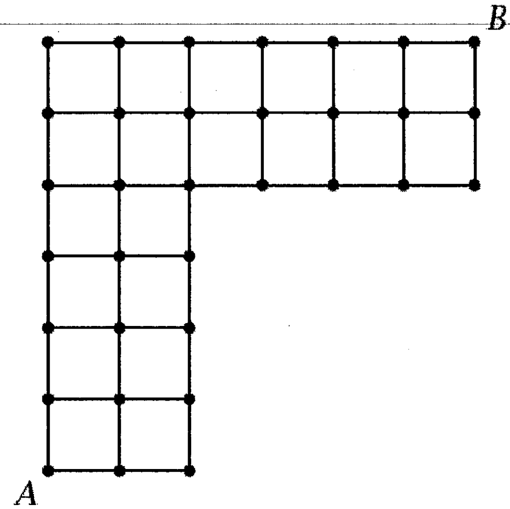
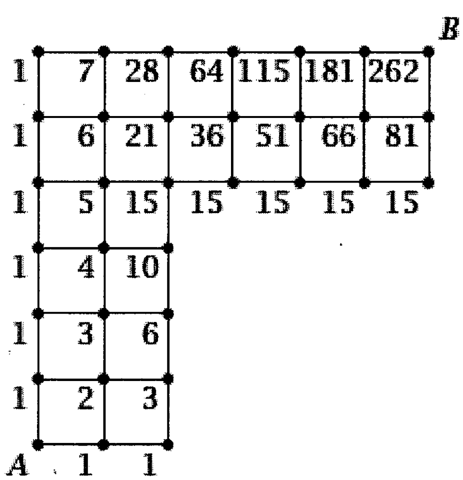
Key

6. The street map below shows the only routes in a rectangular coordinate system. Using this map, how many different shortest paths, along streets only, are there from A to B?



(A) 100 **(B)** 262 **(C)** 297 **(D)** 924 **(E)** 927

6. **B** Let's simplify the problem. Suppose B was adjacent to A. Then there would be only one shortest path. Note that this is true whether B is one unit north or one unit east of A. If B was one unit north *and* one unit east, then there would be two ways to get to B: north then east, or east then north. But this is the sum of the ways to get to the adjacent points. So we continue in this manner, summing the number of ways to get to points which are south and west of the present point. Overlaid on the grid are all the summations in the figure below.



Hence there are 262 shortest paths.

Heron's formula states that the area of a triangle whose sides have lengths a, b, and c is

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter of the triangle; that is,

$$s = \frac{a+b+c}{2}.$$

10. Compute the area of a triangle whose side lengths are 9, 11, and 14.

(A) $11\sqrt{17}$ (B) $12\sqrt{17}$ (C) $11\sqrt{34}$ (D) 77 (E) 99

$s = \frac{9+11+14}{2} = \frac{34}{2} = 17$

$A = \sqrt{17(17-9)(17-11)(17-14)}$

$= \sqrt{17 \cdot 8 \cdot 6 \cdot 3} = \boxed{12\sqrt{17}}$

10. **B** We use Heron's formula. The semiperimeter is $s = \frac{1}{2}(9+11+14) = 17$. Then the area is

$$\sqrt{17(17-9)(17-11)(17-14)} = \sqrt{17 \cdot 8 \cdot 6 \cdot 3}$$
$$= \sqrt{144 \cdot 17} = 12\sqrt{17}.$$

12. Let N be the base-3 number 221221. When N is written in base-9, what is its left-most digit?

(A) 1 (B) 2 (C) 6 (D) 7 (E) 8

12. **E** Since $9 = 3^2$, we can simply group the base-3 digits in pairs and convert each pair: 22 12 21. However, we are only interested in the first pair, since this gives us the leading digit in base-9. Hence, $2 \cdot 3 + 2 = 8$ is the leading digit.

22 12 21

$2 \times 3^0 = 2$
$2 \times 3^1 = 6$
$= \boxed{8}$

* Convert from $base_6$ to base 10

$258_6 = 8 \times 6^0$
$+ 5 \times 6^1$
$+ 2 \times 6^2$

$258_5 \rightarrow 8 \times 5^0$
5×5^1
$+ 2 \times 5^2$

* Convert base 10 to other bases: Divide by base 6 and keep the remainder

140 → to base 8

$8\overline{)140}$ R4
$8\overline{)17}$ R1
$8\overline{)2}$ R2
0

8
60

$\boxed{214_8}$

25. Evaluate $\dfrac{1}{20+\dfrac{1}{15+\dfrac{1}{20+\dfrac{1}{15+\cdots}}}}$ to the nearest thousandth.

(A) 0.047 (B) 0.048 (C) 0.049 $\boxed{\text{(D) } 0.050}$ (E) 0.051

25. **D** Call the expression x. Then

$$\frac{1}{20+\frac{1}{15+x}} = x$$

$$1 = 20x + \frac{x}{15+x}$$

$$15 + x = 20x(x+15) + x$$

$$15 = 20x^2 + 300x$$

$$4x^2 + 60x - 3 = 0.$$

By the quadratic formula, the positive root is $x = -\frac{15}{2} + \sqrt{57} \approx 0.0498344353$, which rounds to 0.050.

Alternately, you could just use your calculator to carefully evaluate

$$\frac{1}{20+\frac{1}{15+\frac{1}{20}}} \approx 0.0498344371,$$

which also rounds to 0.050.

12. Given $\log_2(\log_2(\log_3 x)) = \log_2(\log_3(\log_2 y)) = 0$, compute the value of $x - y$.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 17

Answer: C The given equation implies $\log_2(\log_3 x) = \log_3(\log_2 y) = 1$, so that $\log_3 x = 2$ and $\log_2 y = 3$. Therefore $x = 3^2 = 9$ and $y = 2^3 = 8$. Hence, $x - y = 9 - 8 = 1$.

$\log_2(\log_2(\log_3 x)) = 0$

$2^0 = \log_2(\log_3 x)$

$1 = \log_2(\log_3 x)$

$2^1 = \log_3 x$

$3^2 = x = \underline{9}$

$\log_2(\log_3(\log_2 y)) = 0$

$2^0 = \log_3(\log_2 y)$

$1 = \log_3(\log_2 y)$

$3^1 = \log_2 y$

$2^3 = y = \underline{8}$

$x - y = 9 - 8 = \boxed{1}$