

PreCalc Trig Review: Solving Trig Equations

In Exercises 31–38, solve the equation for  $\theta$  ( $0 \leq \theta < 2\pi$ ).

31.  $2 \sin^2 \theta = 1$

32.  $\tan^2 \theta = 3$

33.  $\tan^2 \theta - \tan \theta = 0$

34.  $2 \cos^2 \theta - \cos \theta = 1$

35.  $\sec \theta \csc \theta = 2 \csc \theta$

36.  $\sin \theta = \cos \theta$

37.  $\cos^2 \theta + \sin \theta = 1$

38.  $\cos \frac{\theta}{2} - \cos \theta = 1$

In Exercises 31-38, solve the equation for  $\theta$  ( $0 \leq \theta < 2\pi$ ).

31.  $2 \sin^2 \theta = 1$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sqrt{\sin^2 \theta} = \sqrt{\frac{1}{2}}$$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\sin \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

32.  $\tan^2 \theta = 3$

$$\sqrt{\tan^2 \theta} = \pm \sqrt{3}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\tan \frac{\pi}{3} = \frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

33.  $\tan^2 \theta - \tan \theta = 0$

$$\tan \theta (\tan \theta - 1) = 0$$

$$\tan \theta = 0 \quad \tan \theta - 1 = 0$$

$$\theta = 0, \pi \quad \tan \theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

34.  $2 \cos^2 \theta - \cos \theta = 1$

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$2 \cos \theta + 1 = 0 \quad \cos \theta - 1 = 0$$

$$\cos \theta = -\frac{1}{2} \quad \cos \theta = 1$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \theta = 0$$

35.  $\sec \theta \csc \theta = 2 \csc \theta$

$$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{2}{\sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta} = \frac{2}{\sin \theta}$$

$$2 \sin \theta \cos \theta = \sin \theta$$

$$2 \sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (2 \cos \theta - 1) = 0$$

$$\sin \theta (2 \cos \theta - 1) = 0$$

$$\sin \theta = 0 \quad 2 \cos \theta - 1 = 0$$

$$\theta = 0 \quad \cos \theta = \frac{1}{2}$$

Extraneous solution

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

36.  $\sin \theta = \cos \theta$

$$\frac{\sin \theta}{\cos \theta} = \frac{\cos \theta}{\cos \theta}$$

$$\tan \theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

37.  $\cos^2 \theta + \sin \theta = 1$

\* Express in terms of 1 trig function

$$* \cos^2 \theta = 1 - \sin^2 \theta$$

$$(1 - \sin^2 \theta) + \sin \theta = 1$$

$$1 - \sin^2 \theta + \sin \theta - 1 = 0$$

$$\sin \theta - \sin^2 \theta = 0$$

$$\sin \theta (1 - \sin \theta) = 0$$

$$\sin \theta = 0 \quad 1 - \sin \theta = 0$$

$$\theta = 0, \pi \quad \sin \theta = 1$$

$$\theta = \frac{\pi}{2}$$

38.  $\cos \frac{\theta}{2} - \cos \theta = 1$

\* Let  $x = \frac{\theta}{2}$

$$\cos x - \cos(2x) = 1$$

$$\cos x - (2 \cos^2 x - 1) = 1$$

$$\cos x - 2 \cos^2 x + 1 - 1 = 0$$

$$\cos x - 2 \cos^2 x = 0$$

$$\cos x (1 - 2 \cos x) = 0$$

$$\cos x = 0 \quad 1 - 2 \cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{10\pi}{3}$$

$$\theta = \pi, \frac{2\pi}{3}$$