down at $t = 1$ since $v(t)$ and $a(t)$ have opposite signs

$a(1) = -18 \text{ m/s}^2$

(i) Is the particle's velocity increasing or decreasing at t = 2? Justify answer.

$a(2) = -12 \text{ m/s}^2$ Since $a(2) < 0$, velocity is decreasing at $t = 2$.

* In other words, Is particle's $a(t) > 0$ or $a(t) < 0$?